[microreview]

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A semigroup with a right zero equal to a right identity is trivial

Open Mathematics Collaboration^{*†}

June 27, 2020

Abstract

We prove the proposition addressed in the title of this paper.

keywords: semigroup, right zero, right identity, trivial

The most updated version of this paper is available at

<https://osf.io/zpwb5/download>

Notation & Definition

1. $[1, 2]$
2. $\mathcal{S}$ = semigroup
3. $(\forall x \in \mathcal{S} : xz = z) \rightarrow (z = \text{right zero})$
4. $(\forall x \in \mathcal{S} : xe = x) \rightarrow (e = \text{right identity})$
5. $(\exists! x \in \mathcal{S}) \leftrightarrow (\mathcal{S} = \text{trivial})$

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Proposition

6. If $\mathcal{S}$ is a semigroup and $e \in \mathcal{S}$ is both a right zero and a right identity, then $\mathcal{S}$ is trivial [1].

Proof

7. Since e is a *right zero*, then $\forall x \in \mathcal{S} : xe = e$
8. Since e is a *right identity*, then $\forall x \in \mathcal{S} : xe = x$
9. From (7) and (8), $xe = xe$, then $x = e$ for all $x \in \mathcal{S}$.
10. Therefore, $\exists! x \in \mathcal{S}$. □

Final Remarks

11. $((\mathcal{S} = \text{semigroup}) \wedge (e \in \mathcal{S}) \wedge (\forall x \in \mathcal{S} : xe = e = x)) \rightarrow (\exists! x \in \mathcal{S})$

Open Invitation

Review, add content, and co-author this paper [3, 4]. *Join* the **Open Mathematics Collaboration** (<https://bit.ly/ojmp-slack>). Send your contribution to mplobo@uft.edu.br.

Open Science

The **latex file** for this paper together with other *supplementary files* are available [5].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

- [1] Cain, Alan J. *Nine Chapters on the Semigroup Art.* AJC Porto & Lisbon, 2020. <http://www-groups.mcs.st-andrews.ac.uk/%7ealanc/teaching/m431/>
- [2] Lobo, Matheus P. “Open Mathematics Knowledge Base.” *OSF Preprints*, 13 May 2020. <https://doi.org/10.31219/osf.io/evq8a>
- [3] Lobo, Matheus P. “Microarticles.” *OSF Preprints*, 28 Oct. 2019. <https://doi.org/10.31219/osf.io/ejrct>
- [4] Lobo, Matheus P. “Simple Guidelines for Authors: Open Journal of Mathematics and Physics.” *OSF Preprints*, 15 Nov. 2019. <https://doi.org/10.31219/osf.io/fk836>
- [5] Lobo, Matheus P. “Open Journal of Mathematics and Physics (OJMP).” *OSF*, 21 Apr. 2020. <https://doi.org/10.17605/osf.io/6hzyp>
- [6] COS. *Open Science Framework*. <https://osf.io>
- [7] Lobo, Matheus P. “A Semigroup with a Right Zero Equal to a Right Identity Is Trivial.” *OSF Preprints*, 27 June 2020. <https://doi.org/10.31219/osf.io/zpwb5>

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