Introduction of Luo's Theory

- Timbre Evaluation of Musical Instruments

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Luo's Theory and its application were invented by Canadian scientist and musician James L. Luo in 2013, by far the only feasible approach to an ancient problem: how to evaluate the timbre of musical instruments and other acoustic sources. It is the first quantitative analysis that is applied to the sense of art. As a brief introduction, we just present the basic framework of Luo's theory almost without mathematical equations and reasonings. First, the summary of terminologies is as follows:

Category	Index \ Level	Low	High
Tone Quality	Noise Level (NL)	dull and mild	sharp and harsh
	Hardness Degree (HD)	soft	hard
Tone Color	Symmetry Degree (SD)	rich	pure
	Aftersound Intensity (AI)	weak	strong
	Frequency Centre (FC)	heavy	light
	Uniformity Degree (UD)	uneven	even

Indexes of timbre evaluation are classified as two categories: *quality* one and *style* one. By analogy with dress, clothes and tailoring are about the feature of quality, while color and fashion belong to that of style. In general, people often have an uniform criterion of quality (though sometimes it is not the case), for example, we all think silk is more comfortable than cotton and cotton is more comfortable than flax, which means a consistent ordinal evaluation. But for color and fashion, such order differs from person to person – we can not argue that red is more good-looking than green, instead, someone prefer red to green and another does not. The former has an objective standard, and the latter refers to subjective bias. These two are merged in a final judge: this T-shirt is better than another one (suppose they have the same price). So dose timbre evaluation. When we evaluate the sound of an instrument, the different orders of preference and the different weights of indexes may lead to quit different subjective judges, while the personal utility function of evaluation is unknown. Thus in the market of one kind of instrument, there are different types of timbre for different players and playing situations. On the other hand, all indices are dimensionless ratios – they present the intrinsic characteristics of sound irrelevant to the human sense of hearing, that is, the evaluation of Luo's Theory is an objective benchmark.

The quality index is to evaluate *tone quality* that has a primary index: *Noise Level* (NL) and a secondary index: *Hardness Degree* (HD).

Noise Level (NL): the ratio of the high-frequency volume to the whole volume in the harmonic content of a note. The threshold of high-frequency (cut-off frequency of stopband in high-pass filter) depends on pitch – the higher the pitch, the higher the threshold, and its cardinal setting is an open question (as the example of piano, we set 2000Hz as the threshold of middle C). For the note of hammered or plucked string instruments, its NL is determined by the initial attack that lasts about 0.1-0.3 seconds (i.e. the window size of FFT, e.g. we set the beginning 0.30s of a piano note). A low level of NL indicated the high quality, i.e. a dull and mild sound, or “dark/mellow” (Fig.1.A). On the contrary, a high level of NL means the low quality, it sounds sharp and harsh, or “bright” (Fig.1.B). The “noise” of NL refers to the high-frequency signal rather than the background noise in signal-noise ratio (SNR). NL is the most important index in timbre evaluation. In addition, if the idea is not applied to the full band but some certain band, e.g. a middle band, it will make a style index whose property are subject to the kind of instruments.

Fig.1.A

Fig.1.B

NL and the dynamic of playing are obviously linearly proportional, which means the more heavily you play, the higher the NL. It is called “voiced phenomenon of dynamic” as the empirical data of piano shows (Fig.2). Since NL changes with the dynamic of playing, we define that the *standard* NL of piano is measured by dynamics of *mf* ~ *f*, i.e. the medium intensity. For piano manufacturers, the standard dynamic can be fixed by an auto-action.

Fig.2

Another similar one is called “voiced phenomenon of hardness”, that is, for hammered or

plucked string instruments, the harder the hammer or plectrum, the sharper the sound (under the condition of playing with the same dynamic), i.e. the higher level of NL (see the spectrums of flesh- and nail-plucking in Fig.2, plucking at 1/8 ~1/9 of the string length).

Fig.2

Fig.3

These two phenomena can not be explained by D'Alembert's equation of vibrating string

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

First, the dynamic of playing means the initial condition of the equation – the more heavily you play, the larger the initial vertical displacement. But the solution of the equation shows such initial condition changes not the spectrum but the volume. Second, D'Alembert's equation does not involve any variables of the hardness of hammer or plectrum. However, the theoretical solution of D'Alembert's equation only approximates to the empirical data of flesh-plucking, and has a great departure from that of nail-plucking. Fig.3 is the regression analysis of Fig.2. Some current researches try to solve the problem, but their theories are far

from enough to explain the great departure in empirical data. We are speculating a new approach and will present it in a separate paper.

Please note that a piano with high tone quality does not necessarily means a good one, because the level of NL is an objective description, but whether the sound quality is good or bad is a subjective judge, which depends on personal evaluation and playing situation. For example, a piano with high (standard) NL is suitable for playing in a bar because of its far-reaching transmission in a noisy and large room.

Hardness Degree (HD): the decay ratio of NL, which only applies to hammered or plucked string instruments. It shows how fast the high-frequency vibration decays after the initial attack. The lower the HD, the faster the decay, thus it sounds softer; otherwise, it sounds harder. In general, it does not change with the dynamic of playing, i.e. a quasi-quality index.

The style index is to evaluate *tone color*, such as *Symmetry Degree*, *Aftersound Intensity*, *Frequency Centre*, *Volume Intensity* and *Uniformity Degrees* etc. In general, they change little with the dynamic within the normal range.

Symmetry Degree (SD): the dominance of the peak overtone for a certain range in the harmonic content of a note (the peak overtone is also called *dominant frequency*, DF), and its FFT size is the same as NL. For the full range, it is called *global symmetry degree*, and it also could be applied to local bands. For most acoustic instruments, since the amplitudes of low&middle frequencies are much larger than that of high frequencies (but the latter determines NL), the global SD is mainly determined by the former. A higher (global) SD means the amplitude of the dominant frequency is much more larger than those of other overtones, which has a more symmetric waveform that is closer to sine wave, and sounds purer (Fig.4.A). Otherwise, a lower SD shapes a less symmetric waveform, and sounds richer (Fig.4.B). For piano, the dominant frequency is always the fundamental, i.e. the first overtone or harmonic; but it is not the case for other string instruments. For relatively high SD, the position of dominant frequency (the ordinal number of overtone) significantly affects timbre – the further forward the position is, the warmer the sound, otherwise the colder the sound. (The position of) DF and SD can be likened respectively to hue and saturation of color, they are the major style indexes.

Fig.4.A

Fig.4.B

The difference between SD and NL reduces when the pitch increases. For those very high notes, we only hear a few overtones and its dominant frequency is always the fundamental. In such cases, NL is almost equivalent to SD – high NL means low SD and vice versa. Thus, the quality index merges into the style index, which implies the significance of quality index, i.e. the weight of NL, diminishes when the pitch increases. As the example of piano, the weight of NL steps down in the lower and middle registers, until the higher register has no quality indices but style ones.

Frequency Centre (FC): the frequency centre of amplitude for a certain frequency band in the harmonic content of a note (Fig.5). It could be a full band (global FC), or other ranges. A higher FC implies a lighter sound while a lower one indicates a heavier sound. If an (global) SD is relatively high, the dominant frequency will be very close to the corresponding (global) FC, in other words, they will be almost equivalent.

Fig.5

Aftersound Intensity (AI): the decay ratio of volume, describing the envelope of a waveform. It has three or more secondary indices: *Attack AI* (AAI), *Sustain AI* (MAI), *Decay AI* (DAI), sometimes *Release AI* (RAI), respectively corresponding to the note durations: e.g., 0.3 second, 0.7 second, 1.5 second and 3.1 second or longer (Fig.6). The higher the AI, the slower the decay ratio of volume, i.e. a stronger aftersound. On the contrary, a lower AS means a weaker aftersound. AI only applies to hammered or plucked string instruments.

Fig.6

Volume Intensity (VI): the average dB level of one segment of a note under a standard dynamics and recording condition.

Uniformity Degree (UD): the relative standard deviation of one index data for some notes. A high UD means the index data is *even*. For example, players prefer an even feedback of volume. Every index has a corresponding UD.

It is possible to design more indices for different kinds of acoustic source. The more indices, the more comprehensively we understand the timbre. All these indices are coupled to each other, in other words, they are relevant (A few examples have been discussed above: NL and SD, DF and FC). If we implement the same benchmark (i.e. fix the parameters of algorithm) for all kinds of sound, the evaluation of Luo's theory can show the diversity of their timbre features. For example, the NL of wind instruments will be lower than piano, and that of piano will be much lower than other string instruments as well; for the notes of the same pitch, the AI of piano will be much stronger than plucked string instruments etc. For different instruments of a type of instrument, we can tell them apart and analyze the variation numerically.

This methodology roughly applies to sounds with continuous spectrum as well, for instance, the percussion instruments.

In the appendix, we give an example of NL evaluation of piano by the programme Matlab.

Appendix

1. NL test

k = 4410;

for n = 1:u

if Head(n,5)<pt

Fm = Head(n,5);

for m = 1:2

% 1st and 2nd NLs

t1 = 2^{^(m-1)}-1; t2 = 2^{^m}-1;

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t = round(Fs*Head(n,1));
e = t+t1*k+1:t+t2*k;
x1 = file(e);
Rp = 1; Rs = round(10^(1.631-0.1129*log10(Fm)));
A = (10^(3.046+0.1540*log10(Fm)))/Fs;           % cut-off frequency of passband
B = (10^(2.591+0.2339*log10(Fm)))/Fs;           % cut-off frequency of stopband
wp=A*pi; ws=B*pi;
wp1=2*Fs*tan(wp); ws1=2*Fs*tan(ws);
[N,Wn]=buttord(wp1,ws1,Rp,Rs,'s');
[Z,P,K]=buttapp(N);
[Bap,Aap]=zp2tf(Z,P,K);
[b,a]=lp2hp(Bap,Aap,Wn);
[bz,az]=bilinear(b,a,Fs);
x2=filter(bz,az,x1);
t=length(x1);
y=x1.^2; x=[0:1:t-1]; At(m)=trapz(x,y);
y=x2.^2; x=[0:1:t-1]; Ah(m)=trapz(x,y);
Head(n,8+m) = sqrt(Ah(m)/At(m));
end
Head(n,8) = 20*log10(sqrt(sum(At)));
Head(n,19) = sqrt(sum(Ah)/sum(At));
else
t = round(Fs*Head(n,1));
e = t+1:t+3*k;
x1 = file(e);
t = length(x1);
y = x1.^2; x=[0:1:t-1]; AT=trapz(x,y);
Head(n,8) = 20*log10(sqrt(AT));
end
end

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- Results of octave average (weight $W = [0.20; 0.27; 0.23; 0.18; 0.12]$, the weight of octave 1 is lower than that of octave 2 because the former is less played):

index\octave	1	2	3	4	5	average
1 st NL	7.46%	3.42%	4.67%	5.91%	4.93%	5.14%
1 st UD	55.62%	56.54%	63.67%	33.29%	45.32%	71.38%
2 nd NL	6.87%	3.69%	3.90%	6.64%	5.41%	5.11%
2 nd UD	55.63%	64.57%	55.81%	21.74%	25.41%	71.97%
mean NL	7.10%	3.57%	4.29%	6.06%	4.97%	5.06%
mean UD	56.78%	62.57%	62.43%	33.45%	42.54%	72.95%

The empirical data of NL in this table are objective, and they could be transformed into intuitive scores on 100 scale.

2. Scores of NL

```

a = 40; b = 0.1; % parameters of SD
Sa = 112; Sb = 600; % parameters of NL
asc = 1.31; % parameters of AI
for N = 1:u
    Head(N,20) = Sa-Sb*Head(N,9);
    Head(N,21) = Sa-Sb*Head(N,10);
    Head(N,22) = Sa-Sb*Head(N,19);
    lf = log10(Head(N,5));
    cof = 2.386-1.597*lf+0.4730*lf^2;
    Head(N,23) = asc*(27.62+41.59*Head(N,11)*cof);
    Head(N,24) = asc*(27.62+41.59*Head(N,12)*cof);
    Head(N,25) = asc*(27.62+41.59*Head(N,13)*cof);
    Head(N,26) = asc*(27.62+41.59*Head(N,14)*cof);
    Head(N,27) = log10(Head(N,7)/b)/log10(a/b);
    wa = fix(Head(N,15)/100);
    wb = Head(N,15)-100*wa;
    w = (wa-1)*12+wb;
    if Head(N,15)<=201
        wc = -3.881+0.08159*w+0.02294*w^2-0.000962*w^3;
    elseif Head(N,15)>201 && Head(N,15)<=305
        wc = -6.832+0.4466*w-0.01263*w^2+0.000201*w^3;
    elseif Head(N,15)>305 && Head(N,15)<=411
        wc = 5.107-0.8201*w+0.02665*w^2-0.000215*w^3;
    else
        wc = -4.255+0.4122*w-0.01495*w^2+0.000181*w^3;
    end
    Head(N,28) = Head(N,16)-wc; % pitch correction
end

```

- Results of NL scores (higher register such as octave 6, 7, and 8 have not NLs but other style indexes), and HD = 1st NL – 2nd NL:

index\octave	1	2	3	4	5	average
1 st NL	67.2361	91.5093	84.0011	76.5666	82.4046	81.1455
2 nd NL	70.7773	89.8766	88.6069	72.1784	79.5481	81.3396
mean NL	69.3704	90.5799	86.2891	75.6432	82.1597	81.6521
HD	-3.5412	1.6326	-4.6058	4.3882	2.8564	-0.1941

- Results of every note's NL (yellow highlighted means this note has relatively low quality):

note\NL	1 st NL	2 nd NL	mean NL
C1	54.8173	63.7892	59.9301
#C1	65.2801	68.1754	67.1274
D1	50.9923	72.4195	63.5623
#D1	84.8768	88.5598	86.9251
E1	25.5736	40.1669	33.8433
F1	48.8281	39.521	43.3336
#F1	88.5658	88.0389	88.2713
G1	64.7103	71.8027	68.851
#G1	92.2942	96.826	94.8749
A1	67.2991	57.1281	62.5123
#A1	84.9565	81.3882	82.916
B1	78.6396	81.5122	80.297
C2	74.1704	77.024	75.7796
#C2	100.2146	90.6093	94.467
D2	86.5457	83.6911	84.8355
#D2	95.6907	100.9557	98.5501
E2	88.2218	84.4704	86.3766
F2	87.9623	81.7777	84.8642
#F2	105.944	101.4369	103.5519
G2	78.7361	83.5148	81.0443
#G2	94.5122	90.4494	92.1124
A2	96.5124	93.2768	94.8392
#A2	96.1015	96.5602	96.3844
B2	93.4997	94.7535	94.1541
C3	74.0892	81.8552	77.9483
#C3	95.0824	99.341	97.237
D3	85.6518	93.5016	89.6873
#D3	89.7491	85.2759	87.707
E3	93.3198	95.6546	94.2162
F3	78.53	91.042	84.4331
#F3	90.9083	92.9837	91.7551
G3	74.0049	89.455	82.7125
#G3	92.4749	92.2639	92.3679
A3	63.0698	73.1335	66.4591
#A3	92.709	101.9365	97.3658
B3	78.4235	66.8395	73.58
C4	77.7004	55.2362	71.4386
#C4	103.8832	92.12	99.8818
D4	83.5814	76.1725	81.5257
#D4	73.757	89.0772	82.4076
E4	78.1104	85.207	81.1171
F4	78.2188	77.2529	77.7793

#F4	100.7572	100.3119	100.5591
G4	86.4976	63.8266	75.686
#G4	101.1903	97.3756	99.3483
A4	49.8731	61.0116	54.6596
#A4	65.283	84.305	71.5392
B4	19.9469	-15.7558	11.7761
C5	56.989	44.992	52.7683
#C5	84.7939	94.6691	89.0098
D5	88.6421	94.3256	91.3209
#D5	74.3434	92.8726	79.877
E5	68.3355	61.2408	67.07
F5	54.8404	45.8722	53.0305
#F5	104.2049	92.8284	100.6688
G5	87.5552	40.3797	77.3574
#G5	78.2242	83.4605	79.8584
A5	96.5183	99.5503	97.0736
#A5	99.5362	100.5677	99.8675
B5	94.8718	103.8187	98.0145