[microresearch]

Diamond Open Access

Sum of all natural numbers

Open Mathematics Collaboration^{*†}

February 18, 2020

Abstract

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + \dots = ?$$

keywords: series, natural numbers, infinite, divergence, convergence, infinity theorem

Introduction

1. $S = 1 + 2 + 3 + \dots = ?$ [1]
2. One trivial answer for (1) is $S = \infty$.

^{*}All authors with their affiliations appear at the end of this paper.

[†]Corresponding author: mplobo@uft.edu.br | Join the Open Mathematics Collaboration

Sum of the natural numbers

3. $S_1 = 1 - 1 + 1 - 1 + 1 - 1 + \dots = 1/2$ [2]
4. $S_2 = 1 - 2 + 3 - 4 + \dots$
5. $S = 1 + 2 + 3 + \dots$
6. $2S_2 = S_2 + S_2$
7. $2S_2 = (1 - 2 + 3 - 4 + 5 - \dots) + (0 + 1 - 2 + 3 - 4 + \dots)$
8. $2S_2 = 1 - 1 + 1 - 1 + 1 - \dots = S_1$
9. $2S_2 = 1/2$
10. $S_2 = 1/4$
11. $S - S_2 = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + \dots) - (1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - \dots)$
12. $S - S_2 = 0 + 4 + 0 + 8 + 0 + 12 + 0 + 16 + 0 + \dots$
13. $S - S_2 = 4 + 8 + 12 + 16 + \dots$
14. $S - S_2 = 4(1 + 2 + 3 + 4 + \dots)$
15. $S - S_2 = 4S$
16. $S - 1/4 = 4S$
17. $3S = -1/4$
18. $S = -\frac{1}{12}$

Final Remarks

19. $S = 1 + 2 + 3 + \dots = \{-1/12, \infty\}$
20. (19) supports the **Infinity Theorem** [3].

Open Invitation

*Review, add content, and co-author this article [4, 5]. Join the **Open Mathematics Collaboration**. Send your contribution to mplobo@uft.edu.br.*

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

- [1] Youtube, Numberphile. Astounding: $1 + 2 + 3 + 4 + 5 + \dots = -1/12$. <https://youtu.be/w-I6XTVZXww>
- [2] Lobo, Matheus P. “The Convergence of Grandi’s Series.” *OSF Preprints*, 4 Jan. 2020. <https://doi.org/10.31219/osf.io/aq5xz>
- [3] Lobo, Matheus P. “The Infinity Theorem.” *OSF Preprints*, 4 Feb. 2020. <https://doi.org/10.31219/osf.io/9zm6b>
- [4] Lobo, Matheus P. “Microarticles.” *OSF Preprints*, 28 Oct. 2019. <https://doi.org/10.31219/osf.io/ejrct>
- [5] Lobo, Matheus P. “Simple Guidelines for Authors: Open Journal of Mathematics and Physics.” *OSF Preprints*, 15 Nov. 2019. <https://doi.org/10.31219/osf.io/fk836>
- [6] COS. *Open Science Framework*. <https://osf.io>
- [7] Lobo, Matheus P. “Sum of All Natural Numbers.” *OSF Preprints*, 17 Feb. 2020. <https://doi.org/10.31219/osf.io/yx28b>

The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br)^{1,2}

¹Federal University of Tocantins (Brazil); ²Universidade Aberta (UAb, Portugal)