



[white paper: pedagogical]

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Two non-isomorphic structures with the same number of elements

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Abstract

For pedagogical purposes, we define a simple language to show that two different structures with the same number of elements in their universes are not isomorphic.

keywords: language, structure, isomorphism, first-order logic

The most updated version of this white paper is available at
<https://osf.io/ytcru/download>
<https://zenodo.org/record/5459328>

Introduction

1. This is a pedagogical white paper on *first-order logic*.
2. Our purpose is to discuss a result in [1] which is licensed under [2].
3. We use minimal notation but preserving all relevant mathematical information.

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Meta-linguistic symbols

- 4. $:=$ means that what is on the left is defined by what is on the right.
- 5. \equiv means that the strings on both sides are identical.
- 6. $\mathbf{a}, \mathbf{b} \vdash \mathbf{c}$ means \mathbf{a}, \mathbf{b} proves \mathbf{c} .

Proposition

- 7. *There exists two non-isomorphic structures for a particular language with the same number of elements in their universes [1].*

Some definitions

- 8. Let $\mathcal{L} = \{+\}$.
- 9. $\mathcal{L} :=$ language
- 10. $+$ $:=$ binary function symbol
- 11. $\mathfrak{A}, \mathfrak{B} := \mathcal{L}$ -structures with universe \mathbb{N}
- 12. $+^{\mathfrak{A}} :=$ standard addition on \mathbb{N}
- 13. $x +^{\mathfrak{B}} y = \min(x, y)$ for any $x, y \in \mathbb{N}$

Proof of (7)

- 14. Assume for the sake of *contradiction* that $\iota : A \rightarrow B$ is an isomorphism from \mathfrak{A} to \mathfrak{B} .
- 15. From (12), (14), and (13), respectively,

$$\iota(2) = \iota(1 +^{\mathfrak{A}} 1) = \iota(1) +^{\mathfrak{B}} \iota(1) = \iota(1).$$

16. $\iota(2) = \iota(1)$

17. (16) contradicts (14), namely, that ι is a bijection. □

Final Remarks

18. We proved proposition (7) in a very simple language.

Open Invitation

Review, add content, and co-author this white paper [3, 4].

Join the **Open Mathematics Collaboration**.

Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [5, 6].

How to cite this paper?

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Agreement

All authors agree with [4].

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