



[white paper: pedagogical]

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# Two non-isomorphic structures with the same number of elements

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## Abstract

For pedagogical purposes, we define a simple language to show that two different structures with the same number of elements in their universes are not isomorphic.

keywords: language, structure, isomorphism, first-order logic

*The most updated version of this white paper is available at*  
<https://osf.io/ytcr/download>  
<https://zenodo.org/record/5459328>

## Introduction

1. This is a pedagogical white paper on *first-order logic*.
2. Our purpose is to discuss a result in [1] which is licensed under [2].
3. We use minimal notation but preserving all relevant mathematical information.

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## Meta-linguistic symbols

4.  $:=$  means that what is on the left is defined by what is on the right.
5.  $\equiv$  means that the strings on both sides are identical.
6.  $a, b \vdash c$  means  $a, b$  proves  $c$ .

## Proposition

7. *There exists two non-isomorphic structures for a particular language with the same number of elements in their universes [1].*

## Some definitions

8. Let  $\mathcal{L} = \{+\}$ .
9.  $\mathcal{L} :=$  language
10.  $+$   $:=$  binary function symbol
11.  $\mathfrak{A}, \mathfrak{B} := \mathcal{L}$ -structures with universe  $\mathbb{N}$
12.  $+^{\mathfrak{A}} :=$  standard addition on  $\mathbb{N}$
13.  $x +^{\mathfrak{B}} y = \min(x, y)$  for any  $x, y \in \mathbb{N}$

## Proof of (7)

14. Assume for the sake of *contradiction* that  $\iota : A \rightarrow B$  is an isomorphism from  $\mathfrak{A}$  to  $\mathfrak{B}$ .
15. From (12), (14), and (13), respectively,

$$\iota(2) = \iota(1 +^{\mathfrak{A}} 1) = \iota(1) +^{\mathfrak{B}} \iota(1) = \iota(1).$$

16.  $\iota(2) = \iota(1)$

17. (16) contradicts (14), namely, that  $\iota$  is a bijection. □

## Final Remarks

18. We proved proposition (7) in a very simple language.

## Open Invitation

*Review, add content, and co-author* this white paper [3, 4].

*Join* the **Open Mathematics Collaboration**.

Send your contribution to `mplobo@uft.edu.br`.

## Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [5, 6].

## How to cite this paper?

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+ **Center for Open Science**

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## Agreement

All authors agree with [4].

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## References

- [1] Leary, Christopher C., and Lars Kristiansen. *A friendly introduction to mathematical logic*, 2nd edition, 2015.  
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- [2] CC. Creative Commons. *CC-By Attribution 4.0 International*.  
<https://creativecommons.org/licenses/by/4.0>
- [3] Lobo, Matheus P. “Microarticles.” *OSF Preprints*, 28 Oct. 2019.  
<https://doi.org/10.31219/osf.io/ejrct>
- [4] Lobo, Matheus P. “Simple Guidelines for Authors: Open Journal of Mathematics and Physics.” *OSF Preprints*, 15 Nov. 2019.  
<https://doi.org/10.31219/osf.io/fk836>
- [5] Lobo, Matheus P. “Open Journal of Mathematics and Physics (OJMP).” *OSF*, 21 Apr. 2020. <https://osf.io/6hzyf/files>
- [6] <https://zenodo.org/record/5459328>

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