

# Bayesian Workshop

$P = .049$

Artwork by Viktor Beekman • [instagram.com/viktordepictor](https://www.instagram.com/viktordepictor)

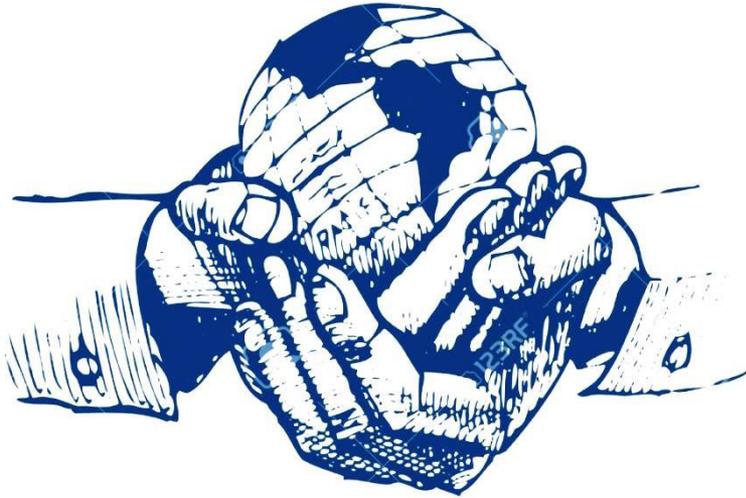
5<sup>th</sup> meeting [01.06.2018] **Refresher session!**

# Today's Game plan

- Intro to **Bayesian Inference** reading  
**McElreath (2015):** “Statistical Rethinking”  
[Chap 2, up to & including section 2.3.4, p.19 – 37]
- Video lectures  
**Daniël Lakens:** “Improving your statistical inferences”  
Lecture 2.1 [16mins] **Likelihoods**  
Lecture 2.2 [14mins] **Binomial Bayesian Inference**  
Lecture 2.3 [11mins] **Bayesian Thinking**
- Announcement of **upcoming events!**  
**OSF homepage:** <https://osf.io/hcm7p/wiki/home/>

# Small Worlds & Large Worlds

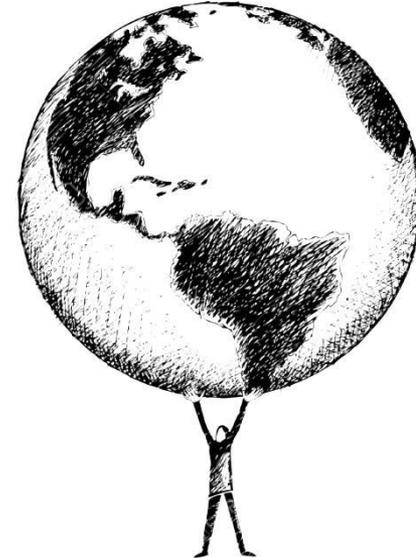
## 2 frames of statistical modeling



**SMALL WORLD**

**model**

self-contained  
logical consistency



**LARGE WORLD**

**reality**

broader context in which  
one deploys a model



model assumptions *may or may not* approximate reality

# The garden of forking data



Jorge Luis Borges

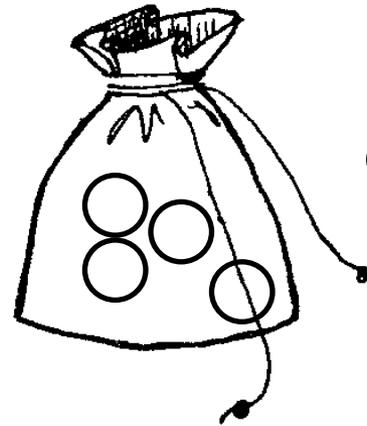
# The garden of forking data

- In order to make good inference about what actually happened, it helps to consider everything that could have happened.
- Bayesian analysis → garden of forking data  
→ alternative sequences of events
- As we learn what did happen, some of these alternative sequences are pruned.
- In the end, what remains is only what is logically consistent with our knowledge.

# Counting possibilities

- Probability theory
- Marble example

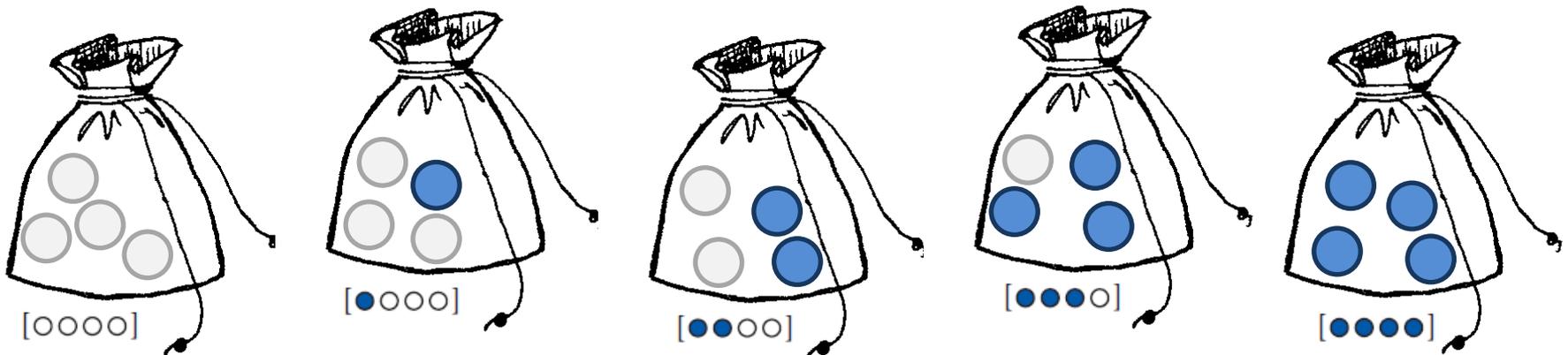
Given:



either



- All possibilities consistent with what we know about the bag  
→ 5 conjectures

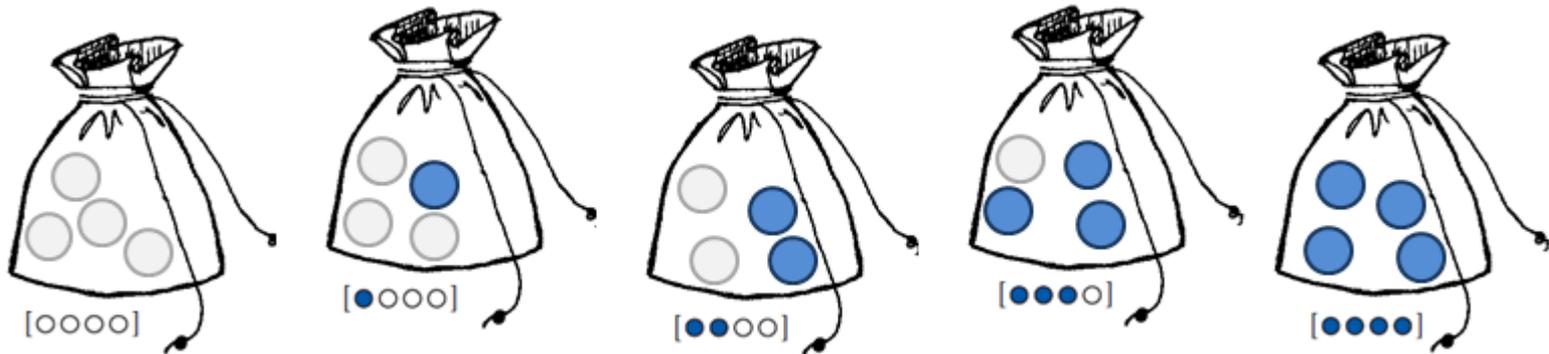
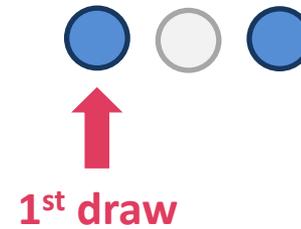


**GOAL:** Determine which conjecture is the most plausible, given some **evidence**.

# Counting possibilities

**Evidence (i.e. data):**

*Draw 3 marbles from the bag (with replacement)*

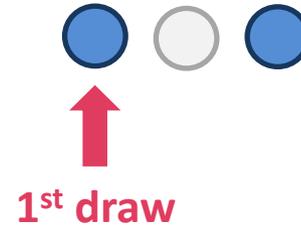
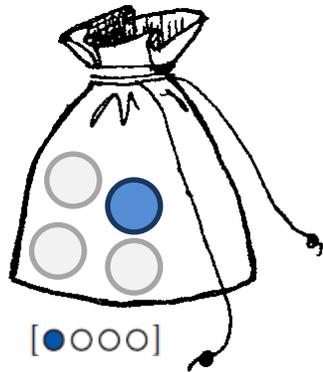


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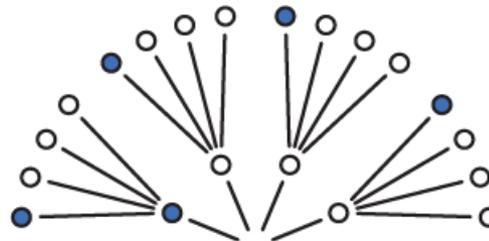
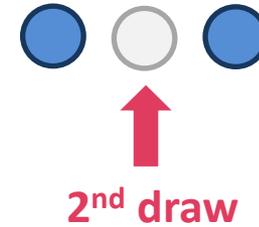
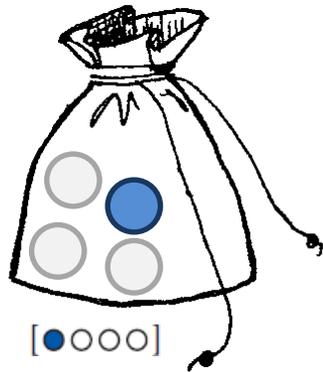
*Draw 3 marbles from the bag (with replacement)*



# Counting possibilities

**Evidence (i.e. data):**

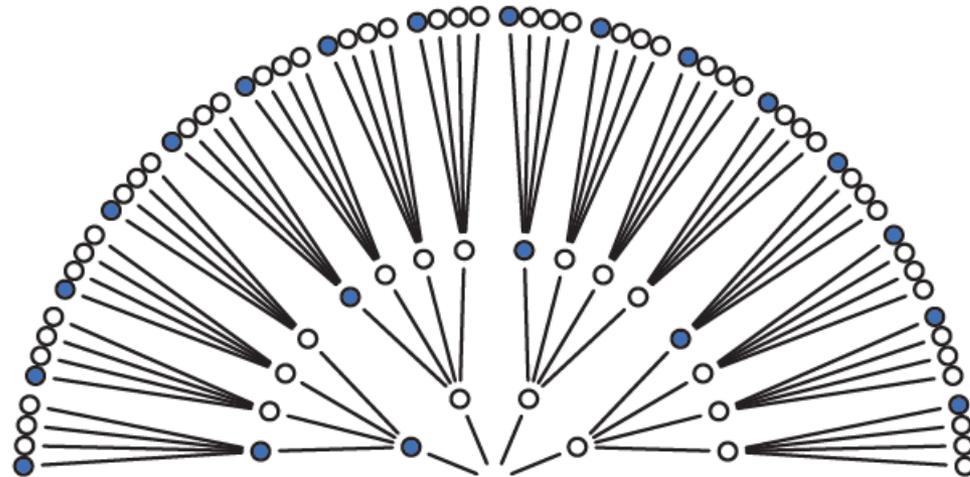
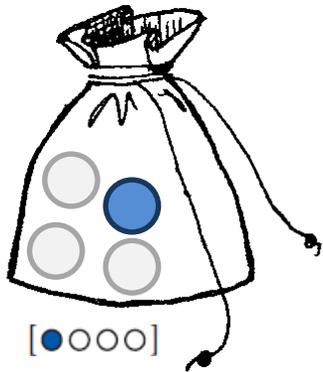
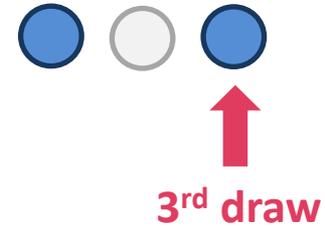
*Draw 3 marbles from the bag (with replacement)*



# Counting possibilities

**Evidence (i.e. data):**

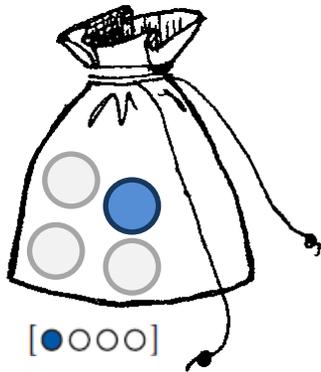
*Draw 3 marbles from the bag (with replacement)*



# Counting possibilities

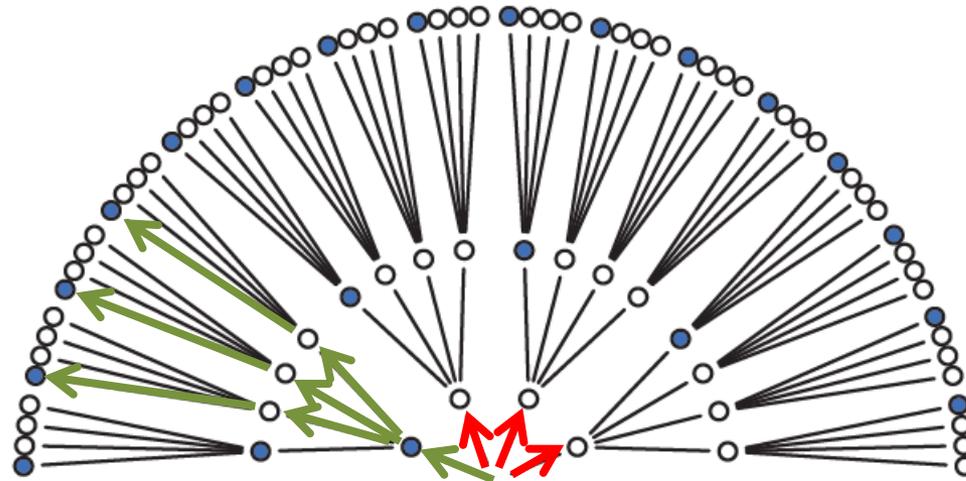
**Evidence (i.e. data):**

*Draw 3 marbles from the bag (with replacement)*



**64 possible paths:**

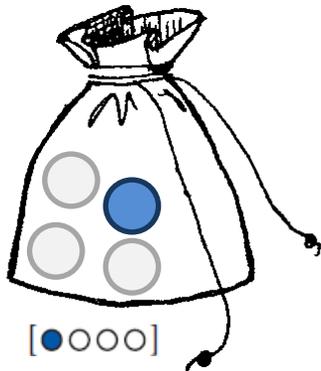
- Some are **logically consistent** with our data
- Some can be **eliminated**



# Counting possibilities

**Evidence (i.e. data):**

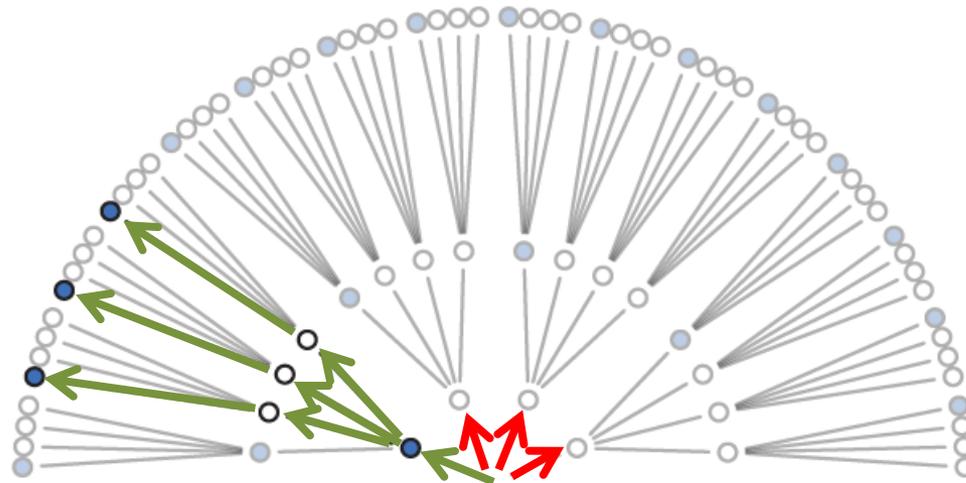
*Draw 3 marbles from the bag (with replacement)*



**64 possible paths:**

- Some are **logically consistent** with our data
- Some can be **eliminated**

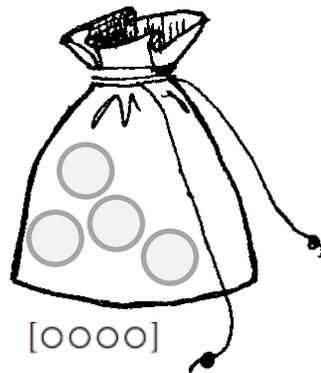
**3 paths remain**



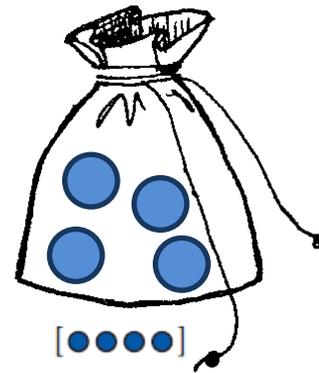
# Counting possibilities

**Evidence (i.e. data):**

*Draw 3 marbles from the bag (with replacement)*



**0 paths**

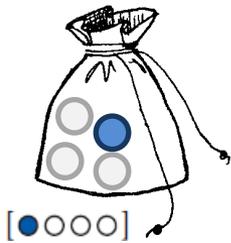


**0 paths**

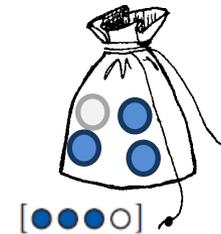
# Counting possibilities

**Evidence (i.e. data):**

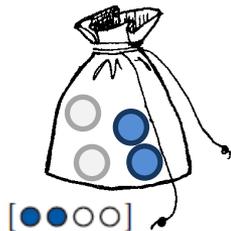
*Draw 3 marbles from the bag (with replacement)*



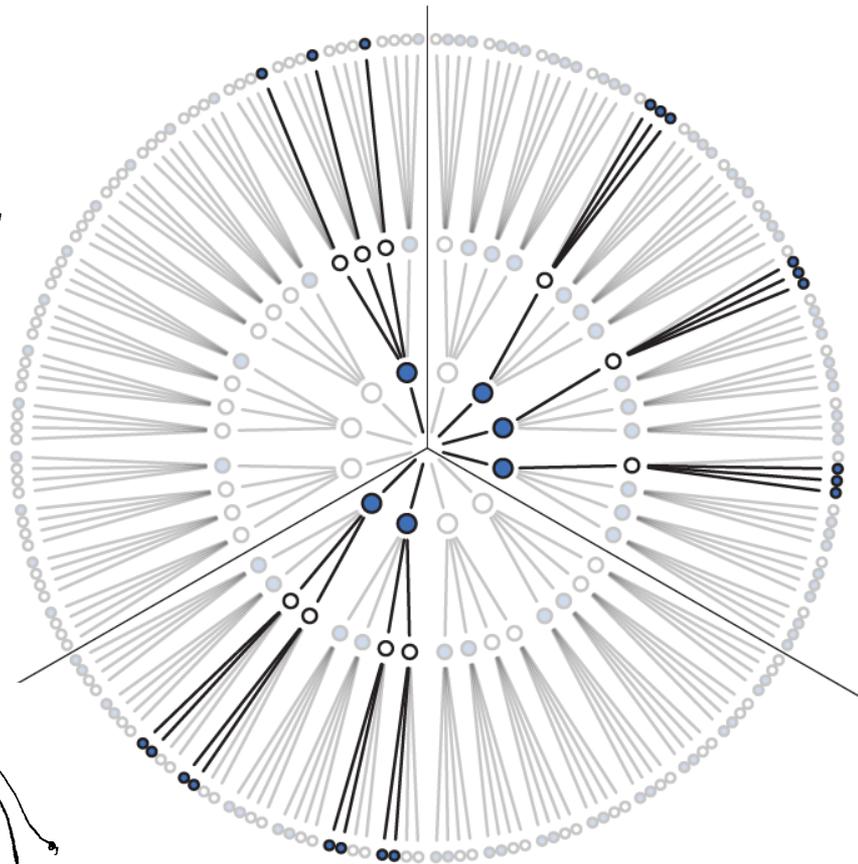
**3 paths**



**9 paths**



**8 paths**

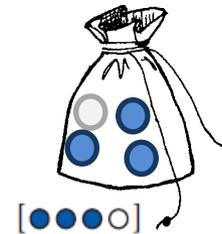
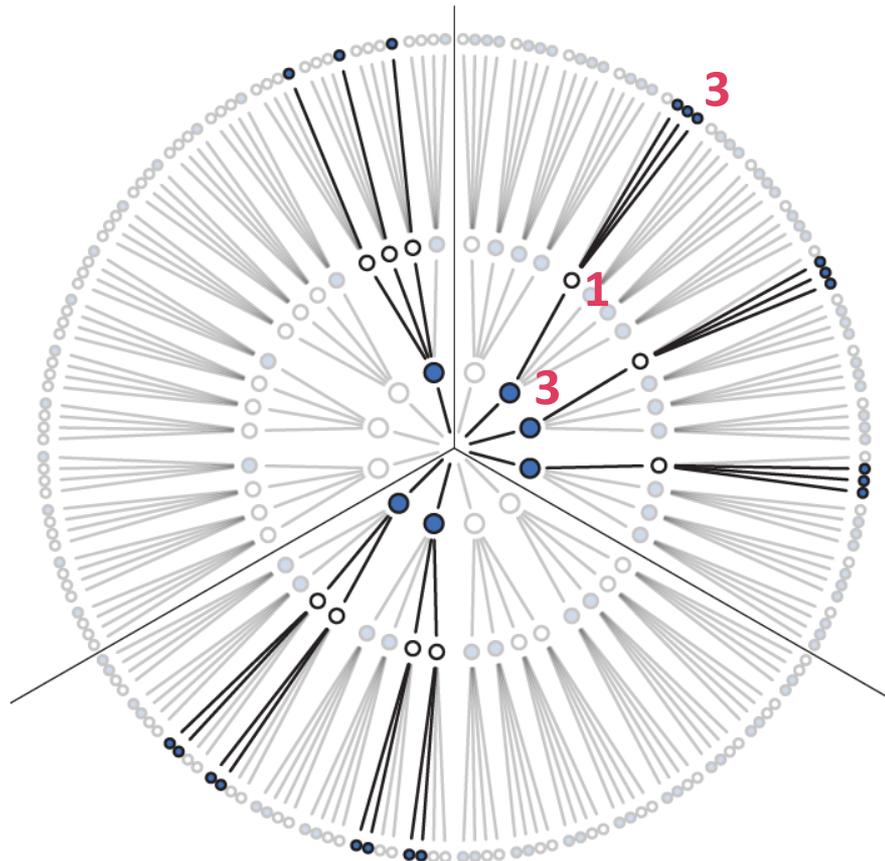


Conjecture	Ways to produce ●○○●
[○○○○]	$0 \times 4 \times 0 = 0$
[●○○○]	$1 \times 3 \times 1 = 3$
[●●○○]	$2 \times 2 \times 2 = 8$
[●●●○]	$3 \times 1 \times 3 = 9$
[●●●●]	$4 \times 0 \times 4 = 0_{14}$

# Counting possibilities

**Evidence (i.e. data):**

*Draw 3 marbles from the bag (with replacement)*



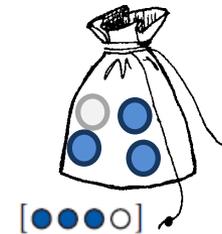
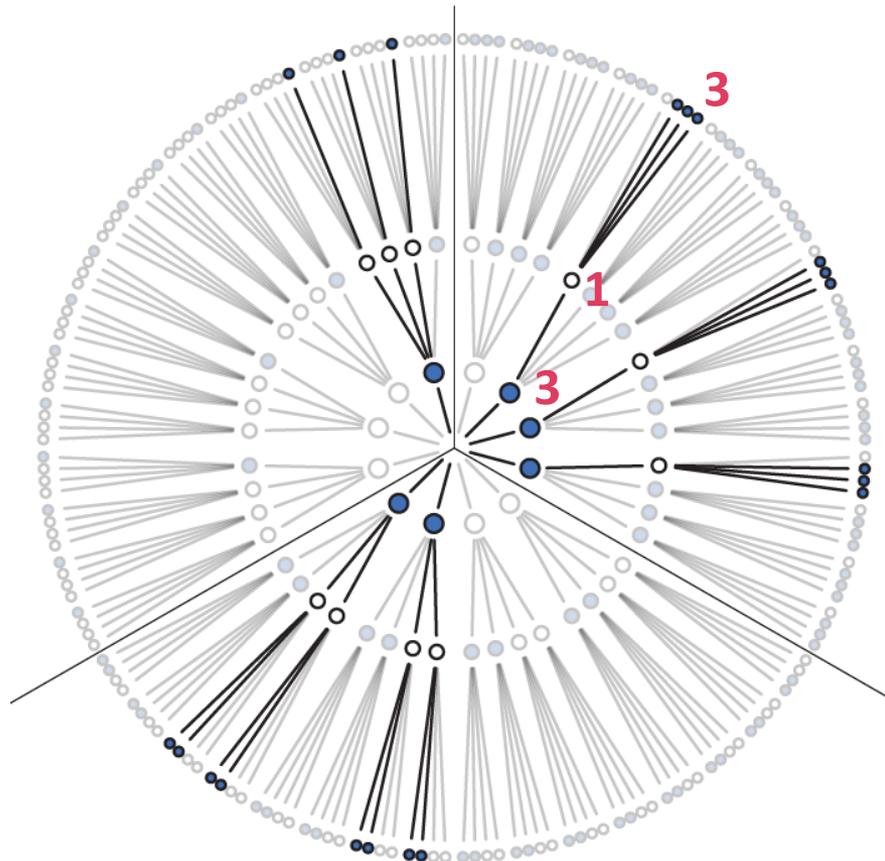
**9 paths**

Conjecture	Ways to produce ●○○
[○○○○]	$0 \times 4 \times 0 = 0$
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[●●○○]	$2 \times 2 \times 2 = 8$
[●●●○]	$3 \times 1 \times 3 = 9$
[●●●●]	$4 \times 0 \times 4 = 0_{15}$

# Counting possibilities

**Evidence (i.e. data):**

*Draw 3 marbles from the bag (with replacement)*



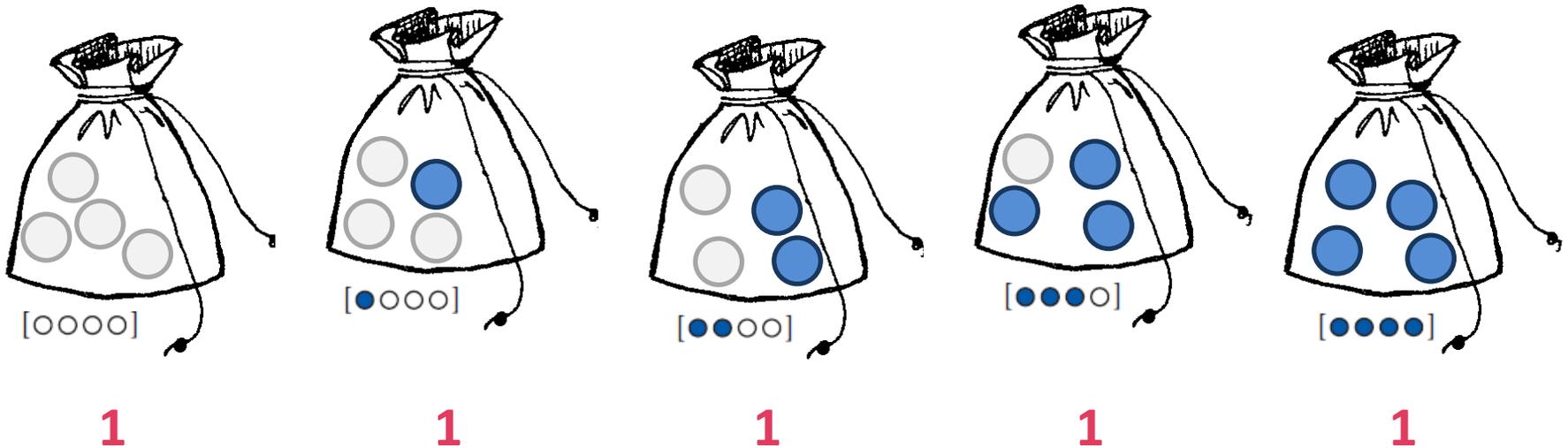
**9 paths**

→ update by multiplying  
new count by old count

Conjecture	Ways to produce ●○○
[○○○○]	$0 \times 4 \times 0 = 0$
[●○○○]	$1 \times 3 \times 1 = 3$
[●●○○]	$2 \times 2 \times 2 = 8$
[●●●○]	$3 \times 1 \times 3 = 9 \times 3$
[●●●●]	$4 \times 0 \times 4 = 0_{16}$

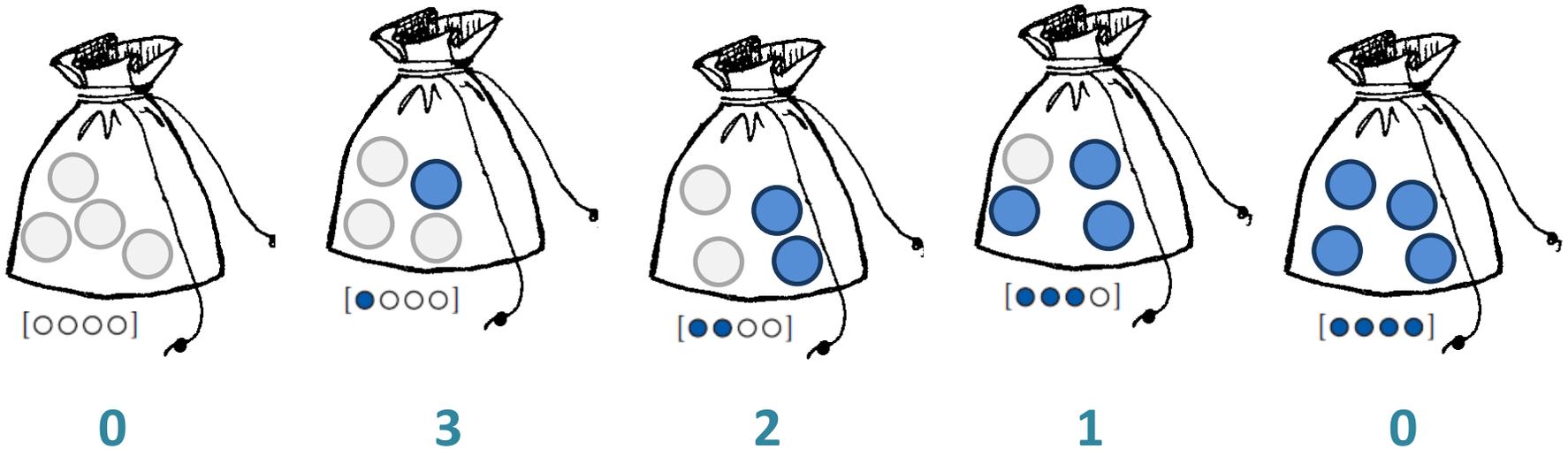
# Counting possibilities

**Principle of indifference:** When there is no reason to say that one conjecture is more plausible than another, weigh all of the conjectures equally.



# Counting possibilities

**Prior information:** Each bag contains at least 1 blue & 1 white marble.  
Blue marbles are rare.



Conjecture	Prior count	Factory count	New count
[○○○○]	0	0	$0 \times 0 = 0$
[●○○○]	3	3	$3 \times 3 = 9$
[●●○○]	16	2	$16 \times 2 = 32$
[●●●○]	27	1	$27 \times 1 = 27$
[●●●●]	0	0	$0 \times 0 = 0$



Relative plausibility of each conjecture in terms of **raw counts**.

# Counting possibilities

Conjecture	Prior count	Factory count	New count
[○○○○]	0	0	$0 \times 0 = 0$
[●○○○]	3	3	$3 \times 3 = 9$
[●●○○]	16	2	$16 \times 2 = 32$
[●●●○]	27	1	$27 \times 1 = 27$
[●●●●]	0	0	$0 \times 0 = 0$



Relative plausibility of each conjecture in terms of **raw counts**.



Can compute these plausibilities as *proportions (standardized)*.

$$\text{plausibility of } p \text{ after } D_{\text{new}} = \frac{\text{ways } p \text{ can produce } D_{\text{new}} \times \text{prior plausibility } p}{\text{sum of products}} = \frac{16 \times 2}{(9 + 32 + 27)}$$

$$[\bullet \circ \circ \circ] \quad 9/68 \sim 0.13$$

$$[\bullet \bullet \circ \circ] \quad 32/68 \sim 0.47$$

$$[\bullet \bullet \bullet \circ] \quad 27/68 \sim 0.40$$

SUM to 1.

# Bayesian Updating

## Probability theory terminology:

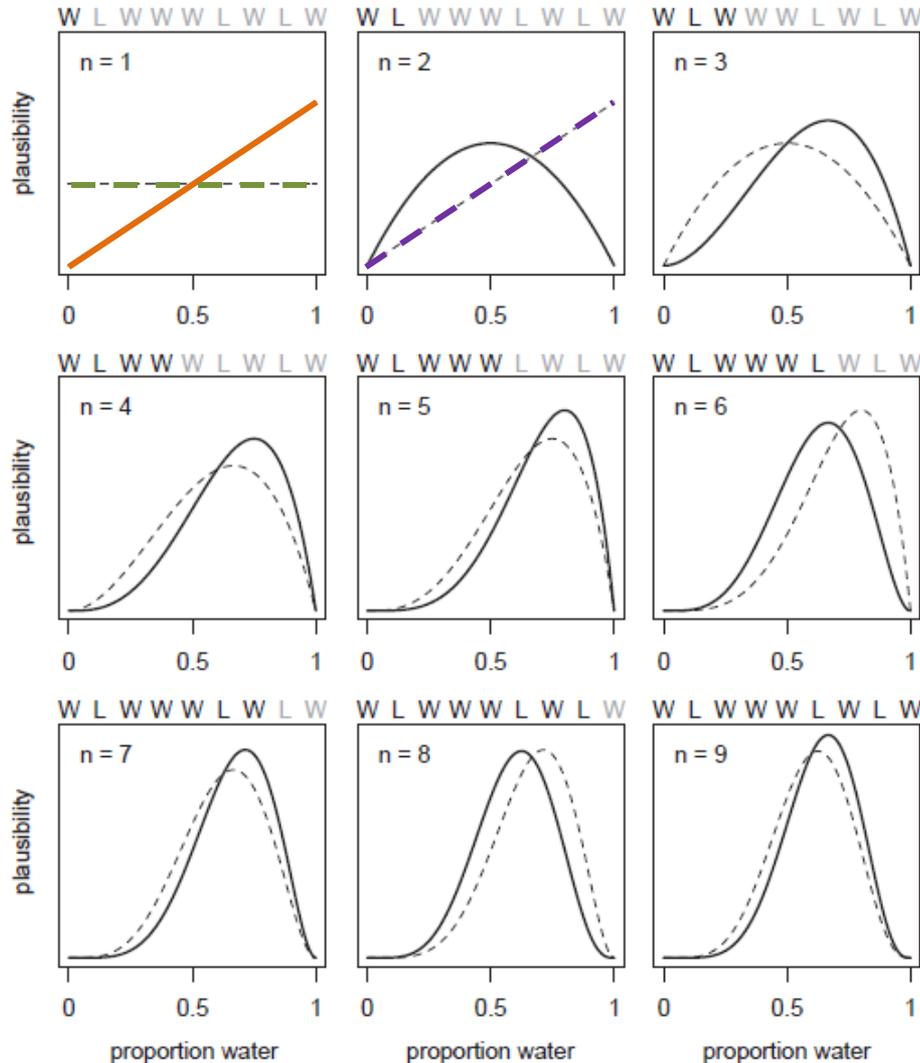
**PARAMETERS:** Represent the different *conjectures* for causes or explanations of the data.

$$\text{POSTERIOR PROBABILITY} \\ \text{plausibility of } p \text{ after } D_{\text{new}} = \frac{\text{LIKELIHOOD} \quad \times \quad \text{PRIOR PROBABILITY}}{\text{ways } p \text{ can produce } D_{\text{new}} \times \text{prior plausibility } p} \\ \text{sum of products}$$

- BAYESIAN UPDATING:**
- Bayesian model begins with one set of plausibilities (*prior probabilities*)
  - Model updates in light of new data (*likelihood*)
  - This produces *posterior probabilities*
  - Every updated set of plausibilities (*posteriors*) becomes the initial plausibilities (*priors*) for the next observation

# Bayesian Updating

## How a Bayesian model learns



---  
uniform  
(uninformative)  
prior

- Each toss of the globe produces an observation of water (W) or land (L).
- With each new observation, the plausibility (i.e. estimate of the proportion of water) is updated.
- **Priors:** dashed curves
- **Posteriors:** solid curves

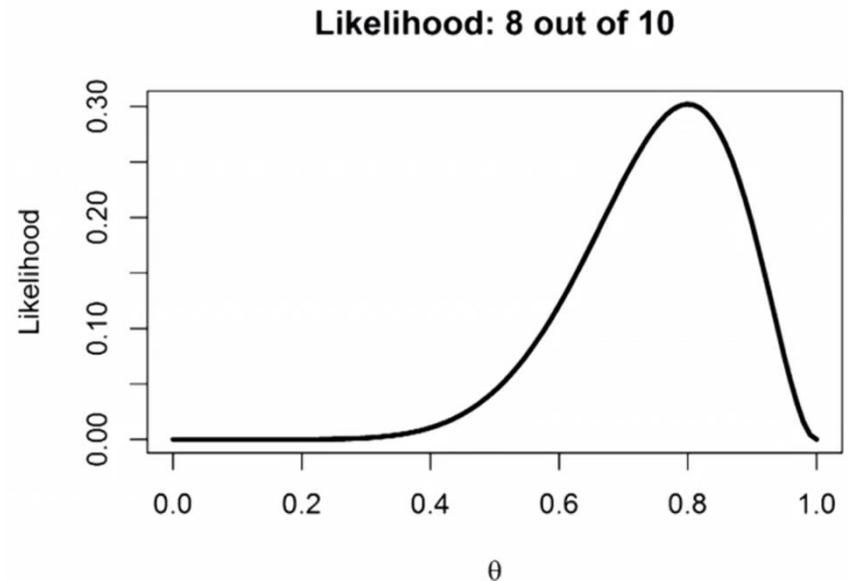
# Bayesian Statistics

## Key concepts

**LIKELIHOOD:** A likelihood gives you the function of a parameter given the data.

*“So when you’ve observed some data, you can plot the accompanying likelihood function, and you can check how likely each hypothesis you may have is.”*

You flip 8 out of 10 heads. The likelihood of  $\theta = 0.8$  is 0.30.



# Bayesian Statistics

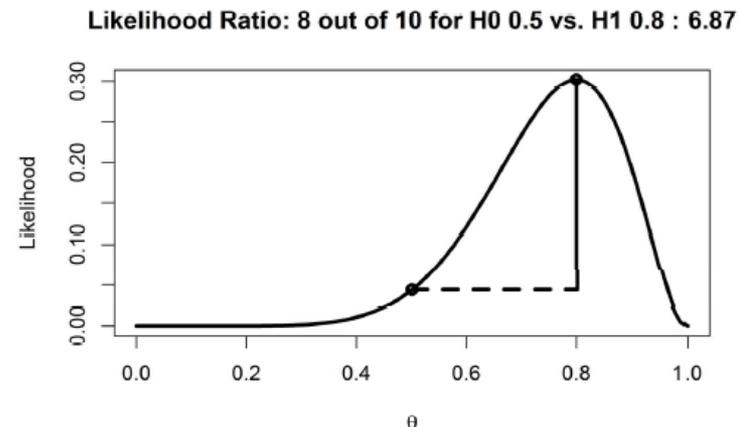
## Key concepts

**LIKELIHOOD RATIO:** “We can use the likelihood under the **null hypothesis** and the likelihood under the **alternative hypothesis**, and calculate the likelihood ratio.”

→ We are taking the **relative evidence** of the one hypothesis (e.g. **H0**) and the **relative evidence** of the other hypothesis (**H1**), and calculating the **odds** of one over the other:

$$\frac{\text{odds}(H1)}{\text{odds}(H0)} \quad \frac{L(H1)}{L(H0)} \quad \frac{L(\theta=0.8)}{L(\theta=0.5)}$$

We can use the likelihood under H0 and H1 to calculate the **likelihood ratio**



→ Given the observed data, we have support for the hypothesis that the coin ‘unfair’.

# Bayesian Statistics

## Key concepts

**LIKELIHOOD RATIO:** “We can use the likelihood under the **null hypothesis** and the likelihood under the **alternative hypothesis**, and calculate the likelihood ratio.”

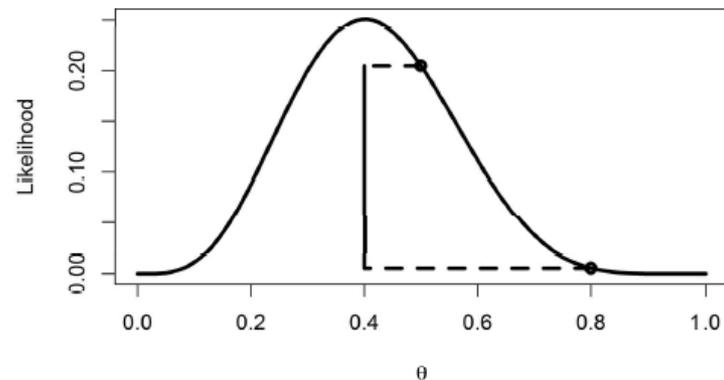
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$$\frac{\text{odds}(H1)}{\text{odds}(H0)} \quad \frac{L(H1)}{L(H0)} \quad \frac{L(\theta=0.8)}{L(\theta=0.5)}$$

Given a different observation:  
e.g., 4 out of 10 flips are HEADS

→ Now, we have support for the hypothesis that the coin is ‘fair’.

Likelihood Ratio: 4 out of 10 for H0 0.5 vs. H1 0.8 : 37.25



# Bayesian Statistics

## Key concepts

### LIKELIHOODS & LIKELIHOOD RATIOS

2 important notes!

Likelihood ratios of **8** and **32** are moderately strong and strong evidence.

(Royall, 1997)

Likelihoods are **relative** evidence for H1 vs. H0. H0 and H1 might be unlikely.

# Bayesian Statistics

## Key concepts

### PRIORS & POSTERIOR (Bayesian updating):

*“You have some **prior belief**, & you have some **data**, & you combine these into a **posterior belief**.”*

Prior Belief +  
Data =  
Posterior Belief

We can calculate **posterior odds** that the alternative hypothesis is true (given the data), compared to the probability that the null hypothesis is true (given the data).

Posterior odds:

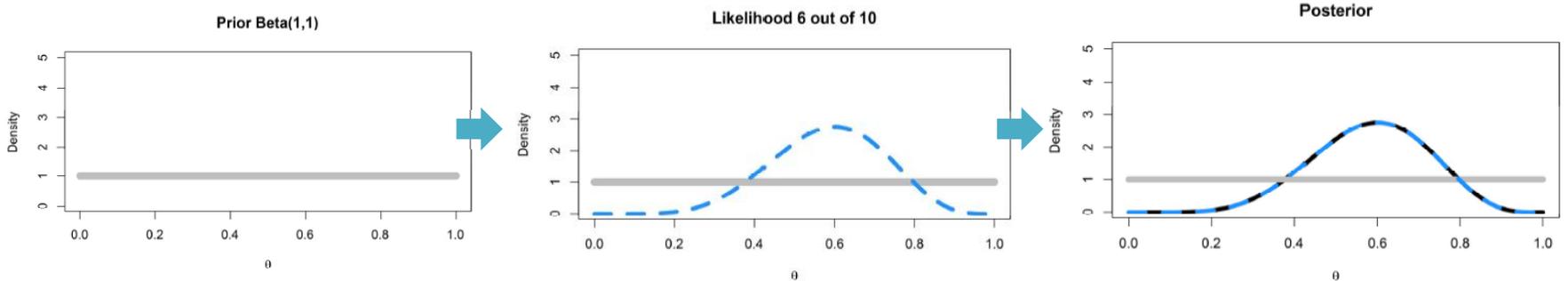
$$\frac{P(H1|D)}{P(H0|D)} = \frac{P(D|H1)}{P(D|H0)} \times \frac{P(H1)}{P(H0)}$$

$$\text{Posterior} = \text{Likelihood Ratio} \times \text{Prior}$$

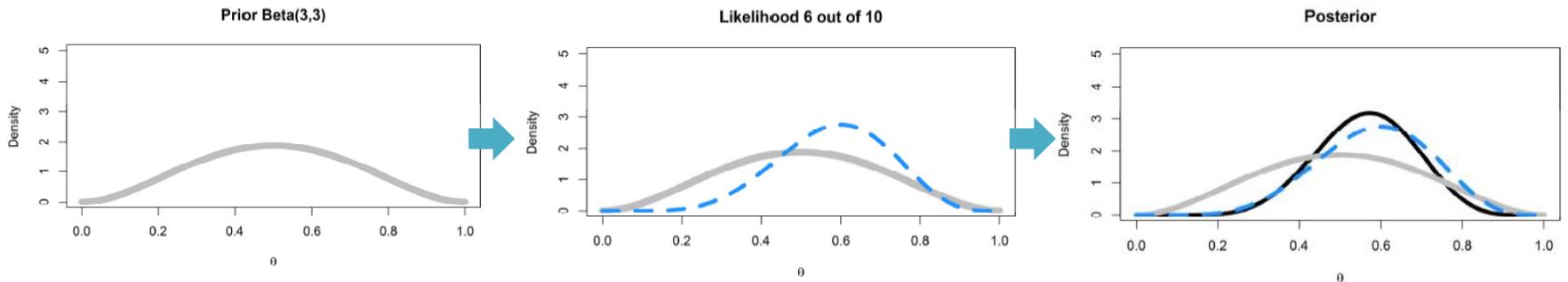
# Bayesian Statistics

## Key concepts

### PRIORS & POSTERIORS (Bayesian updating): *Examples*



\*with a uniform (uninformative) prior, only the likelihood influences our posterior beliefs.



\*given a non-uniform prior, the posterior is a combination of the likelihood + the prior.

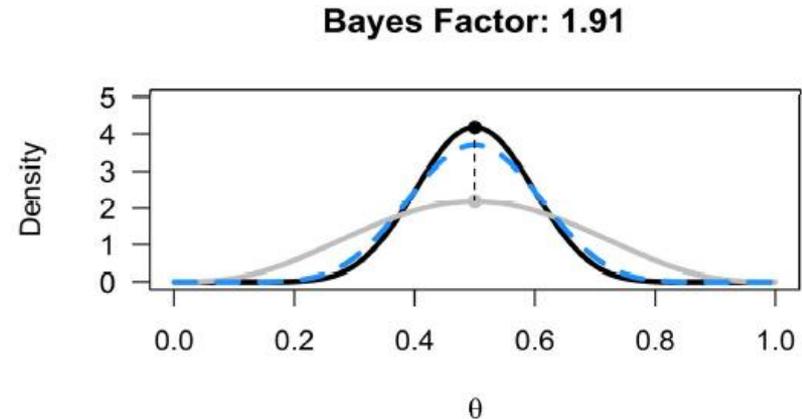
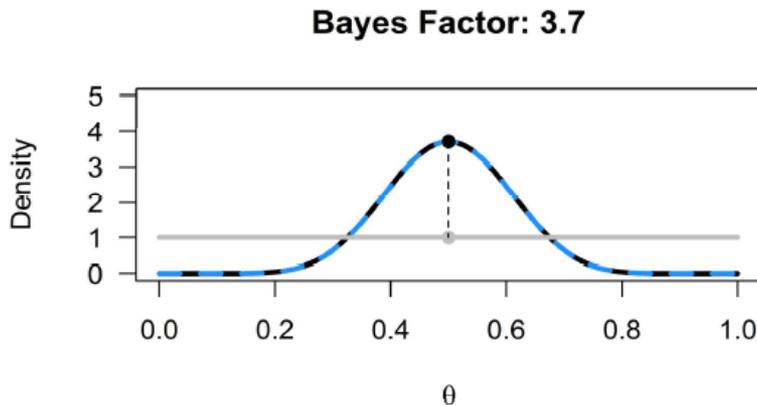
# Bayesian Statistics

## Key concepts

**BAYES FACTORS:** *Relative evidence of one model compared to another.*

→ We can compare the **prior distribution** to the **posterior distribution** to see how much our beliefs have changed by collecting some data.

→ **Larger the BF, larger the change in belief** (from prior to posterior)



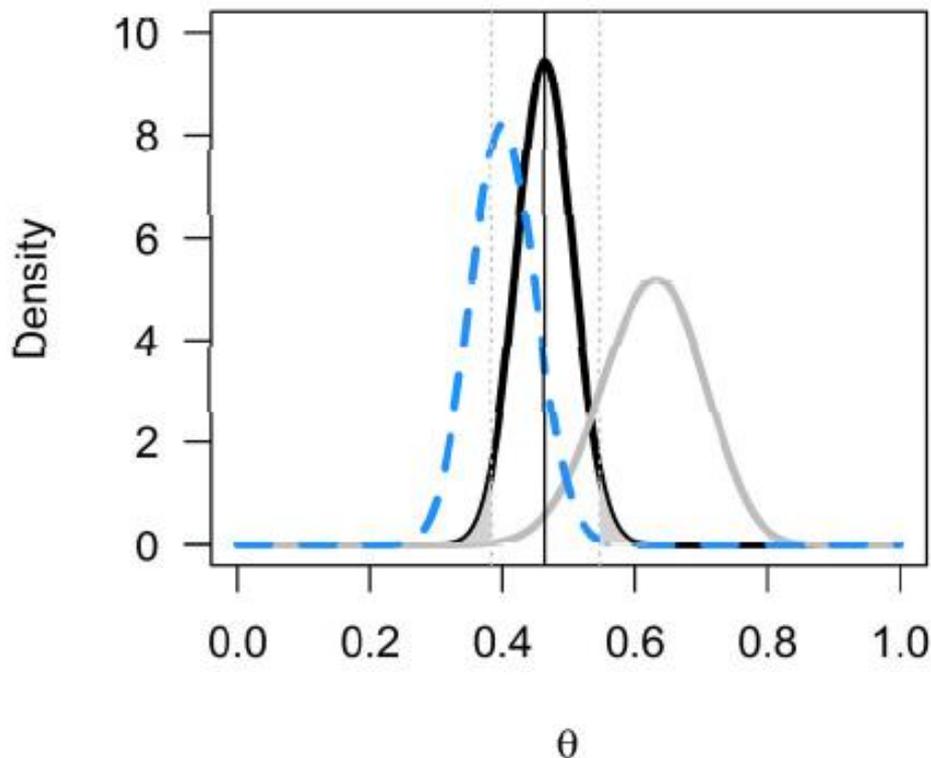
After looking at the data,  $\theta=0.5$  has become **3.70** or **1.91** times more likely, depending on the prior.

# Bayesian Statistics

## Key concepts

**BAYES FACTORS:** *Relative evidence of one model compared to another.*

**Mean posterior: 0.46429 ,  
95% Credible Interval: 0.38 ; 0.55**



**BF = 1 to 3**

*Inconclusive evidence*

**BF > 3, BF < 1/3**

*substantial*

**BF > 10, BF < 1/10**

**STRONG**

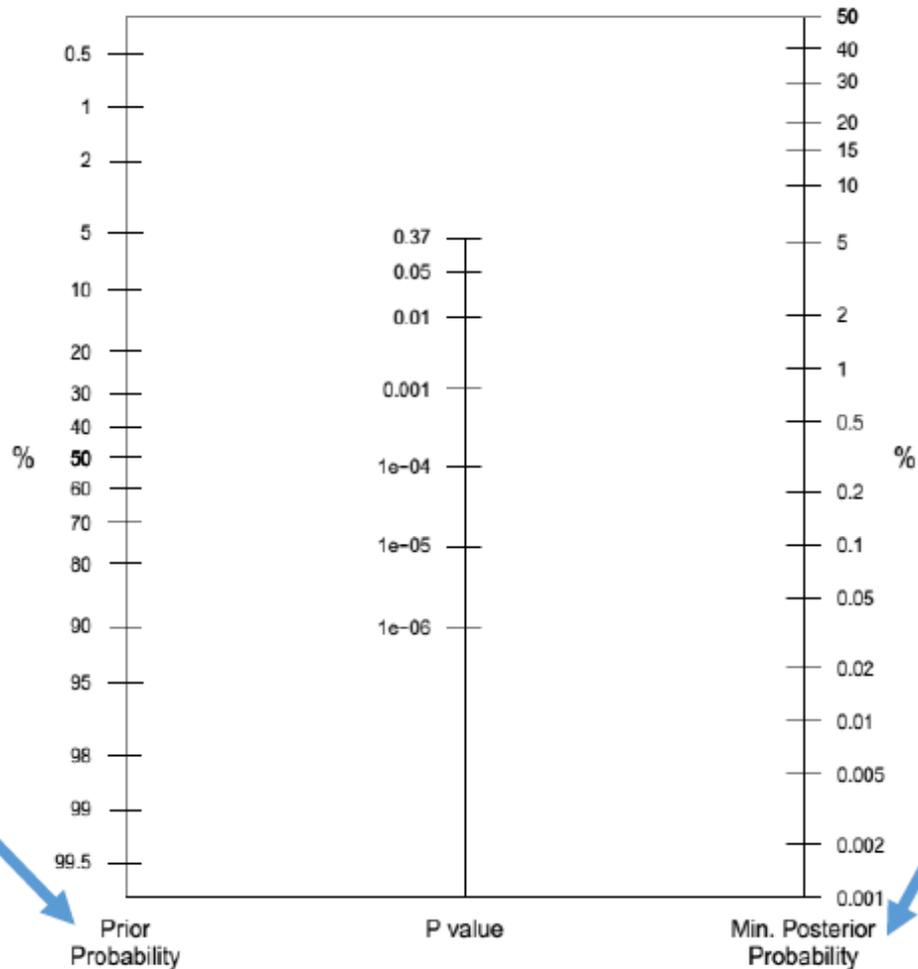
# Bayesian THINKING

**Bayesian  
Thinking**

outside the context of formal  
Bayesian statistics



# Bayesian THINKING

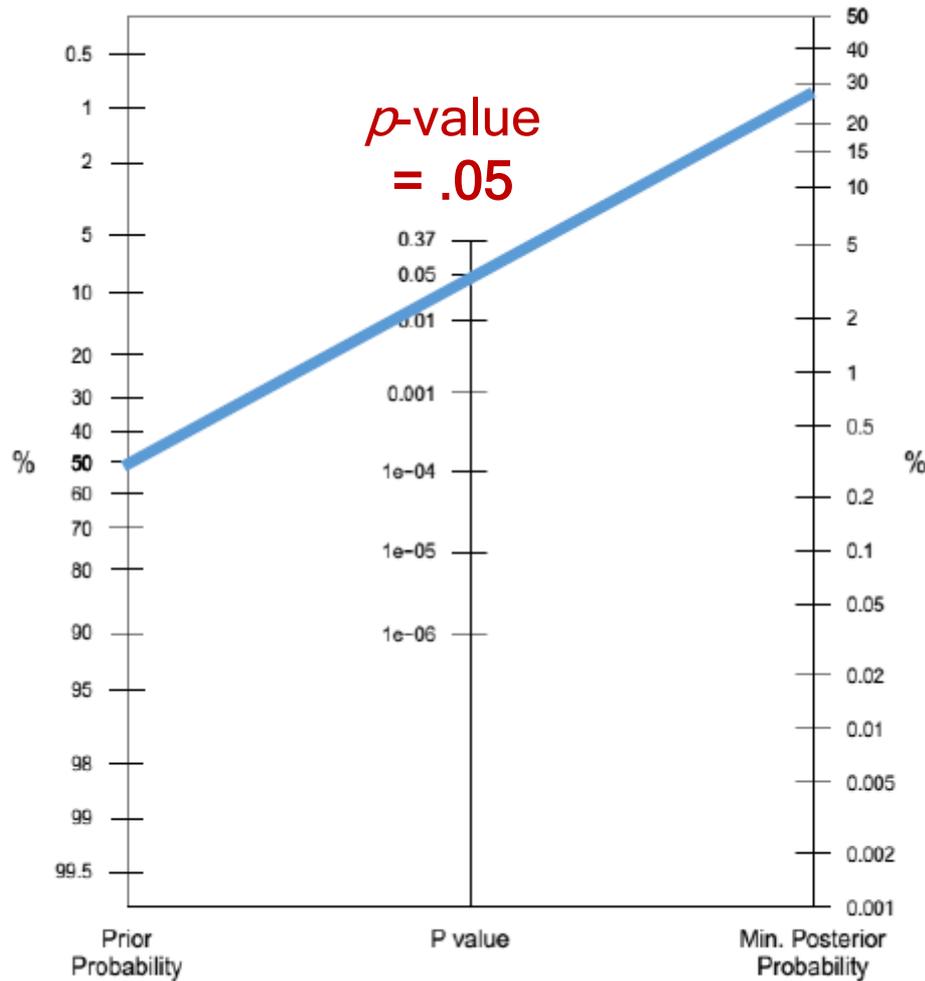


Prior  
Probability  
H0 is True

Posterior  
Probability  
H0 is True

# Bayesian THINKING

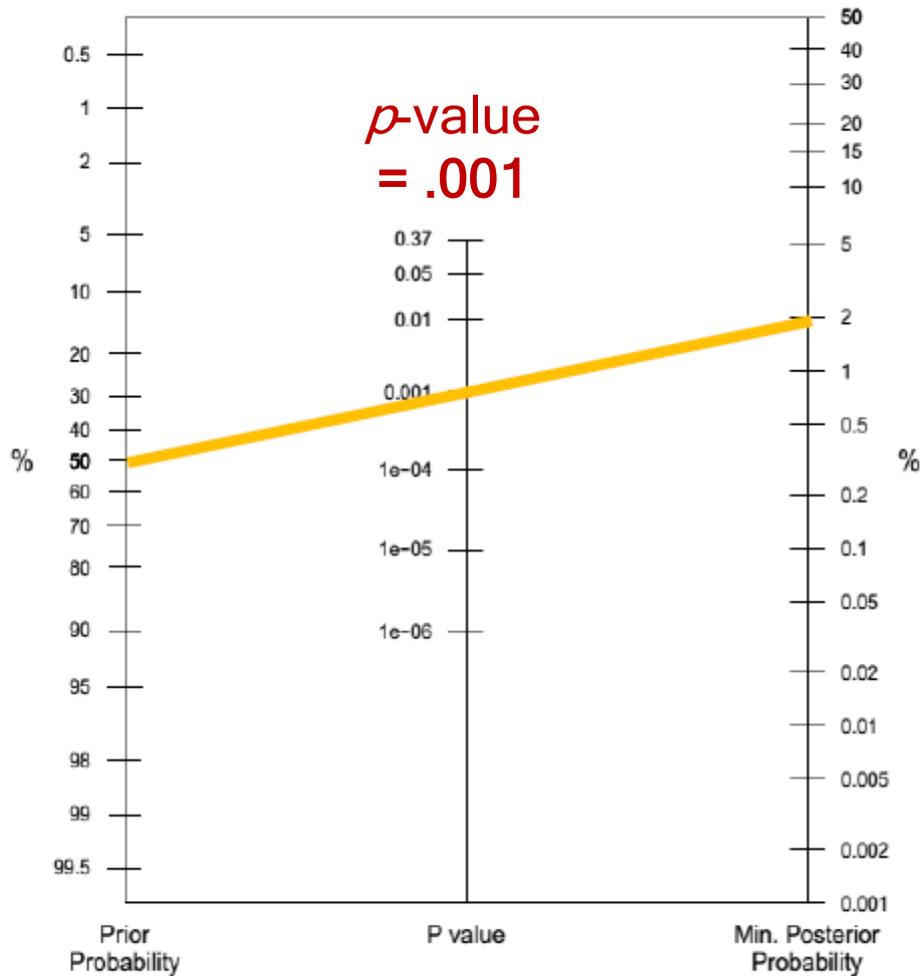
Prior belief  
( $H_0=$ True)  
50%



Posterior belief  
( $H_0=$ True)  
30%

# Bayesian THINKING

Prior belief  
( $H_0=$ True)  
50%



Posterior belief  
( $H_0=$ True)  
2%

# Bayesian THINKING

Taking prior probabilities into account is often smart thinking.

# REASON initiatives

Sarah Bichler &  
Arianne Herrera-Bennett  
(+ Daniel Sommerhoff &  
Ansgar Opitz)

The screenshot shows the OSFHOME interface. At the top, there is a dark header with the OSFHOME logo and a navigation menu. Below this is a light gray bar with 'Project Navigation' and another menu icon. The main content area has a 'Home' header with a house icon and a 'Toggle view: View Edit Compare' dropdown. On the left, a sidebar titled 'Project Wiki Pages' lists various meeting pages, with 'Home' selected. The main content area displays the title 'Bayesian Workshop: Crash course on Bayesian inference' and a section for 'Learning Objectives'. The objectives are listed as bullet points, each with a date and a brief description of the meeting's focus, along with suggested preparation.

OSFHOME

Project Navigation

Home

Toggle view: View Edit Compare

+ New

Project Wiki Pages

- Home
- 1st Meeting [25.09.2017]
- 2nd Meeting [24.10.2017]
- 3rd Meeting [28.11.2017]
- 4th Meeting [30.01.2018]
- 5th Meeting [TBD.05.2018]
- 6th Meeting [20.06.2018]
- 7th Meeting [XX.07.2018]
- Project Ideas & Collab
- Supplementary Read
- The Bayes Factor poc

View Wiki Version:  
(Current) Arianne Constance Herrera-Bennett: 2018-05-15 13:55

## Bayesian Workshop: *Crash course on Bayesian inference*

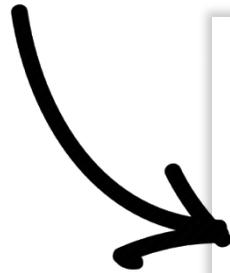
### Learning Objectives

Crash course is meant to serve as a workshop on Bayesian inference, at the introductory / beginner level. Meetings are geared toward anyone who is interested in gaining exposure to the basics of Bayesian inference.

- **1st Meeting objectives [25.09.2017]:** Learn the basics (likelihoods, priors, posterior distributions, Bayesian updating, Bayes' theorem). [Suggested preparation](#) (Arianne)
- **2nd Meeting objectives [24.10.2017]:** Updates on Bayesian events held in September. [Suggested preparation](#) (Karsten, Sarah, Arianne)
- **3rd Meeting objectives [28.11.2017]:** Intro to JASP software (basic analyses, example data set). [Suggested preparation](#) (Daniel)
- **4th Meeting objectives [30.01.2018]:** Cover the concept of sequential testing (NHST & Bayesian approach). [Suggested preparation](#) (Arianne)
- **5th Meeting objectives [01.06.2018]:** REFRESHER session! Review the basics (likelihoods, priors, posterior distributions, Bayesian updating, Bayes' theorem). [Suggested preparation](#) (Arianne)
- **6th Meeting objectives [20.06.2018]:** Bayesian priors: How are they determined? [Suggested preparation](#) (Arianne)
- **7th Meeting objectives [TBD.06.2018]:** Cover the concept of Bayesian credible intervals. [Suggested preparation](#) (Ansgar)
- **NO MEETING Aug 2018,** Bayesian events to resume in Sept. 2018.

<https://osf.io/hcm7p/wiki/home/>

# Upcoming Guest Talk & 2-day Workshop on Bayesian analysis



## Bayesian Events & News

### Upcoming

- **Guest Talk (24.09.2018, LMU, Munich):** Mark Andrews & Thom Baguley offer a talk on the general issues surrounding teaching Bayesian data analysis to social scientists, what they've learned from their experience, and how they think things will evolve in the future. [any & all welcome]
- **2-day Bayesian Workshop (25-26.09.2018, LMU, Munich):** Day 1 "*Bayes for beginners*" aims to be a general introduction to Bayesian data analysis and how it differs from the more familiar classical approaches to data analysis. Day 2 "*Doing Bayesian data analysis*" aims to provide a solid theoretical and practical foundation for real-world Bayesian data analysis in psychology and social sciences. [details about workshop capacity / sign-up to be announced shortly]

### Ongoing

- "*The Bayes Factor*": New podcast, interviewing the people behind Bayesian statistics and other hot methodological issues in psychological research, hosted by JP de Ruiter (@jpderuiter) & Alex Etz (@alxetz). First season interview lineup just announced [5.11.2017]: [click here to see!](#)
- *Episodes:* See "*The Bayes Factor podcast*" wiki tab for more info on all posted episodes.

### Past

- **Free Seminar (26.09.2017, Munich) 'Bayesian Networks: Artificial Intelligence for Research, Analytics, and Reasoning':** In this seminar, we illustrate how scientists in many fields of study - rather than only computer scientists - can employ Bayesian networks as a very practical form of A.I. for exploring complex problems.
- **ESRC funded conference workshop (29.09.2017, Nottingham) 'Bayesian Data Analysis in the Social Sciences Curriculum':** Conference considers how and why we should aim to bring Bayesian methods into the statistics curriculum in the social sciences.

## REASON initiatives

Sarah Bichler &  
Arianne Herrera-Bennett  
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<https://osf.io/hcm7p/wiki/home/>

# Upcoming **Guest Talk & 2-day Workshop** on Bayesian analysis

## **Guest Talk** (Monday, Sept. 24<sup>th</sup>, 2018)

Mark Andrews & Thom Baguley offer a talk on the general issues surrounding teaching Bayesian data analysis to social scientists, what they've learned from their experience, and how they think things will evolve in the future.

## **Workshop Day 1** (Tuesday, Sept. 25<sup>th</sup>, 2018) – *“Bayes for beginners”*

This workshop aims to be a general introduction to Bayesian data analysis and how it differs from the more familiar classical approaches to data analysis.

## **Workshop Day 2** (Wednesday, Sept. 26<sup>th</sup>, 2018) – *“Doing Bayesian data analysis”*

This workshop aims to provide a solid theoretical and practical foundation for real-world Bayesian data analysis in psychology and social sciences.

REASON  
initiatives

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<http://www.priorexposure.org.uk/>

# References

## **Contents borrowed from:**

McElreath, R. (2012). Rethinking: statistical Rethinking book package. *R package version, 1*.

Daniel Laken's Coursera MOOC: *Improving your statistical inferences*.