SEARCHING FOR 0 – 1 ASSIGNMENT

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ABSTRACT. We are interested in the complexity of a problem involving linear 0 – 1 assignments. We describe the relevant definitions for our problem in details.

1. NOTATIONS AND PROBLEM STATEMENT

First, it is convenient to delineate our problem on linear 0 – 1 assignments as follows.

Suppose we have a linear equation in n^2 variables with integer (negatives allowed) coefficients of at most m bits each.

Partition Π_1 the variables into n disjoint sets of n variables each.

Make another partition Π_2 the following way. Pick 1 variable in every disjoint set of variables in Π_1 form a new subset of variable of size n . Pick 1 variable in every disjoint set of variables in Π_1 not picked before to form a new subset of variable of size n . So we have another disjoint partition into n disjoint sets of n variables each.

Our problems are the following two similar ones.

Definition 1. (1a problem):

Is it NP-complete to decide if there is a valid (valid here means vanishing) 0/1 assignment with exactly n of assignments to be 1 with exactly 1 assignment per disjoint set?

Definition 2. (1b problem):

Is it NP-complete to decide if there is a valid (valid here means vanishing) 0/1 assignment that satisfies exactly n of assignments to be 1 with exactly 1 assignment per disjoint set Π_1 and with exactly 1 assignment per disjoint set Π_2 ?

We reduce subset sum to both of these.

2. REDUCING Subset Sum TO Problem 1a

Given a set of m positive integers $A = \{a_1, \dots, a_m\}$, and a positive integer B , the **Subset Sum** asks if there is a subset of $A' \subseteq A$ such that $\sum_{a_i \in A'} a_i = B$.

We simply pick $n = (m + 1)$ and build our set in this way:

- add $a_1, a_2, \dots, a_m$
- add m zeros
- add the only negative number $-B$
- "fill" the other remaining $n^2 - 2m - 1$ elements with $C = \sum_{a_i} a_i + B + 1$

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($\Rightarrow$) If the original subset sum has a solution A' then it is sufficient to use this partition Π_1

$$\begin{bmatrix} a_1 & C & C & \dots & C & 0 & C \\ 0 & a_2 & C & \dots & C & C & C \\ \dots & & & & & & \end{bmatrix}$$

$$\begin{bmatrix} C & C & \dots & 0 & a_m & C \\ C & C & \dots & C & C & -B \end{bmatrix}$$

and for each "row" pick a_i if $a_i \in A'$, 0 otherwise (and pick the final $-B$)

($\Leftarrow$) If there is a partition Π_1 with a vanishing 0/1 assignment then note that:

- (i) none of the C s can be picked (they are too large)
- (ii) at least one row must contain all elements different from 0 (because there are $m+1$ rows and only m zeros); so at least one nonzero element must be picked
- (iii) in every case the $-B$ element must be picked to vanish the sum

So the elements a_i picked, no matter how the elements are partitioned between the rows, sum exactly to B ; i.e. they are a valid solution for the original **Subset Sum** problem.

3. REDUCING TO **Problem 1b**

The ($\Leftarrow$) direction it uses only the constraints on Π_1 which are the same of case 1a (exactly 1 "pick" from each row).

The ($\Rightarrow$) direction is also still valid; intuitively, just build the main diagonal of Π_1 with the elements of $a_i \in A'$, the element $-B$, while the remaining elements of the diagonal are 0s.

For example if $m = 5$ and the solution of the subset sum is $A' = \{a_1, a_2, a_4\}$, Π_1 is:

$$\begin{bmatrix} a_1 & 0 & C & C & C & C \\ C & a_2 & 0 & C & C & C \\ C & C & 0 & a_3 & C & C \\ C & C & C & a_4 & 0 & C \\ C & C & C & C & 0 & a_5 \\ C & C & C & C & c & -B \end{bmatrix}$$

In this way, we can build Π_2 setting the element j of row i -th equal to element j of column i in Π_1 . The assignment clearly vanishes both Π_1 and Π_2 and the constraints are met.

4. CONCLUSION

Garey and Johnson [1] shapes their theory based on previous primal works of Cook, Levin and Karp. Johnson [2] moves on with the guide to this theory. As long as we study a mathematical conjecture, we should encourage ourselves of having enough labouring hours on popular maths books like these. Then, reading some articles on theory of computing like [3] is a good practice. Only after that, could we think of the ultimate final for all mathematics sciences.

REFERENCES

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