Parallelization of Karcsmarz Algorithm using Graphic Cards for Seismic Imaging

Muhamad Ilyas¹, Agah Drajat Garnadi¹ & Sri Nurdianti¹

Department of Mathematics, Bogor Agricultural University, Bogor,
Indonesia 16680

Email: reiken7@gmail.com, agah.garnadi@gmail.com, s.nurdianti@gmail.com

Abstract. We present a SCILAB implementation of algebraic iterative reconstruction methods for discretizations of inverse problems in seismic imaging. These so-called row action methods rely on semi-convergence for achieving the necessary regularization of the problem. We parallelize this method using NVIDIA graphics cards and SCILAB toolbox. We provide a few simplified test problems in seismic imaging to test those solver.

Keywords: *Algebraic reconstruction methods, Parallelization, Seismic imaging, Systems of linear equations.*

1 Introduction

Iterative reconstruction method for computing solutions to discretization of inverse problems have been used for decades in medical imaging, geophysics, material science and many other disciplines that involve 2D and 3D imaging [1]. This paper presents algebraic reconstruction techniques to solve large linear systems of the form

$$Ax \cong b, A \in \mathbb{R}^{m \times n}$$

used in tomography and many other inverse problems. We assume that the elements of matrix A are nonnegative and contains no zero rows or columns. There are no restriction on the dimensions.

We can write the system of linear equations with M equations with N unknown variables:

$$30 \quad \mathbf{a}_i^T \mathbf{x} = b_i, \quad i = 1, 2, \dots, M$$

31 with $\mathbf{a}_i$ and $\mathbf{x}$ are column vectors. Each of M equation will be called hyperplane
 32 in N-dimension Euclidean space, $\mathbb{R}^N$.

33 **2 Parallel Computing**

34 The idea of parallel computing is to share a calculation to some processing unit
 35 and calculate it in each processing unit independently. The restriction of this
 36 technique is the fully sequential method, for example a calculation that based on
 37 previous calculation. In this case, it's based on programmer judgment when and
 38 where parallel computing will be used.

39 Parallel computing used in this paper based on NVIDIA GPU. NVIDIA is well-
 40 known graphic cards manufacturer and with its most recent innovation, graphic
 41 processing unit (GPU) which can be accessed by CUDA (Compute Unified
 42 Device Architecture), programmer can easily use GPU to calculate some
 43 mathematical problems. Programming language used in CUDA are similar to
 44 C/C++ with some specific instruction to access the task to processing unit.

45 Parallel computing with CUDA could be used with sciGPGPU package, a
 46 toolbox in SCILAB. With this package, programmer can access CUDA
 47 commands along with SCILAB command, which makes calculation easier and
 48 more flexible.

49 **3 Algebraic Reconstruction Techniques**

50 The Algebraic Reconstruction Technique (ART) is one of the method
 51 used in commercial machine. This iterative method presented by S.
 52 Kaczmarz in 1937 in his paper "*Angenäherte auflösung von systemen*
 53 *linearer gleichungen*".

54 The core of ART method is the row-action method that treat the equation
 55 one at a time during the iterations. ART method find orthogonal
 56 projection $\mathbf{x}_p$ from $\mathbf{x}^*$ to the hyperplane $\mathbf{a}_k^T \mathbf{x} = b_k$ for $k = 1, 2, \dots, M$. For
 57 each iteration, $\mathbf{x}_p$ will convergent to the solution if the vectors $\mathbf{a}_1, \mathbf{a}_2, \dots,$
 58 $\mathbf{a}_M$ span $\mathbb{R}^N$. Therefore, in each iteration, we calculate the iterative
 59 vector:

$$\mathbf{x}_p = \mathbf{x}^* + \frac{\mathbf{b} - \mathbf{a}_k^T \mathbf{x}}{\mathbf{a}_k^T \mathbf{a}_k} \mathbf{a}_k$$

61 **4 Seismic Imaging**

62 Seismic tomography is a technique for imaging earth sub surface characteristics.
 63 We placed the sources on the right side of the area and the receivers on the left
 64 side and the surface of the area. The imaging burst a ray from each sources to
 65 every receivers on observation area. This ray pierces the observation area and
 66 received as a series of number. The output from the experiment is a series of
 67 numbers listed in vector form which is a value from each source and receiver
 68 pairwise. Our problem is we have to look for earth sub surface form and
 69 characteristic from given matrix and the location of ray sources and receivers.
 70 Below is the figure for the illustration of seismic tomography. In this
 71 images, green point indicates sources and blue point indicates receivers.

72

Figure 1 Illustration of seismic tomography.

We will also present the images of our seismic tomography model problem known as tectonic phantom model. We need this model as the value in x matrix for ART problems.

Figure 2 Illustration of Tectonic Phantom

Characteristics of matrix used in experiment is given below.

Table 1 MATRIX CHARACTERISTICS

	Seismic tomography
Size of Coefficient Matrix	5000 x 2500
Number of nonzero elements of Coefficient Matrix	288400 (2.31 %)
Maximum Value of Coefficient Matrix	1.4001
Minimum Value of Coefficient Matrix	0.0072
Maximum Value of RHS Vector	51.0157
Minimum Value of RHS Vector	0

86 These seismic tomography coefficient matrix is based on *AIR Tools – A*
 87 *MATLAB package of algebraic iterative reconstruction methods* [1]. By
 88 multiply the desired coefficient matrix with the seismic imaging model
 89 problem.

90 **5 Computational Result**

91 All the computational experiment in this paper is conducted in computer
 92 laboratory, Department of Mathematics, Faculty of Mathematics and Natural
 93 Sciences, Bogor Agricultural University. Computer specifications used are:
 94 Processor Intel Core i5-3317U 1.7 GHz, RAM 4 GB, VGA NVIDIA GeForce
 95 GT 635M. Experiment done in Ubuntu OS 10.10 with sciGPGPU 1.3, SCILAB
 96 5.4.0 and CUDA 4.0.

97 Algebraic reconstruction techniques (ART) produces an iterative solution with
 98 given coefficient matrix and right hand side vector. We reproduce approximate
 99 solution with increased iteration numbers by using original coefficient matrix
 100 and right hand side vector. Both tomography techniques produces different
 101 results below.

102 Additionally, we test the problems by giving some perturbation to the right hand
 103 side vector in the form of random error vector. With an additional vector, we
 104 also reproduce approximate solution with increased iteration numbers.

$$105 \quad \mathbf{b}_{new} = \mathbf{b} + \eta \frac{\|\mathbf{b}\|}{\|\mathbf{e}\|} \mathbf{e}$$

106 where $\mathbf{e}$ is a random error vector and η is some small number (in this
 107 experiment we use $\eta = 0.05$).

108 Below are approximate solution to the seismic tomography problems with
109 increased ART iterations, with original and perturbed right hand side vector.

110

111

1 ART iteration (original)	1 ART iteration (perturbed)
$\ Ax^* - b\ = 1002.028$	$\ Ax^* - b\ = 1016.642$
Time = 0.608s	Time = 0.749s

112

113

10 ART iteration (original)	10 ART iteration (perturbed)
$\ Ax^* - b\ = 54.019$	$\ Ax^* - b\ = 97.599$
Time = 4.898s	Time = 3.744s

114

115

100 ART iteration (original)	100 ART iteration (perturbed)
$\ Ax^* - b\ = 1.387$	$\ Ax^* - b\ = 79.016$
Time = 33.166s	Time = 37.331s

116

117

1000 ART iteration (original)	1000 ART iteration (perturbed)
$\ Ax^* - b\ = 0.1688$	$\ Ax^* - b\ = 77.796$
Time = 385.447s	Time = 386.243s

118 From above result for seismic tomography, by additional ART iterations,
119 approximate solutions is closer to exact solution, shown by smaller value of

120 $\|Ax^* - b\|$. This is also matched with approximation data resized to matrix
121 form and represented by scaled image.

122 On the other hand, perturbed right hand side also give approximate solution that
123 is closer to exact solution, shown by smaller value of $\|Ax^* - b\|$, even with
124 slower rate of convergence. The more ART iterations performed, approximation
125 data represented by scaled images is worse than small ART iterations. As we
126 could see that given some random normal perturbation matrix, the images is
127 blur and can't be recognized as a tectonic phantom model.

128 **6 Conclusion**

129 Seismic tomography are widely used in earth sub surface imaging. This
130 experiment shows that the parallelization of algebraic reconstruction techniques
131 to solve system of linear equations is successful. By adding more ART
132 iterations, approximate solutions is closer to exact solution shown by smaller
133 value of norm $\|Ax^* - b\|$, which is also matched with approximation data that
134 represented by scaled image.

135 With an additional error vector in hand right side vector, which we called
136 perturbed systems, the approximate solution is harder to converge, shown by the
137 value of norm $\|Ax^* - b\|$. In the end, even with increased ART iteration, the
138 scaled image produced by perturbed system become more blurred and can't be
139 recognized as a tectonic phantom model, even though time elapsed for solving
140 both equation is not significantly different.

141 **Acknowledgement**

142 This work was funded by the Ministry of Education and Culture, Republic of
143 Indonesia, through the BOPTN Fiscal Year 2013, with contract no:
144 232/IT3.41.2/L2/SPK/2013.

145 **7 Nomenclature (if necessary)**

146 List the nomenclature in alphabetical order. List Roman letters followed by
147 Greek symbols followed by subscript and superscripts.

A	=	matrices of size $m \times n$
x_i	=	vector of length n
b_i	=	vector of length m
K_i	=	modification factor
γ	=	wave number
ψ	=	Complex wave number

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149 **References**

- 150 [1] Hansen, P.C & Saxild-Hansen, M., *AIR Tools – A MATLAB package of*
151 *algebraic iterative reconstruction methods*. Journal of Computational and
152 Applied Mathematics, 2012.
153 [2] sciGPGPU QuickStart, <http://forge.scilab.org/index.php/p/sciCuda>.
154 [3] Grama, A., Gupta A., Karypis, G., & Kumar, V., *Introduction to Parallel*
155 *Computing*, London: Addison Wesley, 2003.
156 [4] Kaczmarz, S. 1937. Angenäherte Auflösung von Systemen linearer
157 Gleichungen, Bulletin de l'Académie Polonaise des Sciences et Lettres,
158 A35 (1937), pp. 355–357.

159 **8 Manuscript Content**

160 The contents of the paper should be in the following order:

- 161 1. Title of Paper
162 2. Author names and affiliation
163 3. Abstract
164 4. Body of the text (Introduction Conclusion)
165 5. Acknowledgements
166 6. Nomenclature
167 7. References
168