Incremental Risk Charge Model

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ABSTRACT

This paper presents a framework to model constant level of risk and incremental risk charge. The model simulates default, migration, and concentration in an integrated way. We compute the loss distribution at the first liquidity horizon. Then we generate loss distribution at the other liquidity horizon based on random draws following the constant level of risk assumption. It convolutes the copies of the single loss distribution to produce one year loss distribution. The aggregation of different portfolios with different liquidity horizons is addressed.

Keywords: Incremental risk charge (IRC), constant level of risk, liquidity horizon, constant loss distribution, concentration.

1 Introduction

The Basel Committee on Banking Supervision (see Basel [2009 a]) released the new guidelines for Incremental Risk Charge (IRC) that are part of the new rules developed in response to the financial crisis and is a key part of a series of regulatory enhancements being rolled out by regulators.

Incremental risk charge (IRC) is a new capital charge in addition to existing Value-at-Risk (VaR). It captures the loss due to default and migration events at a 99.9% confidence level over a one-year capital horizon. IRC is a key part of a series of regulatory enhancements being rolled out by regulators.

The liquidity of position is explicitly modeled in IRC through liquidity horizon and constant level of risk. The constant level of risk assumption in IRC reflects the view that securities and derivatives held in the trading book are generally more liquid than those in the banking book and may be rebalanced more frequently than once a year (see Aimone [2018]).

This constant level of risk assumption implies that a bank would rebalance, or rollover, its positions over the one-year capital horizon in a manner that maintains the initial risk level, as indicated by the profile of exposure by credit rating and concentration.

The current market risk capital rule is:

$$\begin{aligned} \text{Total market risk capital} &= \text{general market risk capital} \\ &+ \text{basic specific risk capital} \\ &+ \text{specific risk surcharge} \end{aligned} \tag{1}$$

where

$$\text{General market risk capital} = 3 \times \text{General_VaR}_{99\%}^{10\text{-day}}$$

$$\text{Basic specific risk capital} = 3 \times \text{Specific_VaR}_{99\%}^{10\text{-day}}$$

$$\text{Specific risk surcharge} = (m - 3) \times \text{Specific_VaR}_{99\%}^{10\text{-day}}$$

The new market risk capital standard will be:

$$\begin{aligned}
\text{Total market risk capital} &= \text{general market risk capital} \\
&+ \text{basic specific risk capital} \\
&+ \text{incremental risk charge}
\end{aligned} \tag{2}$$

where Incremental risk charge = $IRC - VaR_{99.9\%}^{1-year}$

In this paper, we present a methodology for calculating IRC. We first simulate default and migration. A simple approach to determine market data in response to default and credit migration is presented. Next, a methodology toward constant level of risk is described. The details of applying the constant level of risk assumption and aggregating different portfolios are addressed. Finally, the empirical and numerical results are presented.

2 Simulation of Default and Credit Migration

The IRC encompasses all positions subject to a capital charge for specific interest rate risk according to the internal models with exception of securitization and nth-to-default credit derivatives. Equity is optional. For IRC-covered positions, the IRC captures default risk and credit migration risk only.

2.1 Simulation Model

Most of the portfolio models of credit risk used in the banking industry is based on the conditional independence framework. In these models, defaults and credit migration of individual borrowers depend on a set of common systematic risk factors describing the state of the economy.

$$z_i = \beta_i \phi + \sqrt{1 - \beta_i^2} \varepsilon_i \tag{3}$$

where

ϕ, ε_i The independent standard normally random variables

ϕ The systematic risk

ε_i	The idiosyncratic risk for issuer/obligor i
β_i	The weighted correlation reflecting the impact of systematic risk factor on issuer/obligor i.
z_i	The normalized asset return or creditworthiness indicator for issuer/obligor i

This model becomes the most popular one in default and migration risk modeling and yields the core of the Basel II capital rule

This model assumes that the default and migration only happens at the end, which achieves significant simplification.

2.2 Simulation model for multiple-liquidity-horizon portfolios

Liquidity horizons are determined for each position to reflect actual practice and experience during periods of both systematic and idiosyncratic stresses. The total portfolio shall be divided into the portfolios based on different liquidity horizons. Let's assume that there are two portfolios with different liquidity horizons: 3 month and 6 months. To model different liquidity periods, one can use the above model (3) but calibrate different β_i 's, such as, β_{3m_i} and β_{6m_i} , for different periods.

Alternatively, one can also use a multiple-period model as:

$$z_{3m} = \beta_i \phi_{3m} + \sqrt{1 - \beta_i^2} \varepsilon_{3m_i} \quad \text{For 3 month} \quad (4)$$

$$z_{6m} = \beta_i \frac{\phi_{6m} + \gamma \cdot \phi_{3m}}{\sqrt{1 + \gamma^2}} + \sqrt{1 - \beta_i^2} \varepsilon_{6m_i} \quad \text{For 6 month} \quad (5)$$

where β_i is unique for different periods under issuer i and γ is an exponentially declining weight (see Dunn [2008]).

2.3 Calibration of β_i

The most popular approaches to calibrate the asset correlation are Maximum Likelihood Estimation or regression based on time series default data. Alternatively, in the new Basel Capital Accord, a formula for derivation of risk weighted asset correlation for corporate, sovereign, and bank exposures is given as:

$$\beta_i = 0.12 \times \lambda_i + 0.24 \times (1 - \lambda_i) \quad (6)$$

Where $\lambda_i = \frac{1 - e^{-50 \times PD_i}}{1 - e^{-50}}$

2.4 Calibration of ρ_i

The calibration is based on credit migration matrix. It can be derived using either analytic closed-form or Monte-Carlo simulation. In theory, one can use Pearson's product moment or Kendall's τ .

2.5 Determination of default and credit migration

The simulated asset return z_i , combined with migration/default thresholds, is used to ascertain when default or migration is deemed to occur. The calculation of the thresholds of credit migration and default is based on credit migration probability (see JP Morgan [1997]). Using a BBB issuer as an example and given migration matrix, we can calculate the thresholds as:

$z_D^{BBB}, z_{CCC}^{BBB}, z_B^{BBB}, z_{BB}^{BBB}, z_{BBB}^{BBB}, z_A^{BBB}, z_{AA}^{BBB}$. The rating bands and thresholds are shown in Figure 1

Figure 1 Credit migration rating thresholds (for BBB)

If the normalized asset of the issuer is smaller than z_D^{BBB} , it defaults. If the normalized asset is between z_D^{BBB} and z_{CCC}^{BBB} , it migrates to CCC, and so on. We use an effective middle value to represent each band:

$$\begin{aligned}
 u_D^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_D^{BBB}) + 0)) \right] \\
 u_{CCC}^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_{CCC}^{BBB}) + \Phi(z_D^{BBB}))) \right] \\
 u_B^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_{CCC}^{BBB}) + \Phi(z_B^{BBB}))) \right] \\
 u_{BB}^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_B^{BBB}) + \Phi(z_{BB}^{BBB}))) \right] \\
 u_{BBB}^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_{BB}^{BBB}) + \Phi(z_{BBB}^{BBB}))) \right] \\
 u_A^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_{BBB}^{BBB}) + \Phi(z_A^{BBB}))) \right] \\
 u_{AA}^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_A^{BBB}) + \Phi(z_{AA}^{BBB}))) \right] \\
 u_{AAA}^{BBB} &= \Phi^{-1} \left[\frac{1}{2} ((\Phi(z_{AA}^{BBB}) + 1)) \right]
 \end{aligned} \tag{8}$$

2.6 Calibration of transition matrix, default probability (PD), and loss given default (LGD)

The straight forward cohort approach is used to estimate transition matrices based on obligors' rating history, which has become the industry standard. The PD estimate is based on EDF data that is used for calculation of PD benchmarked against internal default history. Internal data is used for LGD parameter benchmarked against relevant external proxy data.

3 Credit Spreads

After simulating default and migration of all issuers/obligors, we need to price every instrument in order to generate loss distributions. The question is whether we should simulate market data or not?

The earlier version of Basel IRC paper (see Basel [2008]) requires financial institutes to capture four risks: default, credit migration, significant credit spread changes, and significant equity price changes. However, the new guideline (see Basel [2009 a]) limits the risks to default and credit migration only. In addition, a separate Basel paper (see Basel [2009 b]) further states that IRC contains only incremental default and migration risks, and all price risks belong to the comprehensive risk. These messages give us a clear indication that only default and credit migration are risk factors in IRC and all market prices/data are not. Therefore, we recommend simulating default and migration only but not simulating any market prices/data.

We assume all market prices/data are deterministic (flat) and use forward prices/data for valuation. The fat tail behavior and market correlations are embedded in the market. Keeping these parameters constant ensures we measure only P&L variation due to credit rating changes (migration or default) per IRC requirements. The selection of credit spreads or equity prices, however, should reflect the credit quality changes.

All issuers/obligors shall be divided into credit groups based on geographies and sectors. Assume that the credit spreads for different ratings under each group are available. Then we can select associated credit spreads to price a bond or a CDS according to the creditworthiness simulation of the issuer/obligor.

4 Constant Level of Risk

The constant level of risk reflects recognition by regulators that securities/derivatives held in the trading book are generally much more liquid than those in the banking book, where a buy-and-hold assumption over one year may be reasonable. It implies that IRC should be modeled under

the assumption that banks rebalance their portfolio several times over the capital horizon in order to maintain a constant risk profile as market conditions evolve. Of course, we do not suggest that the constant level of risk framework be taken literally as a model of banks' behavior: clearly portfolios are altered on a daily basis, not simply held constant for some period then instantaneously rebalanced. Rather, we regard the rollover interpretation as being a **reasonable approximation** to the way banks manage their trading portfolios over a certain horizon. In general, one should model constant level of risk instead of constant portfolio over one year capital horizon.

There are several ways to interpret constant level of risk: constant loss distribution or constant risk metrics (e.g. VaR). We believe the constant loss distribution assumption is the most rigorous. Under this assumption, the same metrics (e.g. VaR, moments, etc.) can be achieved for each liquidity horizon.

The liquidity horizon for a position or set of positions has a floor of **three months**. Let us use three months as an example. We interpret constant level of risk to mean that the bank holds its portfolio constant for the liquidity horizon, then rebalances by selling any default, downgraded, or upgraded positions and replaces them so that the portfolio is returned to the level of risk it had at the beginning. The process is repeated 4 times over the capital horizon resulting 4 independent and identical loss distributions. The one year constant level of risk loss distribution is the convolution of 4 copies of the three month loss distribution. **In Monte Carlo context, this can be modeled by drawing 4 times from the single period loss distribution measured over the liquidity horizon.**

The total PnL is the summary of these 4 random draws.

Figure 2 Constant level of risk

In addition, one needs to consider the reinvestment of all cash flows realized during the liquidity horizon and rollover of expired deals.

5 Aggregation and Time Horizon Correlation

First we need to divide the portfolio into the portfolios based on liquidity horizons. If there is only one single-liquidity-horizon portfolio, the rebalance at the end of each liquidity horizon washes out the time horizon correlation. However, if there are multiple portfolios, the time horizon correlations need to be addressed.

To elaborate the details, we assume there are two portfolios with liquidity horizons: 3 months and 6 months. Based on the default and migration simulation and full re-valuation, we can generate loss distributions at first liquidity horizons for 3-month and 6-month portfolios as PL_{3m} , and PL_{6m} .

There are two approaches to achieve the correlative aggregation: copula approach or correlation matrix approach.

We conduct the second Monte Carlo simulation by generate 4 standard normal random draws for scenario j : $x_1^j, x_2^j, x_3^j, x_4^j$. These random draws represent a Monte-Carlo path.

5.1 Correlation matrix approach

Based on the four 3-month independent identical loss distributions: $PL_{3m}, PL_{3m}, PL_{3m}, PL_{3m}$, and two 6-month independent identical loss distributions: PL_{6m}, PL_{6m} , we can construct a 6×6 pair-wise sample correlation matrix Σ . Applying the Cholesky decomposition to the correlation matrix Σ , we have $\Sigma = LL^T$, where L is a lower triangular matrix.

We conduct the second Monte Carlo simulation by generating 4 independent standard normal random draws: $x_1^j, x_2^j, x_3^j, x_4^j$ for the four 3-month periods in a year and 2 independent standard normal random draws x_5^j, x_6^j for the two 6-month periods to construct a path/scenario j . The random draw vector is $X = [x_1^j \ x_2^j \ x_3^j \ x_4^j \ x_5^j \ x_6^j]$. We can obtain correlative random draw vector

$$\tilde{X} = [\tilde{x}_1^j \ \tilde{x}_2^j \ \tilde{x}_3^j \ \tilde{x}_4^j \ \tilde{x}_5^j \ \tilde{x}_6^j] \text{ by } \tilde{X}^T = L \times X^T \quad (21)$$

The total P&L for scenario j is

$$PL_{total}^j = PL_{total_3m}^j + PL_{total_6m}^j = \sum_{i=1}^4 PL_{3m}(\Phi(\tilde{x}_i^j)) + \sum_{i=5}^6 PL_{6m}(\Phi(\tilde{x}_i^j)) \quad (22)$$

The final IRC will be 99.9% VaR based on distribution PL_{total}^j . In general, the correlation matrix approach is more generic and can be easily extended to any number of portfolios.

6 Numerical and Empirical Results

The above methodology has been implemented. The empirical study shows the results on P&L distributions, numerical stability & convergence, concentration effect, and capital impact.

6.1 Convergence study

People normally believe that 50,000 simulations provide sufficient stability to measure the 99.9th percentile loss required for the regulatory IRC measure. However, our study shows that

50,000 paths are not convergent. Actually 100,000 simulations are needed to archive a better numerical stability and convergence. **Concentration study**

The purpose of this section is to demonstrate that the model (7) can reflect issuer and market concentrations. To simplify our tests, we assign all issuers with the same concentration factor ρ .

Capital impact

The capital impact can be measured as the ratio between IRC and specific risk surcharge. The results significantly depend on the composition of a portfolio and the specific risk multiplier of a financial institution set by the regulator. The ratio of our testing portfolio is 5.8.

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