Proof of the Goldbach's Conjecture

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Abstract

Goldbach's Conjecture states that every even number greater than 3, can be written as a summation of two prime numbers. This conjecture is roughly 300 years old and a very famous unsolved mathematics problem.

To prove the Goldbach's Conjecture, I use the contradiction method in mathematics as below.

Keywords

prime; contradiction; summation ; greater than ; integer ; value

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Acknowledgement

I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

Author's Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of the Goldbach's Conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee is 32 years old and Geerasee graduated with BSc (Hons) in Physical Science from the University of Colombo, Sri Lanka in 2014. And now she is following her PhD in Astrophysics/Cosmology.

Geerasee has been received invitations from several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, Geerasee has already received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Department of Mathematics, Faculty of Engineering, University of Moratuwa, Sri Lanka. Now she is a research scientist in Physics as her career. Furthermore she has achieved several other scientific achievements already.

Introduction

Goldbach's Conjecture states that every even number greater than 3, can be written as a summation of two prime numbers. This conjecture is roughly 300 years old and a very famous unsolved mathematics problem.

To prove the Goldbach's Conjecture, I use the contradiction method in mathematics.

A prime number is a natural number which divides only by 1 and it self. For an example, the number 2 divides by 1 and 2 only. That means 2 is divisible by 1 and itself only. But 3 is also a prime number. Because 3 divides by 1 and itself only. But 6 is not a prime number. Because 6 is divisible by 1, 2, 3 and itself. Therefore 6 is not a prime number.

And even number is a natural number which is divisible by 2.

If we use the contradiction method in some mathematical result, we initially assume the negation of the requited result. Then I proceed with the mathematical steps with the usage of the well

verified mathematical results. Also I should use the assumption that I assumed initially. Then after series of mathematical steps, I will ended up with a contradiction. But I know that in order to get the final contradiction, I didn't use any invalid mathematical result except our initial assumption. Therefore I can conclude that the only issue is our initial assumption. Therefore it is possible to conclude that our initial assumption is wrong. Therefore mathematically I can conclude that the negation of our initial assumption is correct. Because in mathematics, we use only the propositions. That means I can say we got the required result finally. That's how mathematical contradiction method works.

This research paper will use that contradiction method to prove that every even natural number greater than 3 can be written as a summation of two prime numbers.

Literature Review

The question whether every even natural number greater than 3, can be written as a summation of two prime numbers is an unsolved mathematics problem in number theory since roughly 300 years in the mathematics world.

On 1742 June 07th, a very famous German mathematician Christian Goldbach wrote a letter by proposing that 'Every even natural number greater than 3 can be written as the summation of two prime numbers'. That is the general version of the Goldbach's Conjecture. But there are few other versions of Goldbach Conjecture proposed by other several mathematicians.

But with this paper I am intending to prove the general version of the Goldbach's Conjecture.

Lots of mathematicians around the world were trying to prove this conjecture since hundreds years, very hardly.

But mathematician Olivier Ramare proved in 1995 that every even number greater than or equals to 4 is in fact the sum of at most 6 primes. But later it is proven that every even number greater than or equal to 4, is the sum of at most 4 primes.

But in 1973, Chen Jingrun showed that every sufficiently large even number can be written as the sum of either two prime numbers or a prime and a semi-prime. Semi-prime is the product of two primes.

In 1951, Yuri Linnik showed that the existence of a constant K such that every sufficiently large even number is the sum of two primes and at most K powers of 2. But Roger Heath-Brown and Jan-Christoph Schlage-Puchta proved that in 2002 that $K = 13$ works very well.

Although, there are a number of proposed proofs for Goldbach Conjecture already, none of them have accepted by the Mathematical community yet.

Assumption

Let's assume that there exists at least one even number in the set $\{ (2a) \}$; where $2a$ is greater than or equal to 4 (a is a natural number) such that each $2a$ in the set $\{ (2a) \}$ cannot be written as an addition of two prime numbers.....(1)

Proposed methodology

Here in the methodology, I consider some integers with some special properties as below. And after that, I use our above assumption (1) and apply the assumption (1) to those integer numbers with the chosen special properties. Then this will try to get a contradiction. But to get the contradiction, this does not use any unverified mathematical theories. Therefore if this is capable to get a contradiction finally, it is possible to conclude that ‘the only issue is our initial assumption’ (1). Since our assumption (1) is a proposition, if so, it is possible to conclude that the negation of assumption (1) is true. That’s how this mathematics proof will be working.

Let’s consider some even number $2a$ in the set $\{ (2a) \}$. But since $2a$ is a natural number, we can write $2a$ as below:

$$2a = P_N + Q \dots\dots\dots(2)$$

Where P_N is a positive prime number such that $(P_N + 2)$ is also a prime number.

There exists infinitely many twin prime numbers as I have already proven through my previous WORLD RECOGNIZED research paper. Then for each $(2a)$, there exists such a prime number P_N such that $(P_N + 2)$ is also a prime number.

By considering (1) and (2): Q is not a prime number. But Q is a natural number. Then Q can be written as a multiplication of prime numbers. Because each natural number (which is not a prime number) can be written as a multiplication of prime numbers.

Then $Q = \prod_{i=1}^n P_i$ where n is a finite natural number. Where each P_i is a prime number.

$$i \in \{1, 2, 3, \dots\dots n\}$$

$$\text{Then by (2): } 2a = P_N + \prod_{i=1}^n P_i \dots\dots\dots(3)$$

Here P_k and P_{k+1} may be/may not be consecutive prime numbers for some $k \in \{1, 2, 3, \dots\dots n\}$.

Here may be $P_i = P_{i+1}$ or may not be for some $i \in \{1, 2, 3, \dots\dots n\}$.

$$\text{By (3): } 2a = (P_N + 2) + \left(\prod_{i=1}^n P_i - 2 \right) \dots\dots\dots(4)$$

By (4): $2a - (P_N + 2) = \left(\prod_{i=1}^n P_i - 2 \right) = \prod_{i=1}^n P_i \left(1 - \frac{2}{\prod_{i=1}^n P_i} \right)$

Then $1 / \left[\prod_{i=1}^n P_i \left(1 - \frac{2}{\prod_{i=1}^n P_i} \right) \right] = 1 / [2a - (P_N + 2)] \dots\dots\dots(5)$

Since $\prod_{i=1}^n P_i$ is not a prime, $\prod_{i=1}^n P_i > 2$. Then $\frac{2}{\prod_{i=1}^n P_i} < 1$.

Then $1 / \left[\prod_{i=1}^n P_i \left(1 - \frac{2}{\prod_{i=1}^n P_i} \right) \right] = \frac{1}{\prod_{i=1}^n P_i} * \left\{ 1 + \frac{2}{\prod_{i=1}^n P_i} + \left(\frac{2}{\prod_{i=1}^n P_i} \right)^2 \frac{1}{2!} + \left(\frac{2}{\prod_{i=1}^n P_i} \right)^3 \frac{1}{3!} + \dots \right\}$
 $= \sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \dots\dots\dots(6)$

By (5) and (6): $[2a - (P_N + 2)] = 1 / \left[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \right] \dots\dots\dots(7)$

Since $(P_N + 2)$ is a prime, by (1): $[2a - (P_N + 2)]$ is not a prime.

Thus $[2a - (P_N + 2)] = \prod_{j=1}^m P_j$; where P_j is a prime number.

Here P_k and P_{k+1} may be/may not be consecutive prime numbers for some $k \in \{1, 2, 3, \dots, m\}$.

Here may be $P_j = P_{j+1}$ or may not be for some $j \in \{1, 2, 3, \dots, m\}$.

Then by (7): $\prod_{j=1}^m P_j = [2a - (P_N + 2)] = 1 / \left[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \right]$

Then $\prod_{j=1}^m P_j = 1 / \left[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \right] \dots\dots\dots(8)$

But in the series $\left[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \right]$, $\left\{ 2 / [(l+1) \cdot \prod_{i=1}^n P_i] \right\}$ is the multiplication factor.

But since $\prod_{i=1}^n P_i$ is not a prime, $\prod_{i=1}^n P_i > 2$. Then $\frac{2}{\prod_{i=1}^n P_i} < 1$. Since $l \geq 0$, $(l+1) \geq 1$.

Thus the multiplication factor of $\left[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \right]$, which is $\left\{ 2 / [(l+1) \cdot \prod_{i=1}^n P_i] \right\}$ is less than 1.

Then $s = \left\{ 2 / [(l+1) \cdot \prod_{i=1}^n P_i] \right\} < 1$. But in the series $\left[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}} \right]$, the first element which is denoted by $b = 1 / [\prod_{i=1}^n P_i]$.

Then by the theories in mathematics: $[\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}}] = b / (1 - s)$; since $s < 1$.

$$\text{Then } [\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}}] = b / (1 - s)$$

$$= \lim_{l \rightarrow \infty} 1 / [\prod_{i=1}^n P_i] / [1 - \{2 / [(l+1) \cdot \prod_{i=1}^n P_i]\}]$$

$$= \{1 / [\prod_{i=1}^n P_i]\} \cdot \{1 / (1 - [2 / \infty])\}$$

$$= 1 / [\prod_{i=1}^n P_i]$$

$$\text{Then } [\sum_{l=0}^{\infty} \frac{2^l}{l! \cdot (\prod_{i=1}^n P_i)^{l+1}}] = 1 / [\prod_{i=1}^n P_i] \dots\dots\dots(9)$$

$$\text{By (8) and (9): } \prod_{j=1}^m P_j = 1 / [1 / [\prod_{i=1}^n P_i]] = \prod_{i=1}^n P_i$$

$$\text{Then } \prod_{i=1}^n P_i = \prod_{j=1}^m P_j \dots\dots\dots(10)$$

$$\text{But } \prod_{j=1}^m P_j = [2a - (P_N + 2)] .$$

$$\text{And by (4): } 2a = (P_N + 2) + (\prod_{i=1}^n P_i - 2) .$$

$$\text{Thus } \prod_{j=1}^m P_j = (\prod_{i=1}^n P_i - 2) \dots\dots\dots(11)$$

$$\text{Thus by (10) and (11): We get a contradiction} \dots\dots\dots(12)$$

But to get the above contradiction as in (12), I used our initial assumption (1) and other well-verified mathematical theories. Therefore it is possible to conclude that our initial assumption (1) is not true. But our initial assumption (1) is a proposition. Therefore mathematically, it is possible to conclude that the negation of our initial assumption of (1) is true.

But the negation of our initial assumption is:

For all even numbers (2a), (2a) can be written as an addition of two prime numbers. Where (2a) is greater than or equal to 4 (a is a natural number)

Thus every even number greater than 3 can be written as an addition of two prime numbers.

Thus the Goldbach's Conjecture is proved !

Results

Therefore I have used our assumption (1) to get the contradiction finally, as showed in (12). Therefore it is possible to conclude that our assumption (1) is false. But we know that our assumption (1) is a proposition. Thus it is possible to say that the negation of the assumption (1) is true. But the negation of our assumption (1) is:

Every even natural number greater than 3 can be written as a summation of two prime numbers.

Conclusion

We assumed initially that there exists at least one even number (which is greater than 3) which cannot be written as a summation of two prime numbers. After proceeding with that, I finally received a contradiction. But in order to get the contradiction, I used our initial assumption (1) in the mathematical steps. But rather than initial assumption (1), I didn't use any other un-verified mathematical theories and steps. Therefore to get the contradiction as in (12), I have used the facts got from our assumption (1). Then the only possibility is our assumption (1) is false. Since our assumption (1) is a proposition, it is possible to state that the negation of our assumption (1) is true. But the negation of our assumption (1) is: every even number greater than 3 can be written as a summation of two prime numbers'.

Appendix

Prime number: A natural number which is divisible by 1 and itself only.

Even number is a (natural) number which is divisible by 2.

If S_n is the summation of a geometric series, $S_\infty = a / (1 - r)$; if the multiplication ratio (r) of the series is less than 1. Where 'a' is the first term of the series S_∞ .

If $x < 1$ and if x is a real number : $1 / (1 - x) = \sum_{i=0}^{\infty} \frac{x^i}{i!}$

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