Off key or off beat? Searching for common ground between tonality and meter

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Introduction

Much of the appeal of music is due to its work with tension (Lehne & Koelsch, 2014). To regulate tension in listeners, musicians often exploit two cognitive phenomena: tonality and meter. The former underlies the well known tendency of the leading tone to resolve to the tonic (Krumhansl, 2004) and the latter forms the basis of syncopation (Fitch, 2013). Even though tonality and meter are conveyed by distinct musical features – pitch and rhythm, respectively – they share many similarities. For example, from the music-theoretical point of view, both pitch and rhythm are hierarchical in nature (Lerdahl, 2001): In the same way as strong beats are subdivided by half-weak beats, which, in turn, are subdivided by weak beats, a tonic triad is subdivided by a diatonic scale, which, in turn, is subdivided by the chromatic scale. Furthermore, the same model of a neural resonator (Large, Almonte, & Velasco, 2010) was proposed as a basis for explaining both phenomena (Large, 2010; Large, Herrera, & Velasco, 2015; Large, Kim, Flaig, Bharucha, Krumhansl, & Lynne, 2016), the only difference being the time constant, which has to match the widely different time scales of pitch and rhythm.¹ Even though the model does not perform particularly well in comparison to other models of tonality (Milne, Laney, & Sharp, 2015), this is probably due to the simplifications made on the way from the full neurodynamical model to the simple formula for predicting tonal stability rather

¹ The model relates preferences for simple frequency ratios in pitch and rhythm to generic properties of oscillatory systems which can be described in terms of frequency ratios and their simplicity.

than due to shortcomings of the model itself (Large et al., 2016). On the other hand, the model is attractive because it is based on a generic mechanism (i.e., Hopf bifurcation) which is actually at work in neural circuitry (Prescott, De Koninck, & Sejnowski, 2008; Prescott, Ratté, et al., 2008). If tonality actually relies on such a generic time-scale invariant mechanism, one could ask whether it pertains to rhythms, too. To the best of our knowledge, such a question has not been addressed experimentally, yet. However, the idea of relating rhythm to pitch is by no means new: For instance, some analyses of contemporary music draw a direct analogy between a distance between two beats (duration) and that between two pitches (interval; Roeder, 2003; Colannino et al. 2009).

In our study, we explore whether the ranking of pitches according to their fit to a tonal context (e.g., a major/minor chord or a major/minor scale) generalizes to isochronous rhythms presented in an analogous context. For example, consider two pitches, G and B, and the C major triad as their tonal context. Previous studies (e.g., Krumhansl & Kessler 1982) have shown that the G pitch is judged to fit better to the C major context than the B pitch. In the present study, we explore if the ranking of the two pitches is preserved if we transform each constituent pitch of the triad as well as the G and B pitches to isochronous rhythms by scaling all frequencies down by the same constant (thus preserving their frequency ratios; see Figure 1).

Figure 1: Transformation of the C major context chord together with the B pitch to rhythms. For each stage of the transformation, the context tones/rhythms are shown in black and the probe tone/rhythm in red. The waveform resulting from the superposition of the constituent tones/rhythms in each stage is shown at the bottom.

The present study adopts a method used in so-called probe-tone studies, which provide the most thorough account of the phenomenon of tonality to date (Krumhansl 1990), and adjusts it to rhythms. In a typical probe-tone study, participants are presented a *context stimulus* (e.g., a chord, scale, or chord progression) followed by a *probe tone* and asked how well the latter fits into the former. However, it is not possible to replicate the design of these studies directly with all frequencies transformed to rhythms. Transformation of each constituent frequency of even a

single complex tone to a rhythm results in a complicated polyrhythm rare in Western music practice and the polyrhythms resulting from transformation of chords, which consist of multiple complex tones, are even more complicated. Consequently, whereas in the original probe-tone studies each pitch was rendered as an octave-complex tone, in our study, each pitch is rendered as a single isochronous rhythm – more precisely, a train of metronome ticks. Additionally, to make the task as simple as possible for our subjects, we have opted for the two-alternative forced choice paradigm. Specifically, we asked participants to choose between two polyrhythms which differed in a single constituent isochronous rhythm (*probe rhythm*) and shared a set of three isochronous rhythms corresponding to a major triad (*context polyrhythm*). We hypothesized that participants would prefer the polyrhythm which contains a probe rhythm corresponding to a pitch with a higher preference rating within the major key profile from Krumhansl and Kessler (1982).

Methods²

Participants

We recruited 204 participants³ (68% female, 79% students) from our subject pool. The experiment was administered on a computer within a larger set of experiments. The participants received 100 CZK (~4USD) for their participation in the whole set of experiments. The experiment was administered in a lab in groups of up to 16 participants.

Stimuli

The stimuli had a form of short samples (8-16 s) of metronome ticks. Each stimulus was created as a superposition of four isochronous rhythms (see Figures 1 and 2): three of them, constituting the context polyrhythm, had frequencies corresponding to the tonic, mediant, and dominant in a major key (e.g., C, E, and G in C major). The fourth rhythm (the probe rhythm) had a frequency corresponding to one of the twelve scale degrees of the chromatic scale. Frequencies of all probe rhythms were located within an octave above the corresponding tonic rhythm. The probe rhythms corresponding to the first, third, and fifth scale degrees (the tonic, mediant, and dominant, respectively) shared the same stimulus because the probe rhythms were already present within the context polyrhythm. Therefore, we included the shared stimulus only once, resulting in ten stimuli for each context polyrhythm. Note that by design, this polyrhythm only consisted of three isochronous rhythms.

² Pre-registration as well as data, materials, and analysis scripts can be found on osf.io/b9xpf/.

³ A slight discrepancy between the actual number of participants and the 200 participants we have in our pre-registered protocol is because the participants were recruited in groups and the last group exceeded the planned 200 participants.

Figure 2: Some of the polyrhythms used in the experiment. Context rhythms are shown in black and probe rhythms in red. Each tick is connected to the closest tick in the rhythm immediately below with a curve whose opacity is inversely proportional to the distance. This way, the points of synchronization are easily discerned as black vertical lines. Note that in contrast to polyrhythms in Farey tuning, there are no vertical lines in polyrhythms in the 12TET tuning, except for the onset. However, the points of synchronization from the Farey tuning can still be discerned in the 12TET tuning. Also note that as we move down towards the less preferred polyrhythms in our experiment, there are fewer points of synchronization (the frequency ratios are more complex).

Each polyrhythm was produced in two different tunings: the standard Western twelve-tone equal temperament (12TET) and a just tuning (Farey; Large et al., 2016).⁴ Additionally, each polyrhythm appeared in the stimulus set in four different absolute frequencies of the constituent rhythms. The variation of frequencies and tunings was used to minimize a possible influence of familiarity, i.e., the mere-exposure effect caused by previously presented stimuli (Zajonc, 1968). The duration of each polyrhythm was limited to 16 beats of the tonic rhythm.

The stimuli were prepared using temporal-music-notation (Version 0.4.1; Kholomiov, 2015), csound-expression (Version 4.9.1; Kholomiov, 2015), and csound-catalog (Version 0.4.0; Kholomiov, 2015) packages for the Haskell programming language and synthesized with the Csound (Version 6.06.0 for OS X; 2015) compiler. Each polyrhythm was rendered as a train of metronome ticks and written to both channels of a stereo wav file.

⁴ The tuning approximates each irrational frequency ratio of the 12TET tuning with the simplest rational one such that the error is below 1%.

Procedure

The whole experiment was administered on a computer using a custom written Python program. At the beginning of the experiment, participants were asked to put on headphones and adjust the volume on their headphones using a calibration rhythm not included in the test stimuli. Afterwards, they proceeded to the experiment which consisted of 8 trials.

During each trial, participants were presented two polyrhythms in succession and asked to choose which of the two polyrhythms they liked more. Each participant was given all combinations of the four absolute frequencies of the tonic rhythm and the two tunings in a random order, resulting in the total of 8 trials. For each combination of the absolute frequency and tuning – that is, in each trial – two random probe rhythms were selected. After hearing both polyrhythms, participants indicated their preference and proceeded to the next trial.

Results

Confirmatory analysis

To compare the relative preferences of probe rhythms, we conducted a logistic regression where the choice of the first presented polyrhythm served as a binary dependent variable. For each probe rhythm, we created a dummy variable which served as a predictor in the regression. The dummy variable was coded as: 1 if the probe rhythm was presented first, -1 if the probe rhythm was presented second, and 0 if the probe rhythm was not in the pair of the presented polyrhythms. Due to the collinearity of predictors coded this way, we had to select one probe rhythm as a baseline condition and the reported estimates are therefore always in comparison to this baseline probe rhythm. In this analysis, we assumed that all probe rhythms corresponding to a given scale degree would influence preferences similarly, independently of tuning and absolute frequencies. Therefore, we included in the analysis all data from the eight combinations of absolute frequencies and tunings. The 8 observations from each participant were treated as independent, given that it was not possible to estimate participant effects because all pairs of polyrhythms differed in their context polyrhythm and each participant was given each probe rhythm only a small number of times. A different analysis where the participant effect could be estimated is reported in Exploratory analysis section below. The results of the analysis are shown in Figure 3 for each possible baseline probe rhythm (in columns). It is possible to see that the C⁵ probe rhythm⁶ was preferred to all other

⁵ From now on, we will designate scale degrees by their labels in C major (e.g., C and F will stand for the tonic and the subdominant, respectively).

probe rhythms. Additionally, the D probe rhythm was significantly preferred to the C#, D#, F, G#, and A# probe rhythms. No other effects were significant.

	C	C#	D	D#	F	F#	G#	A	A#	B
C		2.93 [2.14, 4.03]	2.07 [1.51, 2.85]	2.81 [2.06, 3.85]	2.87 [2.10, 3.94]	2.37 [1.72, 3.28]	2.89 [2.09, 4.02]	2.57 [1.89, 3.53]	2.82 [2.07, 3.88]	2.27 [1.66, 3.11]
C#	0.34 [0.25, 0.47]		0.71 [0.53, 0.95]	0.96 [0.72, 1.28]	0.98 [0.73, 1.31]	0.81 [0.60, 1.08]	0.99 [0.73, 1.33]	0.88 [0.66, 1.17]	0.96 [0.72, 1.29]	0.77 [0.58, 1.04]
D	0.48 [0.35, 0.66]	1.41 [1.05, 1.90]		1.35 [1.01, 1.83]	1.38 [1.02, 1.87]	1.14 [0.85, 1.54]	1.39 [1.02, 1.90]	1.24 [0.93, 1.66]	1.36 [1.01, 1.84]	1.09 [0.81, 1.48]
D#	0.36 [0.26, 0.49]	1.04 [0.78, 1.40]	0.74 [0.55, 0.99]		1.02 [0.76, 1.37]	0.84 [0.63, 1.13]	1.03 [0.76, 1.39]	0.92 [0.69, 1.22]	1.00 [0.75, 1.34]	0.81 [0.60, 1.09]
F	0.35 [0.25, 0.48]	1.02 [0.76, 1.37]	0.72 [0.53, 0.98]	0.98 [0.73, 1.31]		0.83 [0.62, 1.11]	1.01 [0.74, 1.37]	0.90 [0.67, 1.20]	0.98 [0.74, 1.32]	0.79 [0.59, 1.06]
F#	0.42 [0.30, 0.58]	1.24 [0.92, 1.66]	0.87 [0.65, 1.17]	1.18 [0.89, 1.59]	1.21 [0.90, 1.62]		1.22 [0.91, 1.64]	1.09 [0.81, 1.45]	1.19 [0.89, 1.60]	0.96 [0.71, 1.29]
G#	0.35 [0.25, 0.48]	1.02 [0.75, 1.37]	0.72 [0.53, 0.98]	0.97 [0.72, 1.31]	0.99 [0.73, 1.35]	0.82 [0.61, 1.10]		0.89 [0.66, 1.20]	0.98 [0.72, 1.32]	0.79 [0.58, 1.07]
A	0.39 [0.28, 0.53]	1.14 [0.85, 1.52]	0.81 [0.60, 1.08]	1.09 [0.82, 1.45]	1.11 [0.84, 1.48]	0.92 [0.69, 1.23]	1.12 [0.83, 1.51]		1.10 [0.83, 1.45]	0.88 [0.66, 1.18]
A#	0.35 [0.26, 0.48]	1.04 [0.77, 1.39]	0.73 [0.54, 0.99]	1.00 [0.75, 1.33]	1.02 [0.76, 1.36]	0.84 [0.62, 1.13]	1.02 [0.76, 1.38]	0.91 [0.69, 1.21]		0.80 [0.59, 1.09]
B	0.44 [0.32, 0.60]	1.29 [0.96, 1.74]	0.91 [0.68, 1.23]	1.24 [0.92, 1.67]	1.26 [0.94, 1.70]	1.04 [0.78, 1.41]	1.27 [0.94, 1.73]	1.13 [0.85, 1.52]	1.24 [0.92, 1.68]	

Figure 3: The figure shows the result of a comparison of each probe rhythm with all other probe rhythms described in the text (computed across all absolute frequencies and both tunings). The numbers represent odds ratios computed using logistic regressions with a different baseline probe rhythm in each column. That is, the numbers represent the odds ratios for the preference of a probe rhythm labelled next to the row over probe rhythms in columns (e.g., the first row suggests that the C probe rhythm is preferred to all the other probe rhythms). The numbers in brackets represent 95% confidence intervals for the odds ratios. Statistically significant results are shown in bold.

Exploratory analysis

Apart from the analysis of the relative preference of various probe rhythms within a polyrhythmic context, we compared the results with previous findings regarding the relative preference of probe tones within an analogous tonal context from Krumhansl and Kessler (1982). First, as an analogy to the major tonal profile, we computed a

⁶ By “C probe rhythm” we mean the C, E, or G probe rhythm. Recall that the tonic, mediant, and dominant probe rhythms shared the same stimulus in our experiment.

rhythmic profile. The latter gives for each scale degree the proportion of times the probe rhythm corresponding to that scale degree was preferred to the other probe rhythm in the pair of presented probe rhythms. We computed the rhythmic profile also separately for both tunings and for the four absolute frequencies as well as for all combinations of the absolute frequencies and tunings. Next, we computed Spearman correlation coefficients between each rhythmic profile and the major tonal profile reported in Krumhansl and Kessler (1982).⁷ The results (Table 1) show similarities between rhythmic profiles and the major tonal profile. Interestingly, the similarities seem to be more pronounced with the Farey tuning than with the 12TET tuning (Table 1, Figure 4) and with absolute frequencies 1.000 Hz and 2.000 Hz than with absolute frequencies 1.260 Hz and 1.587 Hz (Table 1, Figure 4).

	TET	Farey	Both tunings
1.000 Hz	.27 [-.54, .81]	.66 [.01, .92]	.78 [.33, .96]
1.260 Hz	.32 [-.47, .87]	.14 [-.64, .86]	.12 [-.70, .81]
1.587 Hz	-.20 [-.86, .60]	.13 [-.77, .80]	-.07 [-.80, .71]
2.000 Hz	.61 [.01, .92]	.65 [-.09, .99]	.80 [.32, 1.00]
All frequencies	.13 [-.63, .73]	.70 [.05, .99]	.53 [-.25, .95]

Table 1: The table shows Spearman correlation coefficients (with 95% confidence intervals in brackets) computed between relative preferences of probe tones reported in Krumhansl and Kessler (1982) and proportions of times a corresponding probe rhythm was preferred in the present study. Correlations are reported separately for the two tunings and four absolute frequencies as well as for data pooled across the tunings and frequencies.

⁷ We used values for the C probe tone as corresponding to the C/E/G probe rhythm. However, the ratings for G and E probe tones were the second and third highest rated in Krumhansl and Kessler (1982), so the results of the analysis using Spearman correlation would have been unchanged if we had used the ratings for the other two probe tones instead.

Figure 4: The figure shows proportions of trials when a polyrhythm with a given probe rhythm was selected separately for the tunings (top) and frequencies (bottom). The black points represent preferences pooled across both tunings and all frequencies. The error bars represent 95% confidence intervals.

A possible concern of the reported results is that the C probe rhythm differed from the other probe rhythms in that the corresponding polyrhythm consisted only of three rhythms instead of four as was the case for the other probe rhythms. Since the C probe rhythm was preferred to all other probe rhythms and it also had the highest rating in the study by Krumhansl and Kessler (1982), any correspondence of our results to the results of Krumhansl and Kessler could have been driven by this feature of our stimuli. We therefore conducted an analysis in which we used the ratings from Krumhansl and Kessler as a predictor, which enabled us to test the possibility of correspondence between preferences of probe rhythms and probe tones more directly. In particular, we conducted a logistic mixed-effect regression with the preference of the first presented polyrhythm as the dependent variable. The difference between corresponding ratings from Krumhansl and Kessler (1982) for the first presented and the second presented probe rhythms served as an independent

variable. We also included a random intercept for participants which took into account the dependence in data and random slope for the difference in ratings. Participants were more likely to say that they prefer the polyrhythms with higher ratings from Krumhansl and Kessler, $z = 6.55$, $p < .001$, $OR = 1.24$, 95% CI = [1.17, 1.33]. However, when the same analysis was conducted without the data for the C probe rhythm, the effect disappeared, $z = 0.71$, $p = .48$, $OR = 1.04$, 95% CI = [0.93, 1.17]. The association between the preference of polyrhythms and probe tone ratings from Krumhansl and Kessler seemed to be driven mainly by the C probe rhythm (see also Table 2).

	TET	Farey	Both tunings
1.000 Hz	.00 [-.73, .62]	.68 [.03, .95]	.70 [.11, .90]
1.260 Hz	.07 [-.74, .69]	-.18 [-.81, .71]	-.22 [-.89, .69]
1.587 Hz	-.65 [-.95, .13]	-.20 [-.86, .64]	-.47 [-.86, .34]
2.000 Hz	.47 [-.23, .85]	.52 [-.30, .95]	.73 [.11, 1.00]
All frequencies	-.20 [-.82, .50]	.58 [-.18, .94]	.35 [-.53, .90]

Table 2: The table shows the same results as Table 1, but without the C probe rhythm. The values in the table are Spearman correlation coefficients (with 95% confidence intervals in brackets) computed between relative preferences of probe tones reported in Krumhansl and Kessler (1982) and proportions of times a corresponding probe rhythm was preferred in the present study.

Next, we used a similar analysis to examine whether preferences can be predicted by the lowest number of cycles of the slowest rhythm (i.e., tonic) after which all constituting rhythms in a polyrhythm synchronize (henceforth “period” for simplicity). We used a binary logarithm of a proportion of the period of the first and the second presented polyrhythm as a predictor. The reported odds ratios therefore show the effect of doubling the proportion by two. Note that the logarithm of the period correlates highly with the ratings from Krumhansl and Kessler when the C probe rhythm is included, $r_s = -.86$, 95% CI = [-1.00, -.33], $p = .001$, as well as without the C probe rhythm, $r_s = -.80$, 95% CI = [-1.00, -.14], $p = .03$. The analysis showed that participants were more likely to prefer polyrhythms that synchronized faster, $z = -6.06$, $p < .001$, $OR = 0.81$, 95% CI = [0.75, 0.86]. This association held even when the analysis was conducted without the C probe rhythm, $z = -2.39$, $p = .02$, $OR = 0.90$, 95% CI = [0.83, 0.98]. Since the polyrhythms do not synchronize exactly in the 12TET tuning, we were interested whether the effect of the predictor might be stronger for the Farey tuning. When we included in analysis the interaction between tuning and the logarithm of the proportion of the period of the first and the second presented polyrhythm, we did not find a significant interaction when all the probe rhythms were analyzed, $z = 1.37$, $p = .17$, ratio of OR = 1.10, 95% CI = [0.96, 1.25],

but the interaction was significant when the C probe rhythm was not included, $z = 2.04$, $p = .04$, ratio of OR = 1.19, 95% CI = [1.01, 1.41], suggesting that the effect may be stronger (i.e., more negative) for the Farey tuning. This can be seen when the analysis is conducted for the two tunings separately. While the effect of the logarithm of a proportion of the period of the first and the second presented polyrhythm is significant for the Farey tuning, $z = -3.01$, $p = .003$, OR = 0.84, 95% CI = [0.75, 0.94],⁸ it is not for the 12TET tuning, $z = -0.12$, $p = .91$, OR = 0.99, 95% CI = [0.87, 1.13].

Discussion

The rhythmic profile computed across different absolute frequencies and tunings correlated strongly with the major tonal profile in a study by Krumhansl and Kessler (1982). However, the correlation seemed to be driven mainly by the preference of the C probe rhythm over all other probe rhythms. The latter preference might simply reflect the lower number of constituent rhythms in the single polyrhythm representing the C, E, and G probe tones in our experiment. To test this explanation, it would be possible to present the context polyrhythm to one ear and the probe rhythm to the other, so that even if the probe rhythm is identical to one of the context rhythms, the two rhythms would be perceived as distinct. This is the strategy used, for example, by Toiviainen and Krumhansl (2003).

An interesting finding is that the synchronization periods of the polyrhythms correlate highly with the major tonal profile. At the same time, there is a significant preference for polyrhythms with a shorter synchronization period. Both effects persist even when the C probe rhythm is excluded from the analysis. All this would point to a common mechanism underlying both tonality and meter, but when the association between the two is tested directly, it seems to rest mostly on the C probe rhythm.

We also found that the preference for shorter synchronization periods is limited only to the Farey tuning. In the Farey tuning, all frequency ratios are rational whereas in the 12TET tuning, frequency ratios are mostly irrational. Generally, two isochronous rhythms with a rational frequency ratio $p:q$ and a simultaneous onset synchronize each p ticks of the first and q ticks of the second rhythm (see the right column of Figure 2). By definition, there are no such integers p , q for an irrational frequency ratio, meaning that there are no points of exact synchronization and hence no well-defined synchronization period (see the left column of Figure 2). However, one can always approximate a given irrational frequency ratio with a rational one. This results in an absolute approximation error which, when expressed in terms of reciprocals of frequencies, is larger when frequencies decrease. We hypothesize that tonality relies

⁸ Despite the high correlation between the ratings from Krumhansl and Kessler (1982) and the period, the ratings from Krumhansl and Kessler are not significantly associated with the preferences even when only the Farey tuning is used, $z = 1.06$, $p = .29$, OR = 1.09, 95% CI = [0.93, 1.28], when the C probe rhythm is not included in analysis.

on patterns of synchronization like those in Figure 2. Because the frequencies of tones are relatively high, the approximation error is relatively small and does not hinder perception of synchronization patterns with a definite period. On the other hand, the 12TET tuning leads to larger inaccuracies in the case of rhythms and it thus exaggerates differences that might not be perceived in tones. Paradoxically, the Farey tuning in rhythms could therefore be perceived as closer to how the 12TET tuning is perceived in pitches than the 12TET tuning itself.

In summary, our data are not entirely conclusive as to whether tonality and meter rely on the same generic mechanism. Hence, we cannot attest to the validity of recent studies which use the same family of models to explain both tonality and meter (e.g., Large, 2010; Large, Herrera, & Velasco, 2015; Parncutt, 1994). However, if there really is a common ground between tonality and meter, we hypothesize that it is related to periodicity, a feature on which several published models of tonality are based (e.g., Parncutt, 1989; Leman, 2000; Parncutt, 2011).

The present study is limited by its dependence on the results of the previous study by Krumhansl and Kessler (1982) and by the limited amount of data that could have been used for the comparison of preferences of pitches and rhythms given the method employed by the present study. Future studies could use a different design to further test the similarities between perception of rhythms and pitches.

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