A distributional approach to measuring lifespan stratification

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Abstract

The study of the mortality differences between groups has traditionally focused on metrics that describe average levels of mortality such as life expectancy and standardized mortality rates. Additional insights can be gained by examining differences in lifespan distributions between groups using statistical distance metrics. Here, we use a distance metric, the non-overlap index, to capture the sociological concept of stratification, which emphasizes the emergence of unique, hierarchically layered social strata. We show an application using Finnish registration data covering the entire population over the period from 1996 to 2017. The results indicate that lifespan stratification and life-expectancy differences between income groups both increased substantially from 1996 to 2008; subsequently, life-expectancy differences declined, whereas stratification stagnated for men and increased for women. We conclude that the non-overlap index uncovers a unique domain of inequalities in mortality, and helps to capture important between-group differences that conventional approaches miss.

Keywords: Finland, income, life expectancy, lifespan inequality, measurement, mortality inequality

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1 Introduction

Mortality reduction and longevity extension are unequal at the global, regional, national, and subnational levels. In 2018, life expectancy at birth among low-income countries was 63 years, 18 years less than that for high-income countries (World Bank 2021). Numerous studies have documented that even within the high-income countries, differences in life expectancy between socioeconomic groups were strikingly large. In the US, for instance, men in the top 1% income distribution on average lived 14.6 years more than those in the bottom 1% during 2001–2014 (Chetty et al. 2016). A large gap in life expectancy between socioeconomic groups has been reported in many other countries and at different times (e.g. Brønnum-Hansen and Baadsgaard 2012; Martikainen et al. 2014; Mackenbach et al. 2018).

Reducing mortality inequalities arguably constitutes one of the major public health challenges, and is important to further advancing human longevity as a whole. To date, a large segment of the literature that examines between-group differences in longevity has focused on the comparison of group averages such as age-standardized mortality rates and life expectancies. While life expectancy is undoubtedly a powerful mortality indicator, other important facets of between-group differences in longevity can be hidden if life expectancy alone is studied.

To better the understanding of between-group differences in longevity, we introduce and quantify a new concept of between-group differences in longevity: lifespan stratification. Measuring lifespan stratification belongs to an approach that has been less frequently taken—comparing the distance and non-overlap between lifespan distributions, which is conceptually different from conventional approaches of comparing central tendencies (e.g. life expectancy) or variabilities. We show that the non-overlap index we use operationalizes the long-standing sociological concept of social stratification and reveals the extent to which society is divided into unique social strata when individual longevity is the outcome variable. Our empirical example of lifespan stratification between Finnish income groups illustrates that this new measure reveals new inequality patterns. Hence, our measure is a useful addition to demographers’ toolbox of analysis on mortality differences.

2 Conventional approaches to measuring mortality differences

Differences in lifespan distributions between population subgroups have generally been approached in one of the following ways: (1) by comparing measures of central tendency, e.g. mean, median, mode; (2) by comparing the spread or prematureness of mortality, e.g. variability, years of life lost (*YLL*); (3) by comparing overall distributions using statistical distance measures, e.g. Kullback-Leiber divergence. We first review the first two approaches.

2.1 Comparing central tendencies

Earlier work on social differences in mortality tended to compare age-standardized mortality rates between social groups using rate ratios (Whitney 1934; Logan 1954; Kitagawa and Hauser 1973; Duleep 1989). Rate ratios show the relative difference between groups, and rate differences show the actual size of inequality. Life expectancy has also long been used by prior studies to examine group differences in mortality (Antonovsky 1967; Brønnum-Hansen and Baadsgaard 2012; Chetty et al. 2016). It is the mean value of the lifespan distribution from the life table, and can be used to indicate the average number of years a group of individuals outlives another group. Other measures of central tendency such as the mode and the median of age at death have long been introduced to mortality analysis and enriched demographers' understanding of the evolution of human longevity (Lexis 1878; Kannisto 2001; Cheung and Robin 2007; Canudas-Romo 2008, 2010; Ouellette and Bourbeau 2011; Cohen and Oppenheim 2012; Horiuchi et al. 2013). Recent research on mortality differences between population subgroups has started to incorporate these additional central tendency measures beyond the traditional comparison of life expectancy (Brown et al. 2012; Zarulli et al. 2012; Diaconu et al. 2021).

Similar to mortality rates, central tendency measures can be also compared by ratios or differences. In practice, life-expectancy ratios are less frequently reported than life-expectancy differences. Absolute and relative measures yield the same pattern of inequalities when population subgroups are examined at a given time, in which case the aim is to understand whether group A performs better than group B. However, absolute and relative measures may contradict each other when the goal is to examine how the magnitude of mortality difference between A and B varies across time or place. For example, whether the gender difference in life expectancy is larger in one

country than in another. Adding a dimension of time or place to social differences in mortality complicates the comparison of central tendency measures.¹ Assessing whether the longevity gap has been growing or diminishing and whether policies have been successfully narrowing the gap can depend on whether absolute or relative measures are used (Carter-Pokras and Baquet 2002; Harper and Lynch 2016). The potential inconsistency in the two types of measures is perhaps a limitation of this approach.

2.2 Comparing variabilities

A recent strand of research has shifted the attention from group differences in means to differences in variabilities (Edwards and Tujarpukar 2005; Brown et al. 2012; Permanyer et al. 2018; van Raalte et al. 2018). Lifespan variability measures the within-group heterogeneity in ages at death. It also captures the degree of lifetime uncertainty that a random individual from that group faces. For individuals, smaller uncertainty conditional on the same expected lifespan means greater security, which, as economists have argued, people would rather have at the expense of losing a few years of expected life (Edwards 2013). A wide range of variability measures are available, due to the well-researched area of income inequality. Lifespan variability measures include the interquartile range, Gini coefficient, coefficient of variation, standard deviation, among others. In demography, metrics such as the life table entropy and the life disparity (e-dagger) have been introduced due to their useful interpretations (Leser 1955; Keyfitz 1977; Goldman and Lord 1986; Vaupel 1986; Vaupel and Canudas-Romo 2003).

The theory of mortality compression predicts that, as mortality reduction continues, lifespans tend to be concentrated around the mean age at death and thus the survival curve evolves towards a more rectangular shape (Fries 1980; Myers and Manton 1984). Indeed, research has

¹ For illustration, we compare male life expectancies of three countries. In 2018, male life expectancies in Nigeria, Pakistan, and Hong Kong were 53, 66, and 82 years, respectively (World Bank 2021). Consider two comparisons: (1) difference in male life expectancy between Nigeria and Pakistan, and (2) difference in male life expectancy between Pakistan and Hong Kong. The mean difference between Pakistan and Nigeria was 13 years and between Hong Kong and Pakistan was 16 years, suggesting that mortality difference was larger between Hong Kong and Pakistan than between Pakistan and Nigeria. However, male life expectancy of Pakistan was 1.25 times that of Nigeria, while the male life expectancy of Hong Kong was 1.24 that of Pakistan, suggesting that mortality difference was more similar between Hong Kong and Pakistan than between Pakistan and Nigeria. Therefore, monitoring life expectancy inequalities using differences and ratios yields inconsistent results. Similarly, slope and relative indices of inequality when examined on mortality rates often lead to different conclusions about the direction of changes in inequalities (e.g., Mackenbach et al. 2018).

shown that historically rising life expectancy was often coupled with falling lifespan variability (Smits and Monden 2009; Vaupel et al. 2011). Cross-sectional data showed that lower socioeconomic groups tended to have larger lifespan variability in addition to lower life expectancy (Edwards and Tujarpukar 2005; Brown et al. 2012; van Raalte et al. 2011). These findings point to the so-called “double burden” among people with lower socioeconomic status (SES; van Raalte et al. 2018); i.e. compared with high SES people, low SES people not only have shorter lives on average, but also face greater uncertainty about the timing of death.

The theoretical prediction and the empirical associations between life expectancy and lifespan variability seem to challenge the motivation of including lifespan variability for research on between-group differences in mortality. However, recent evidence has suggested that, for some populations or population subgroups (e.g. lower SES individuals), increasing life expectancy has been associated with stagnating or increasing lifespan variability (Brønnum-Hansen 2017; Aburto and van Raalte 2018; Permanyer et al. 2018; van Raalte et al. 2018). Hence, comparing lifespan variability between social groups is helpful to detect additional dimensions of inequalities.

3 Comparing distributions

Unlike the approaches of comparing central tendency measures and lifespan variabilities, our approach takes into account differences in the whole range of lifespan distributions, i.e. measuring statistical distance.

3.1 Statistical distance

How big is the difference between two lifespan distributions? This question relates to the concept of *statistical distance*. Measures of statistical distance can quantify the distance between two populations or two probability distributions, and capture how much “effort” is needed to transform one lifespan distribution into another. Statistical distance measures have been widely used in many disciplines including biology, chemistry, information theory, mathematics, mechanical engineering, statistics, etc. (Cha 2007).

All *distance metrics* need to fulfil four conditions (Deza and Deza 2009). Consider a distance metric $D(A, B)$ where A and B are two distributions, the four conditions are expressed as follows:

- (a) $D(A, B) \geq 0$ (non-negativity);
- (b) $D(A, B) = 0$, if and only if A and B are identical (identity of indiscernibility);
- (c) $D(A, B) = D(B, A)$ (symmetry);
- (d) $D(A, B) + D(B, C) \geq D(A, C)$ (triangle inequality).

A related concept to statistical distance is *divergence*, based on the idea of entropy originating in information theory (Shannon 1948). Unlike distance metrics, divergence only has to satisfy conditions (a) and (b), although the term divergence is sometimes used interchangeably with statistical distance (Ullah 1996). A well-known divergence measure is the Kullback–Leibler divergence (*KLD*; Kullback and Leibler 1951).

The distributional approach using measures of distance and divergence offers new research opportunities to the analysis of mortality differences beyond summary measures such as mean, mode, and variance. Numerous distance and divergence measures (see an overview in Cha 2007) can be used to answer the question “how big is the difference between two lifespan distributions”. Thus far, only a few studies have taken this distributional approach. Sasson (2016) used the *KLD* to examine the divergence in age-at-death distributions² between educational groups. Edwards and Tuljapurkar (2005) and d’Albis and colleagues (2014) also used the *KLD* to compare country-specific lifespan distributions. As a divergence measure, the *KLD* does not satisfy the symmetry and triangle inequality conditions.

3.2 Conceptualizing stratification

Different distance and divergence measures have unique properties that provide different information. We use a distance metric to quantify the concept of *stratification* of lifespan. In sociology, stratification is defined as the phenomenon of society being divided into *hierarchically laid-out strata*, characterized by historical, cultural, and economic distinctions (Mann 1984). Social class, education, and income are among the most common social stratifiers of a society

² Throughout the paper, “age-at-death distribution” refers to the life table dx , rather than the observed deaths Dx .

(Grusky and Weisshaar 2014). The use of terminology in the prior literature is wide-ranging. The “stratifier” is often inserted before “stratification”, such as “gender” in “gender stratification”. It is also common use to insert the “outcome” before “stratification”; for instance, “income stratification” is used in prior research where income is the outcome (e.g. Yitzhaki and Lerman 1991; Allanson 2014; Zhou and Wodtke 2019). In our study, lifespan is the outcome, and the stratifier is income. We use the terms “lifespan stratification” and “the stratification of lifespan” interchangeably.

In a broad sense, stratification is often a synonym of *inequality*, yet Yitzhaki and Lerman (1991) argued that stratification is conceptually different from inequality, which is often measured by the distance between group means. Social stratification emphasizes the formation of *observable layers* based on group characteristics (Lasswell 1965). Put differently, stratification reflects the extent to which social groups form *unique strata*, but not how big the differences in group means are. According to this rationale, life-expectancy difference between social groups is a good measure for inequality, but not suitable for stratification. When we discuss stratification, it is strictly this narrower phenomenon of distributional distance that we are referring to. The emphasis of stratification on distinguishable strata and noticeable boundaries coincides partly with the concept of statistical distance. Both stratification and statistical distance relate to the phenomenon of two distributions being apart.

Prior research using the narrow definition of stratification has focused almost exclusively on outcomes such as income and wealth, and little is known about the stratification of lifespan. Admittedly, unlike income, lifespan cannot be redistributed across individuals. But unequal lifespans arise partly from unequal distributions of both monetary and non-monetary resources (Link and Phelan 1995; Lynch et al. 2000; Marmot 2005; Phelan et al. 2010). Examining the stratification of lifespan can therefore inform how health-related resources are stratified between social groups, and shed new light on research in mortality differences.

3.3 Formalizing stratification

Consistent with prior research on the stratification of income (Zhou and Wodtke 2019), we propose to quantify the stratification of lifespan by the proportion of non-overlap of two lifespan distributions. This measure relates to the *Jaccard* index, originally used to measure florula

similarity between districts by ecologists (Jaccard 1912), and the *Tanimoto* index (Tanimoto 1958), a measure widely used in chemical research to quantify molecular similarity. The *Jaccard* and *Tanimoto* similarity metrics are mathematically equivalent. They are calculated as the ratio of the intersection to the union, i.e. $(A \cap B)/(A \cup B)$. The *Jaccard* (*Tanimoto*) distance is calculated as $1 - \text{Jaccard} (\text{Tanimoto}) \text{ similarity}$. For probability distributions, the *Jaccard* (*Tanimoto*) distance is also equivalent to the *Wave Hedges* index (Cha 2007).³ Due to the inconsistent naming of this measure, we refer to it as the *non-overlap index*, with notation S_{ij} for the stratification of lifespan between (sub)populations i and j . Note that here i and j can be two subpopulations or any two populations wherever the comparison is substantively meaningful (e.g. USA vs. Japan). S_{ij} is calculated as follows:

$$S_{ij} = 1 - \frac{\int_{\alpha}^{\omega} \min\{d_i(x), d_j(x)\} dx}{\int_{\alpha}^{\omega} \max\{d_i(x), d_j(x)\} dx}$$

where α is the starting age, ω is the maximum lifespan, and $d_i(x)$ and $d_j(x)$ are life table age-at-death distributions for (sub)populations i and j , respectively.

S_{ij} reaches the maximum of one when two age-at-death distributions do not overlap (maximum stratification), and the minimum of zero when two distributions are identical (no stratification). From zero to one, stratification increases monotonically as S_{ij} increases. It can be easily proved that this index satisfies the four aforementioned conditions of a distance metric (Kosub 2019). This is an advantage as compared to the asymmetric measure Kullback–Leibler divergence (*KLD*) used by earlier research (e.g. Sasson 2016). It is arbitrary whether the *KLD* between groups A and B is measured from A to B or from B to A. The non-overlap index is not just a distance measure, but also captures the sociological concept of stratification, which makes it easier to interpret than other distance measures.

Furthermore, a larger proportion of non-overlapping lifespans means a greater likelihood that a random individual in the group with lower life expectancy will die earlier than another random individual from the group with higher life expectancy. This concept is closely related to

³ We agree with Prasath et al. (2017) that the name “*Wave Hedges*” is questionable, as the formula for the *Wave Hedges* index cannot be traced to the original paper Hedges (1971) as cited in Cha (2007).

the outsurvival probability (Bergeron-Boucher et al. 2020; Vaupel et al. 2020). Stratification and the outsurvival probability both rely on the topological relationship of two lifespan distributions. Yet unlike the outsurvival probability, which focuses interpretations at the *individual* level, stratification reflects the unequal distributions of lifespans at the *societal* level.

Although the non-overlap index always captures the extent to which two distributions *differ* from each other, it is not designed as a measure of the tendency that values from one distribution tend to be *higher* (or *lower*) than those from the other. Thus, the non-overlap index is not suitable for measuring lifespan stratification, unless x and $\{d_i(x) - d_j(x)\}/\{d_i(x) + d_j(x)\}$ are strongly correlated. This way, lifespan distributions can be seen as *hierarchically layered*. Figure 1A shows an example where two distributions do not fulfil this condition. However, for human populations, x and $\{d_i(x) - d_j(x)\}/\{d_i(x) + d_j(x)\}$ tend to be strongly correlated, because human populations follow similar patterns of lifespan distribution and because the central tendency and the dispersion of a lifespan distribution are generally negatively correlated both cross-sectionally and over time. It is worth mentioning is that the same value of the non-overlap index may result from scenarios where the skewness and location of the distributions differ. Figures 1B and 1C show the same proportion of non-overlap (around 2/3), but Figure 1B displays a situation where the two distributions occupy a wider range of lifespans. No individuals from the shorter-lived group survive to ages above 85 where a big part of the non-overlap from the longer-lived group lies. Consequently, compared to Figure 1C, the two distributions in Figure 1B arguably occupy more unique strata, and thus have higher stratification. In this case, the non-overlap index would not be an appropriate metric of lifespan stratification. Again, these cases are unlikely to be observed in human populations.

Figure 1. Special cases where the non-overlap index cannot perform well.

To illustrate the conceptual differences and the possible links between stratification and the two conventional approaches, we show four hypothetical scenarios of the stratification of lifespan in Figures 2A~2D. In Figures 2A~2B, the life expectancies of the two social groups are 70 (left) and 75 (right); thus, the life-expectancy difference between the two groups is five years. As the two groups in Figure 2A have low standard deviations and the two groups in Figure 2B have high standard deviations, the two panels have different stratification levels, with moderate stratification for Figure 2A, and low stratification for Figure 2B. In Figures 2C~2D, there is a ten-year difference in life expectancy between the two groups. Figure 2C has a higher level of mortality stratification due to the low levels of standard deviations of both groups. If we examine these panels vertically, from Figure 2A to Figure 2C or from Figure 2B to Figure 2D, the standard deviations of the two groups are the same, but that as life-expectancy differences increase, stratification also increases. Therefore, stratification can be affected by differences in life expectancy and standard deviation.

Figure 2. Hypothetical scenarios of age-at-death distributions of two groups.

Our stylized examples suggest that the relationships between stratification and the other two types of between-group differences are unclear. The three indicators may respond differently to mortality changes at different locations (i.e. ages) of the distributions; thus, over time, they may produce different or even conflicting trends, as has been observed for the stratification of other types of social goods (Zhou and Wodtke 2019). For instance, in the high-stratification scenario depicted in Figure 2C, individuals in the lower-mortality group all survived to middle age, whereas no one from the higher-mortality group survived to old age. If the oldest people in the lower-mortality group were to live longer, or if those individuals in the higher-mortality group who died at young ages were to die at an even younger age, differences in life expectancy would increase, while stratification would remain invariant.

Unlike the two conventional approaches that have absolute and relative measures, the non-overlap index for the stratification of lifespan is a relative measure and does not have an absolute counterpart. As a non-parametric index, the value of stratification is not dependent on actual lifespan values, but on the relative positions of the two distributions. A monotonic transformation of the lifespans of all individuals (by adding/subtracting some years or including a scaling factor) will not change the value of stratification.

3.4 Multigroup stratification

Calculation of the stratification index becomes less straightforward when one is interested in comparing multiple groups. Constructing a stratification index for multiple groups involves several considerations. First, one needs to decide whether interest lies in the *overall* stratification or stratification with regard to a reference group. We propose to measure the overall stratification by a weighted sum of all pairwise comparisons:

$$S_{Total} = \frac{\sum_{1 \leq i < j \leq N} w_i w_j S_{ij}}{\sum_{1 \leq i < j \leq N} w_i w_j}$$

where S_{Total} denotes the overall stratification, N is the total number of (sub)populations, w_i and w_j are the sizes for (sub)populations i and j , respectively (when $i = j$, $S_{ij} = 0$). It is essential to include all pairwise comparisons as the two-group stratification is not transitive, i.e. one combination of S_{AB} and S_{AC} can lead to different values of S_{BC} . See Figure A1 for an example.

An alternative is to calculate stratification with regard to a reference group, e.g. the highest SES group or the best-performance group. We propose to measure the reference-based stratification as a weighted sum of all comparisons in which one group is always the reference group R :

$$S_R = \frac{\sum_{l \leq i \leq N} w_R w_i S_{Ri}}{\sum_{l \leq i \leq N} w_R w_i}$$

where S_R denotes the multigroup stratification with a reference group R , and w_R and w_i are the sizes for (sub)populations R and i , respectively (when $i = R$, $S_{Ri} = 0$). The selection of a reference group may involve normative judgments (Harper et al. 2010) and has implications for interpretation. Choosing the best-performance group as the reference group quantifies the extent of mortality inequality that could be eradicated if lifespans of those who are in the disadvantaged groups could be extended to that enjoyed by the lowest mortality group. Alternatively, one may also use the population average as the reference group. This could imply that reducing the lifespan of those longer-lived is one way to achieve low stratification. This is unlikely to be acceptable even to egalitarians and does not recognise that lifespans are not directly transferable.

Lastly, the choice of weights also has consequences for interpretation. Weights based on population size may be considered to reflect public health relevance of lifespan differences. However, if a socially disadvantaged group comprises a small number of individuals, should we care less about them? One might argue that each group is equally important (Harper and Lynch 2016). Such justification may lead to giving all compared groups equal weights.

4 An empirical example of lifespan stratification between income groups in Finland

In this section, we present an empirical example of the stratification of lifespan between Finnish income groups. Finland is used because it has an exceptionally long time series of full population data by income percentile, and has comparable mortality inequalities to countries and regions such as the United States and western Europe (Mackenbach et al. 2005; Elo et al. 2006; Mortensen et al. 2016). The results are based on period life tables by sex, year, and income quintile (measured in the year prior to exposure). Individuals aged 25 and above were included. See an overview of the dataset in the appendix.

4.1 Lifespan distributions

We first investigate how the age-at-death distributions have evolved differently for the lowest and the highest 20% income groups. Figure 3 shows the distributions for three periods, 1996–2000, 2006–2010 and 2013–2017. The vertical dashed lines mark the life expectancy values for the two distributions. From 1996–2000 to 2013–2017, there was a general pattern of mortality moving toward older ages for both females and males regardless of income levels, which resulted in increases in life expectancy. However, mortality evolved differently between the low- and the high-income groups at certain ages. For males in the low-income group, there were early humps over ages 55–70 in age-at-death distributions for 2006–2010 and 2013–2017. Accordingly, life-expectancy differences between low- and high-income males rose from 9.0 in 1996–2000 to 12.2 in 2006–2010, and then dropped to 9.8 in 2013–2017. Similarly, stratification increased from 0.42 in 1996–2000 to 0.54 in 2006–2010, and then dropped to 0.49 in 2013–2017. We find lifespan-variability (measured by standard deviation) differences increased in both 2006–2010 and 2013–2017.

Figure 3. Lifespan distributions by income and sex in 1996–2000, 2006–2010 and 2013–2017. *Source:* Authors' own calculation based on Finnish registry data. *Note:* Dashed lines refer to the positions of life expectancy.

Similar to the findings for males, a less pronounced hump in deaths at around retirement age is also observed for low-income females. From 1996–2000 to 2006–2010, life expectancy difference increased from 4.5 to 6.1, and stratification increased from 0.24 to 0.29. In 2013–2017, low-income females caught up with their high-income counterparts in the proportion of deaths occurring at ages above 90. However, a large number of low-income females were still dying at relatively younger ages. It seems that among low-income females, the improvement at older ages more than compensated for the apparent stagnation at younger ages, leading to a counterintuitive result: life-expectancy difference between the two income groups decreased to 4.2, while stratification increased to 0.31. Similar to findings for males, we find lifespan-variability differences increased in both 2006–2010 and 2013–2017.

4.2 Trends of life expectancy and lifespan variability by income

How did life expectancy and lifespan variability evolve for each income group? Before turning to our analysis of mortality stratification, we investigate the two most commonly used health indicators. Figure 4 presents the trends in life expectancy at age 25 and lifespan variability by income and sex from 1996 to 2017. As expected, there was a persistent income gradient in life expectancy for both men and women; however, the differences were larger among men than

women. At the beginning of our study, male life expectancy for different income quintiles ranged between 43.6 and 51.8 years, whereas female life expectancy hovered between 53.3 and 57.5 years. Over the years, life expectancy increased for both sexes in all five income groups, with slight fluctuations in certain years. In 2017, life expectancy had risen to 48.2–57.9 years for males and to 57.3–60.9 years for females.

We find that those groups with higher life expectancy tended to have lower levels of lifespan variability. Females had lower lifespan variability than males, and higher income groups had lower lifespan variability than lower income groups. However, unlike life-expectancy trends, lifespan-variability trends diverged across different income groups. Lifespan variability decreased for the top four income quintiles, but increased for the lowest quintile.

Figure 4. Trends in life expectancy and lifespan inequality by income and sex, 1996–2017. *Source:* Authors' own calculation based on Finnish registry data.

4.3 Trends of mortality stratification, life-expectancy, and lifespan-variability differences

We proceed by displaying the trends in mortality stratification and in the other two between-group differences between the lowest and the highest income quintiles in Figures 5, respectively. The overall mortality stratification across all five quintile groups shows very similar trends to the one we present here (see Figure A2 in the Appendix). Figure 5A shows that mortality stratification between the richest and the poorest 20% (for convenience, hereafter referred to as “mortality stratification”) increased from 1996 to 2017 for both men and women. Male mortality stratification increased from 0.39 in 1996 to 0.55 in 2007, and decreased to 0.50 in 2017; while female mortality stratification increased by 36%, from 0.24 in 1996 to 0.33 in 2017. In more recent years, female stratification continued to increase, whereas male stratification decreased slightly.

Figure 5B indicates that for both males and females, life-expectancy differences increased over the first decade, but decreased between 2008 and 2017, and life expectancy ratios generally show the same pattern (see Figure A3 in the Appendix). The trends for men and women are highly consistent, except that the female life-expectancy differences reached a lower level in 2017 (4.0 years) than the initial level in 1996 (4.2 years). For both mortality stratification and life-expectancy difference, we find that men had larger values than women. Comparing trends in life-expectancy difference with trends in mortality stratification, we observe that the trends were similar for men; but also that, interestingly, the trends were different for women in the most recent years where life-expectancy difference dropped whereas mortality stratification continued rising.

In Figure 5C, we find that the absolute differences in lifespan variability increased consistently over the whole period for both males and females. The trends of relative difference in lifespan variability are very similar (Figure A4). Yet in contrast to the findings in Figures 6 and 7, which show that the gaps in the indicator remained more or less at the same level for men and women, we see a convergence in lifespan-variability difference for men and women. Whereas lifespan-variability differences were much larger for men than for women in the earlier years, this gap had almost disappeared by 2017, especially for lifespan-variability ratios.

Figure 5. Trends in stratification, difference in life expectancy, and difference in lifespan variability between the lowest and the highest income quintiles by sex, 1996–2017. *Source:* Authors' own calculation based on Finnish registry data.

To determine how different age groups contributed to the divergent trends we observed, we could either conduct a decomposition analysis or simply inspect the age-at-death distributions at different periods. Looking back at Figure 3, we note that, for the low-income group, faster mortality improvements at more advanced ages raised life expectancy, and compensated for the lack of improvements at younger ages. However, the excessive deaths at pre-retirement ages (the “hump”) for the low-income group increased their lifespan variability levels as well as the mortality stratification.

How were mortality stratification associated with life-expectancy and lifespan-variability differences? Figure 6 shows these associations using all pairwise comparisons between every two income groups (for each year, there are 10 different pairs out of five income groups for both men and women). Life-expectancy and lifespan-variability differences were strongly associated with mortality stratification for men; however, the associations were much weaker for women. Put differently, for women, there was a large range of mortality-stratification levels given the same levels of life-expectancy or lifespan-variability differences. Among women, mortality stratification was more closely correlated with the lifespan-variability differences than with life-expectancy differences.

Figure 6. Pairwise association between mortality Stratification and (1) life-expectancy differences (panels A & B) and (2) lifespan-variability differences (panels C & D). *Source:* Authors' own calculation based on Finnish registry data. *Note:* Each point refers to a comparison between two income groups (e.g. the lowest and the second lowest income quintile groups, or the third and the highest income quintile groups) in a given year.

4.4 Temporal changes between every two years

On its own, Figure 6 cannot tell us how the indices are connected to each other over time. An interesting question would be whether the direction of change in mortality stratification was consistent with the direction of change in the life-expectancy or lifespan-variability differences. For this reason, Figure 7 shows the association between yearly changes in stratification and yearly changes in life-expectancy (panels A and B) and lifespan-variability (panels C and D) differences, using all of the pairwise comparisons as we did in Figure 8. The first-row panels show that changes in stratification were more consistent with changes in life-expectancy differences for males ($r = 0.85$) than for females ($r = 0.35$). For males, both of these changes simultaneously increased 52% of the time and simultaneously decreased 34% of the time. An increase in stratification coupled with a decrease in life-expectancy differences occurred only 8% of the time (upper-left quadrant); and just 6% of the time the other way around. For females, desirable changes (i.e. decreases in both indicators, bottom-left panel) occurred 28% of the time, while increases in both indicators occurred 39% of the time. It is notable that around 20% of the time, mortality stratification increased while life-expectancy differences decreased (upper-left quadrant). Interestingly, changes in lifespan-variability difference were not associated with changes in mortality stratification for men ($r = -0.01$), and the association for women was weak ($r = 0.22$).

Figure 7. Pairwise association between yearly changes in mortality stratification and (1) yearly changes in life-expectancy differences (panels A & B) and (2) yearly changes in lifespan-variability differences (panels C & D). *Source:* Authors' own calculation based on Finnish registry data. *Note:* Each point refers to a comparison between two income groups (e.g. the lowest and the second lowest income quintile groups, or the third and the highest income quintile groups) in a given year.

5 Discussion

In this paper, we introduced the concept of stratification of lifespan and demonstrated how to measure it. The non-overlap index reflects the extent to which individuals' lifespans are stratified by their social characteristics, and also captures the distance between two lifespan distributions. Monitoring stratification can uncover between-group differences that go unnoticed in the two conventional approaches of comparing life expectancy or lifespan variability, and can help to link these two lines of research. Our empirical application to Finland shows that income has come to play an increasingly important role in the stratification of lifetimes in Finland, while life-expectancy differences have decreased in recent years.

Our contribution is mainly methodological. From a mathematical perspective, the vast majority of research on between-group mortality differences has almost exclusively focused on only two moments of distributions, mean and variance. Measuring distributional differences is conceptually different from comparing central tendency measures or lifespan variabilities. One may argue that because of the strong regularity of the age pattern of mortality, knowing the life-expectancy difference can inform the overall distributional distance. This is only partly true. Given two life expectancies, the range of stratification is relatively predictable. However, how they change over time is much less so. It is possible for distributional distance to diverge when life expectancies converge. This point is clearly illustrated by our empirical examples (see Figure 2D & 2F, and the second quadrants in Figure 8A~8B). Further, although showing similar trends, lifespan-variability difference is conceptually different from our distributional approach, because comparing lifespan variability does not take the positions of the distributions into account. As the distance and stratification depend on the life expectancy and the lifespan variability levels of two distributions, our approach links these two lines of research nicely. Lastly, many regression-based inequality indices such as the relative index of inequality (Regidor 2004) can only be applied to ordinal independent variables. Thus, our index is of particular use when the stratifier is not ordinal, for example, marital status and region, which are of great interest in health inequality research.

Thus far, only a few previous studies have taken the approach of measuring distributional dissimilarity, using the Kullback–Leibler divergence (*KLD*) (Edwards and Tuljapurkar 2005; d'Albis et al. 2014; Sasson 2016), which is asymmetric and less easy to interpret. The non-overlap index has several advantages. First, as a distance metric, it has a simple graphical representation

and interpretation—the more overlap between two lifespan distributions, the smaller the distance. Second, it reflects the sociological concept of social stratification, which focuses on the process of society forming clearer boundaries between social strata. The majority of prior stratification research that emphasizes the geometric distance between two distributions has focused on the distributions of economic outcomes (e.g. Yitzhaki and Lerman 1991; Zhou and Wodtke 2019). To date, little is known about how lifespan distributions are stratified.

The non-overlap index for the stratification of lifespan should be included in the toolkit of future mortality inequality analysis, along with existing metrics such as differences in life expectancy, lifespan variability, years of life lost (*YLL*), cross-sectional average length of life (*CAL*), and other *CAL*-family measures (Brouard 1986; Guillot 2005; Canudas-Romo and Guillot 2015; Sauerberg et al. 2020; Nepomuceno et al. *forthcoming*). This is because we need an array of measures to reflect different dimensions of mortality inequalities. For example, for the low-income group in our data, a mid-life mortality hump emerged over the follow-up, while old-age mortality declined and approached the high-income group. Together these trends resulted in an increasing distance between the two lifespan distributions, and was captured by our index. As alcohol-related mortality is prominent at the ages around this mortality hump for the low-income group (Tarkiainen et al. 2012), we postulate that changes in alcohol consumption may have played an important role in driving the recent increase in stratification. Diverging health behaviours may be explained by changing access to health-related information and technology or important social networks (Link and Phelan 1995; Glied and Lleras-Muney 2008; Montgomery et al. 2020). With survey data, future work could examine whether these factors correlate with lifespan stratification.

A growing literature has documented widening gaps in life expectancy between socioeconomic groups in the last few decades among high-income countries (e.g. Meara et al. 2008; Brønnum-Hansen and Baadsgaard 2012; Tarkiainen et al. 2012; Chetty et al. 2016; Sasson 2016; Permanyer et al. 2018; Sasson and Hayward 2019). Our empirical example shows that for the Finnish population, income-group differences in life expectancy increased over 1996–2008 and decreased over 2008–2017. Yet average lifetimes do not tell the whole story. First, stratification increased even during periods in which life expectancy differences remained stable or decreased, particularly for women. The correlations between changes in mortality stratification and changes in the other two measures were weak, indicating that life-expectancy comparisons did not fully

capture the developments of mortality inequalities that occurred in the last decade. Second, although the trends of lifespan-variability differences and stratification were similar, lifespan-variability differences by income levels were approximately the same for men and women in recent years, whereas mortality stratification was noticeably higher for men than for women. Gender differences in the stratification of lifespan by income were relatively stable over time, a pattern similar to the results from life-expectancy differences. Further decomposition between men and women (Figure A5) shows that temporal mortality changes at older ages accounted for much of the convergence in lifespan-variability differences, while they played a much less important role in driving the gender differences in the other two indices.

Previous work has shown that the ages and the causes of death that drive differences in life expectancy and lifespan variability are not the same in high-income countries (Seligman et al. 2016). Lifespan variation is generally more sensitive to mortality change at younger ages than life expectancy (van Raalte and Caswell 2013). In contemporary low mortality settings, mortality change at young adult ages is particularly important in driving trends in lifespan variability (van Raalte et al. 2014; Aburto et al. 2020), while life expectancy trends tend to be driven by ages where the death density is higher, i.e. older ages (Vaupel 1986; Rau et al. 2018). Additional analyses (Figure A6) indicate that the rise in mortality stratification was mainly driven by ages between 50 and 74⁴—the ages in between the main drivers of lifespan inequality and life expectancy differences. The most prominent causes of death for these ages are circulatory system diseases and neoplasms (GBD 2016 Causes of Death Collaborators 2017). Alcohol-related causes may also play an important role, especially for pre-retirement ages. Indeed, much of the stagnation in life expectancy in the lowest income quintile during the 2000s and the subsequent increase in 2010s were attributable to changes in alcohol-related mortality at pre-retirement ages (Tarkiainen et al. 2012, 2017). Thus, one policy implication is that efforts should be made to reduce alcohol- and smoking-related deaths among low-income people in the middle and early old age groups (Martikainen et al. 2014; Tarkiainen et al. 2012, 2017). Reducing such deaths would not only increase life expectancy and reduce lifespan variability for the low SES group, but also lead to a declining trend in lifespan stratification. However, the role of alcohol consumption and smoking

⁴ We conducted supplementary analyses to disentangle the total change in the three metrics between the lowest and the highest income groups from the first to the last of the five-year periods (i.e., the same periods that we showed in Figure 2; results can be found in Figure A5 in the Appendix).

to life expectancy differences by income varies by country. Their relevance is particularly strong in Finland (Östergren et al. 2019); thus, policies tackling them are particularly called for.

When lifespan is highly stratified in society, disadvantaged individuals are not only more likely to die earlier, but also tend to experience more premature deaths among members of their social and kinship networks over their life course (Daw et al. 2016; Umberson et al. 2017). This becomes more important at older ages when lower SES individuals have fewer strong ties. Research has shown that bereavement and lack of social support are critical determinants for poor health and mortality (Berkman 1995; Unger et al. 1999; Kawachi and Berkman 2001; Stroebe et al. 2007; Holt-Lunstad et al. 2015; Cohen and McKay 2020), and such mortality burden is heavier in the lower socioeconomic strata (Martikainen and Valkonen 1998), further contributing to mortality stratification. In addition, there are sex differences in the effect of bereavement on health and mortality; men are at greater risk of dying than women after experiencing spousal death (Stroebe et al. 2007). This may partly explain the higher mortality stratification levels among males than among females.

Growing stratification of lifespan can also have negative societal consequences. It questions the efficacy of social systems in reducing health inequalities. Even in the absence of empirical data, we might reasonably speculate that when the stratification of lifespan increases, it can cause stress among individuals who belong to higher-mortality strata. Like other types of social stratification, growing stratification of lifespan may trigger social unrest and chaos. While the increasing divergence in health behaviours may be an important explanation, designing better social policies to tackle mortality inequalities has considerable merit.

Finally, increasing stratification of lifespan has further implications. It means that policies targeting people uniformly across SES and age groups are increasingly consequential. For instance, universal state pension ages in many countries are increasingly unfair for people in higher-mortality strata, as the proportion of this group who will have long post-retirement lives will be smaller. However, such inequalities within the population are difficult to detect when only life expectancies are compared. Even pegging retirement age to group-specific remaining life expectancies, which has been touted as a solution to reduce pension inequalities (see discussion in Alvarez et al. 2021), might not be appropriate in highly stratified settings where there are large differences in survival up to such an age.

There are potential extensions to our work. First, recent work has developed estimation methods for significance tests for the non-overlap index (Rahman et al. 2014; Chung et al 2019). Future research may examine the utility of these tests when applying the index. Second, a large number of distance and divergence measures have been extensively used in other disciplines (Deza and Deza 2009). Some of them might also be good candidates to measure lifespan stratification. It is unclear if other measures will lead to similar patterns. Third, we employed a period perspective in our empirical example, so the results pertain to hypothetical cohorts who experience the age-specific mortality rates of a given year. The stratification index itself is not a period measure; hence, future work could examine mortality stratification from a cohort perspective. Researchers may also extend this work to analyse stratification of truncated lifespan distributions or to incorporate a perspective of multiple cohort experiences, similar to the work by Canudas-Romo and Guillot (2015).

6 Conclusion

Life expectancy trends demonstrate that, on average, living a long life is increasingly becoming the domain of the rich and privileged. Rising stratification of lifespans is perhaps an even clearer indication of growing mortality inequalities than traditional indicators, such as life-expectancy or lifespan-variability differences. This is because despite diverging life expectancies, a substantial proportion of the disadvantaged groups could nevertheless be experiencing tremendous progress in longevity on par with that of the most advantaged groups. Our approach takes the divergence in the full age-at-death distribution into account, and is a clearer signal that social groups are effectively experiencing different survival ages. To the extent that individuals surround themselves with others of a similar income level, the poor will have fewer connections to healthy and long-lived adults, while the wealthy will have less experience of premature death within their immediate social networks. For these reasons, we argue that policy-makers should be monitoring the stratification of lifespan alongside other mortality indicators.

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References

- Aburto, José Manuel and Alyson van Raalte. 2018. Lifespan dispersion in times of life expectancy fluctuation: the case of Central and Eastern Europe, *Demography* 55(6): 2071–2096.
- Aburto, José Manuel, Francisco Villavicencio, Ugofilippo Basellini, Søren Kjærgaard, and James W. Vaupel. 2020. Dynamics of life expectancy and life span equality, *Proceedings of the National Academy of Sciences* 117(10): 5250–5259.
- Allanson, Paul. 2014. Income stratification and between-group inequality, *Economics Letters* 124(2): 227–230.
- Alvarez, Jesús-Adrián, Malene Kallestrup-Lamb, and Søren Kjærgaard. 2021. Linking retirement age to life expectancy does not lessen the demographic implications of unequal lifespans, *Insurance: Mathematics and Economics* 99: 363–375.
- Antonovsky, Aaron. 1967. Social class, life expectancy and overall mortality, *The Milbank Memorial Fund Quarterly* 45(2): 31–73.
- Bergeron-Boucher, Marie-Pier, Jesús-Adrian Alvarez, Ilya Kashnitsky, Virginia Zarulli, and James W. Vaupel. 2020. Not all females outlive all males: A new perspective on lifespan inequalities between sexes, *SocArXiv. October, 20*.
- Berkman, Lisa F. 1995. The role of social relations in health promotion, *Psychosomatic Medicine* 57(3): 245–254.
- Berkman, Lisa F., Thomas Glass, Ian Brissette, and Teresa E. Seeman. 2000. From social integration to health: Durkheim in the new millennium, *Social Science & Medicine* 51(6): 843–857.
- Berkman, Lisa F. and S. Leonard Syme. 1979. Social networks, host resistance, and mortality: a nine-year follow-up study of Alameda County residents, *American Journal of Epidemiology* 109(2): 186–204.
- Brønnum-Hansen, Henrik. 2017. Socially disparate trends in lifespan variation: a trend study on income and mortality based on nationwide Danish register data, *BMJ Open* 7(5): e014489.
- Brønnum-Hansen, Henrik and Mikkel Baadsgaard. 2012. Widening social inequality in life expectancy in Denmark. A register-based study on social composition and mortality trends for the Danish population, *BMC Public Health* 12(1): 994.
- Brouard, Nicolas. 1986. Structure et dynamique des populations. La pyramide des années à vivre, aspects nationaux et exemples régionaux [Structure and dynamics of populations. The pyramid of years to live, national aspects and regional examples], *Espace Populations Sociétés* 4(2): 157–168.
- Brown, Dustin C., Mark D. Hayward, Jennifer Karas Montez, Robert A. Hummer, Chi-Tsun Chiu, and Mira M. Hidajat. 2012. The significance of education for mortality compression in the United States, *Demography* 49(3): 819–840.
- Canudas-Romo, Vladimir. 2008. The modal age at death and the shifting mortality hypothesis, *Demographic Research* 19: 1179–1204.
- Canudas-Romo, Vladimir. 2010. Three measures of longevity: Time trends and record values, *Demography* 47(2): 299–312.
- Canudas-Romo, Vladimir and Michel Guillot. 2015. Truncated cross-sectional average length of life: A measure for comparing the mortality history of cohorts, *Population Studies* 69(2): 147–159.

- Carter-Pokras, Olivia and Claudia Baquet. 2002. What is a "health disparity"? *Public Health Reports* 117(5): 426–434.
- Cha, Sung-Hyuk. 2007. Comprehensive survey on distance/similarity measures between probability density functions, *International Journal of Mathematical Models and Methods in Applied Sciences* 1(4): 300–307.
- Chetty, Raj, Michael Stepner, Sarah Abraham, Shelby Lin, Benjamin Scuderi, Nicholas Turner, Augustin Bergeron, and David Cutler. 2016. The association between income and life expectancy in the United States, 2001–2014, *Journal of the American Medical Association*, 315(16):1750–1766.
- Cheung, Siu Lan Karen and Jean-Marie Robine. 2007. Increase in common longevity and the compression of mortality: The case of Japan, *Population Studies* 61(1): 85–97.
- Chung, Neo Christopher, Błażej Miasojedow, Michał Startek, and Anna Gambin. 2019. Jaccard/Tanimoto similarity test and estimation methods for biological presence-absence data, *BMC Bioinformatics* 20(15): 1–11.
- Cohen, Sheldon and Garth McKay. 2020. Social support, stress and the buffering hypothesis: A theoretical analysis, in S. E. Talyor, J. E. Singer, and A. Baum (eds), *Handbook of Psychology and Health (Volume IV)*. Routledge, pp. 253–267.
- Cohen, Joel E. and Jacob Oppenheim.. 2012. Is a limit to the median length of human life imminent?, *Genus* 68(1): 11–40.
- Daw, Jonathan, Ashton M. Verdery, and Rachel Margolis. 2016. Kin count (s): Educational and racial differences in extended kinship in the United States, *Population and Development Review* 42(3): 491–517.
- Deza, Michel Marie and Elena Deza. 2009. *Encyclopedia of Distances*. Springer, Berlin, Heidelberg.
- Diaconu, Viorela, Alyson van Raalte, and Pekka Martikainen. 2021. Why we should monitor disparities in old-age mortality with the modal age at death, *SocArXiv*.
- Duleep, Harriet Orcutt. 1989. Measuring socioeconomic mortality differentials over time, *Demography* 26(2): 345–351.
- d'Albis, Hippolyte, Loesse Jacques Ezzo, and Héctor Pifarré i Arolas. 2014. Persistent differences in mortality patterns across industrialized countries, *PloS One* 9(9): e106176.
- Edwards, Ryan D. and Shripad Tuljapurkar. 2005. Inequality in life spans and a new perspective on mortality convergence across industrialized countries, *Population and Development Review* 31(4): 645–674.
- Edwards, Ryan D. 2013. The cost of uncertain life span. *Journal of Population Economics*, 26(4), 1485–1522.
- Elo, Irma T., Pekka Martikainen, and Kirsten P. Smith. 2006. Socioeconomic differentials in mortality in Finland and the United States: the role of education and income, *European Journal of Population/Revue Européenne de Démographie* 22(2): 179–203.
- Fries, James F. 1980. Aging, natural death, and the compression of morbidity, *New England Journal of Medicine* 303(3):130–135.
- GBD 2016 Causes of Death Collaborators. 2017. Global, regional, and national age-sex specific mortality for 264 causes of death, 1980–2016: a systematic analysis for the Global Burden of Disease Study 2016, *The Lancet* 390(10100): 1151–1210.
- Glied, Sherry and Adriana Lleras-Muney. 2008. Technological innovation and inequality in health, *Demography* 45(3): 741–761.

- Goldman, Noreen and Graham Lord. 1986. A new look at entropy and the life table, *Demography* 23(2): 275-282.
- Grusky, David and Katherine Weisshaar. 2014. The questions we ask about inequality, in D. Grusky and K. Weisshaar (eds), *Social Stratification: Class, Race, and Gender in Sociological Perspective* (4th edition). Boulder, Colorado: Westview Press, pp. 1–16.
- Guillot, Michel. 2003. The cross-sectional average length of life (CAL): A cross-sectional mortality measure that reflects the experience of cohorts, *Population Studies* 57(1): 41–54.
- Hedges, T. S. 1976. An empirical modification to linear wave theory, *Proceedings of the Institution of Civil Engineers* 61(3): 575–579.
- Harper, Sam, Nicholas B. King, Stephen C. Meersman, Marsha E. Reichman, Nancy Breen, and John Lynch. 2010. Implicit value judgments in the measurement of health inequalities, *The Milbank Quarterly* 88(1): 4–29.
- Harper, Sam and John Lynch. 2016. Health inequalities: measurement and decomposition, in J. M. Oakes and J. S. Kaufman (eds), *Methods in Social Epidemiology*. John Wiley & Sons, pp. 91–131.
- Holt-Lunstad, Julianne, Timothy B. Smith, Mark Baker, Tyler Harris, and David Stephenson. 2015. Loneliness and social isolation as risk factors for mortality: a meta-analytic review, *Perspectives on Psychological Science* 10(2): 227–237.
- Horiuchi, Shiro, Nadine Ouellette, Siu Lan Karen Cheung, and Jean-Marie Robine. 2013. Modal age at death: lifespan indicator in the era of longevity extension, *Vienna Yearbook of Population Research*: 37–69.
- Jaccard, Paul. 1912. The distribution of the flora in the alpine zone, *New Phytologist* 11(2):37–50.
- Kannisto, Vaino. 2001. Mode and dispersion of the length of life, *Population: An English Selection* 13(1): 159–171.
- Kawachi, Ichiro and Lisa F. Berkman. 2001. Social ties and mental health. *Journal of Urban Health* 78(3): 458–467.
- Keyfitz, Nathan. 1977. What Difference Does it Make if Cancer Were Eradicated? An Examination of the Taeuber Paradox, *Demography* 14: 411–418.
- Kitagawa, Evelyn M. and Philip M. Hauser. 1973. *Differential Mortality in the United States: A Study in Socioeconomic Epidemiology*. Cambridge, Massachusetts: Harvard University Press.
- Kocher, Mirco and Jacques Savoy. 2017. Distance measures in author profiling, *Information Processing & Management* 53(5): 1103–1119.
- Kosub, Sven. 2019. A note on the triangle inequality for the Jaccard distance, *Pattern Recognition Letters* 120:36–38.
- Kullback, Solomon and Richard A. Leibler. 1951. On information and sufficiency, *The Annals of Mathematical Statistics* 22(1): 79–86.
- Lasswell, Thomas E. 1965. *Class and Stratum*. Boston, Massachusetts: Houghton Mifflin.
- Leser, C. E. V. 1955. Variations in mortality and life expectation, *Population Studies* 9(1): 67–71.
- Lexis, W. 1878. Sur la durée normale de la vie humaine et sur la théorie de la stabilité des rapports statistiques [On the normal length of human life and on the theory of the stability of statistical relations], *Annales de Démographie Internationale* 2(5): 447–460.

- Link, Bruce G. and Jo Phelan. 1995. Social conditions as fundamental causes of disease, *Journal of Health and Social Behavior*: 80–94.
- Lleras-Muney, Adriana and Frank R. Lichtenberg. 2005. Are the more educated more likely to use new drugs?, *Annales d'Économie et de Statistique* no. 79/80: 671–696.
- Logan, William Philip Dowie. 1954. Social class variations in mortality, *British Journal of Preventive & Social Medicine* 8(3): 128–137.
- Lynch, John W., George Davey Smith, George A. Kaplan, and James S. House. 2000. Income inequality and mortality: importance to health of individual income, psychosocial environment, or material conditions, *British Medical Journal* 320(7243): 1200–1204.
- Mackenbach, Johan P., Pekka Martikainen, Caspar WN Looman, Jetty AA Dalstra, Anton E. Kunst, and Eero Lahelma. 2005. The shape of the relationship between income and self-assessed health: an international study, *International Journal of Epidemiology* 34(2): 286–293.
- Mackenbach, Johan P., José Rubio Valverde, Barbara Artnik, Matthias Bopp, Henrik Brønnum-Hansen, Patrick Deboosere, Ramune Kalediene, Katalin Kovács, Mall Leinsalu, Pekka Martikainen, Gwenn Menvielle, Enrique Regidor, Jitka Rychtaríková, Maica Rodriguez-Sanz, Paolo Vineis, Chris White, Bogdan Wojtyniak, Yannan Hu, and Wilma J. Nusselder. 2018. Trends in health inequalities in 27 European countries, *Proceedings of the National Academy of Sciences* 115(25): 6440–6445.
- Mann, Mann (ed). 1984. *The International Encyclopedia of Sociology*. Continuum International Publishing Group.
- Marmot, Michael. 2005. Social determinants of health inequalities. *The Lancet* 365(9464): 1099–1104.
- Marmot, Michael, Jessica Allen, Ruth Bell, Ellen Bloomer, and Peter Goldblatt. 2012. WHO European review of social determinants of health and the health divide, *The Lancet* 380(9846): 1011–1029.
- Martikainen, Pekka, Pia Mäkelä, Riina Peltonen, and Mikko Myrskylä. 2014. Income differences in life expectancy: The changing contribution of harmful consumption of alcohol and smoking, *Epidemiology*: 182–190.
- Martikainen, Pekka, and Tapani Valkonen. 1998. Do education and income buffer the effects of death of spouse on mortality?, *Epidemiology*: 530–534.
- Meara, Ellen R., Seth Richards, and David M. Cutler. 2008. The gap gets bigger: changes in mortality and life expectancy, by education, 1981–2000, *Health Affairs* 27(2): 350–360.
- Montgomery, Shannon C., Michael Donnelly, Prachi Bhatnagar, Angela Carlin, Frank Kee, and Ruth F. Hunter. 2020. Peer social network processes and adolescent health behaviors: A systematic review, *Preventive Medicine* 130: 105900.
- Mortensen, Laust H., Johan Rehnberg, Espen Dahl, Finn Diderichsen, Jon Ivar Elstad, Pekka Martikainen, David Rehkopf, Lasse Tarkiainen, and Johan Fritzell. 2016. Shape of the association between income and mortality: a cohort study of Denmark, Finland, Norway and Sweden in 1995 and 2003, *BMJ Open* 6(12): e010974.
- Myers, George C., and Kenneth G. Manton. 1984. Compression of mortality: myth or reality?, *The Gerontologist* 24(4): 346–353.
- Nepomuceno, Marília, Qi Cui, Alyson van Raalte, José Manuel Aburto, and Vladimir Canudas-Romo. *Forthcoming*. The cross-sectional average inequality in lifespan (CAL†): A lifespan variation measure that reflects the mortality histories of cohorts, *Demography*.

- Östergren, Olof, Pekka Martikainen, Lasse Tarkiainen, Jon Ivar Elstad, and Henrik Brønnum-Hansen. 2019. Contribution of smoking and alcohol consumption to income differences in life expectancy: evidence using Danish, Finnish, Norwegian and Swedish register data, *Journal of Epidemiology and Community Health* 73(4): 334–339.
- Ouellette, Nadine, and Robert Bourbeau. 2011. Changes in the age-at-death distribution in four low mortality countries: A nonparametric approach, *Demographic Research* 25: 595–628.
- Permanyer, Iñaki, Jeroen Spijker, Amand Blanes, and Elisenda Renteria. 2018. Longevity and lifespan variation by educational attainment in Spain: 1960–2015, *Demography* 55(6): 2045–2070.
- Phelan, Jo C., Bruce G. Link, and Parisa Tehranifar. 2010. Social conditions as fundamental causes of health inequalities: Theory, evidence, and policy implications, *Journal of Health and Social Behavior* 51(1_suppl): S28–S40.
- Prasath, V. B., Haneen Arafat Abu Alfeilat, Ahmad Hassanat, Omar Lasassmeh, Ahmad S. Tarawneh, Mahmoud Bashir Alhasanat, and Hamzeh S. Eyal Salman. 2017. Distance and Similarity Measures Effect on the Performance of K-Nearest Neighbor Classifier--A Review, *arXiv preprint arXiv:1708.04321*.
- Rahman, Syed Asad, Sergio Martinez Cuesta, Nicholas Furnham, Gemma L. Holliday, and Janet M. Thornton. 2014. EC-BLAST: a tool to automatically search and compare enzyme reactions, *Nature Methods* 11(2): 171174.
- Rau, Roland, Christina Bohk-Ewald, Magdalena M. Muszyńska, and James W. Vaupel. 2018. Surface Plots of Age-Specific Contributions to the Increase in Life Expectancy, in Rau, Roland, Christina Bohk-Ewald, Magdalena M. Muszyńska, and James W. Vaupel (eds), *Visualizing Mortality Dynamics in the Lexis Diagram*. Springer, Cham, pp. 81–97.
- Regidor, Enrique. (2004). Measures of health inequalities: part 2, *Journal of Epidemiology and Community Health* 58(11), 900.
- Sasson, Isaac. 2016. Trends in life expectancy and lifespan variation by educational attainment: United States, 1990–2010, *Demography* 53(2): 269–293.
- Sasson, Isaac and Mark D. Hayward. 2019. Association between educational attainment and causes of death among white and black US adults, 2010–2017, *Journal of the American Medical Association* 322(8): 756–763.
- Sauerberg, Markus, Michel Guillot, and Marc Luy. 2020. The cross-sectional average length of healthy life (HCAL): a measure that summarizes the history of cohort health and mortality, *Population Health Metrics* 18(1): 1–17.
- Seligman, Benjamin, Gabi Greenberg, and Shripad Tuljapurkar. 2016. Equity and length of lifespan are not the same, *Proceedings of the National Academy of Sciences* 113(30): 8420–8423.
- Shannon, Claude Elwood. 1948. A mathematical theory of communication, *The Bell System Technical Journal* 27(3): 379–423.
- Smits, Jeroen, and Christiaan Monden. 2009. Length of life inequality around the globe, *Social Science & Medicine* 68:1114–1123.
- Stroebe, Margaret, Henk Schut, and Wolfgang Stroebe. 2007. Health outcomes of bereavement, *The Lancet*, 370(9603): 1960–1973.
- Tanimoto, T. T. 1958. *An Elementary Mathematical Theory of Classification and Prediction*. New York: IBM Research Yorktown Heights.

- Tarkiainen, Lasse, Pekka Martikainen, Mikko Laaksonen, and Tapani Valkonen. 2012. Trends in life expectancy by income from 1988 to 2007: decomposition by age and cause of death, *Journal of Epidemiology and Community Health* 66(7): 573–578.
- Tarkiainen, Lasse., Pekka Martikainen, Riina Peltonen, and Hanna Remes. 2017. Pitkään jatkunut sosiaaliryhmien välisten elinajanodote-erojen kasvu on pääosin pysähtynyt 2010-luvulla [The gap in life expectancy between socioeconomic groups has not widened during the 2010s], *Suomen Lääkärilehti* 72(9): 53–59.
- Timmermans, Stefan and Rebecca Kaufman. 2020. Technologies and health inequities, *Annual Review of Sociology* 46: 583–602.
- Ullah, Aman. 1996. Entropy, divergence and distance measures with econometric applications, *Journal of Statistical Planning and Inference* 49(1): 137–162.
- Umberson, Debra, Julie Skalamera Olson, Robert Crosnoe, Hui Liu, Tetyana Pudrovskaya, and Rachel Donnelly. 2017. Death of family members as an overlooked source of racial disadvantage in the United States, *Proceedings of the National Academy of Sciences* 114(5): 915–920.
- Unger, Jennifer B., Gail McAvay, Martha L. Bruce, Lisa Berkman, and Teresa Seeman. 1999. Variation in the impact of social network characteristics on physical functioning in elderly persons: MacArthur Studies of Successful Aging, *The Journals of Gerontology Series B: Psychological Sciences and Social Sciences* 54(5): S245-S251.
- Van Raalte, Alyson A. and Hal Caswell, H. 2013. Perturbation analysis of indices of lifespan variability, *Demography* 50(5): 1615–1640.
- Van Raalte, Alyson A., Anton E. Kunst, Patrick Deboosere, Mall Leinsalu, Olle Lundberg, Pekka Martikainen, Bjørn Heine Strand, Barbara Artnik, Bogdan Wojtyniak, and Johan P. Mackenbach. 2011. More variation in lifespan in lower educated groups: evidence from 10 European countries, *International Journal of Epidemiology* 40(6): 1703–1714.
- van Raalte, Alyson A., Pekka Martikainen, and Mikko Myrskylä. 2014. Lifespan variation by occupational class: Compression or stagnation over time?, *Demography* 51(1): 73–95.
- van Raalte, Alyson A., Sasson, I., & Martikainen, P. 2018. The case for monitoring life-span inequality, *Science* 362(6418): 1002–1004.
- Vaupel, James W. 1986. How change in age-specific mortality affects life expectancy, *Population Studies* 40(1): 147–157.
- Vaupel, James. W., Marie-Pier Bergeron Boucher, and Ilya Kashnitsky. 2021. Outsurvival as a measure of the inequality of lifespans between two populations, *Demographic Research* 44: 853–864.
- Vaupel, James W. and Vladimir Canudas-Romo. 2003. Decomposing change in life expectancy: A bouquet of formulas in honor of Nathan Keyfitz's 90th birthday, *Demography* 40(2): 201–216.
- Vaupel, James W., Zhen Zhang, and Alyson A. van Raalte. 2011. Life expectancy and disparity: an international comparison of life table data, *BMJ Open* 1(1): e000128.
- World Bank, World Development Indicators. 2021. *Life expectancy at birth, male (years)* [Data file]. Available: <https://data.worldbank.org/indicator/SP.DYN.LE00.MA.IN> (accessed 19 October 2021).
- Whitney, J. S. (ed). 1934. *Death Rates by Occupation: Based on Data of the US Census Bureau, 1930*. New York: National Tuberculosis Association.
- Yitzhaki, Shlomo and Robert I. Lerman. 1991. Income stratification and income inequality, *Review of Income and Wealth* 37(3): 313–329.

- Zarulli, Virginia, Domantas Jasilionis, and Dmitri A. Jdanov. 2012. Changes in educational differentials in old-age mortality in Finland and Sweden between 1971-1975 and 1996-2000, *Demographic Research* 26: 489–510.
- Zhou, Xiang, and Geoffrey T. Wodtke. 2019. Income stratification among occupational classes in the United States, *Social Forces* 97(3): 945–972.

Appendix A: Figures

Figure A1. An example for the intransitivity of stratification. *Note:* The distributions for A and C are the same in both scenarios 1 and 2. The only difference in the two scenarios is the location of distribution B. S_{AB} in scenario 1 equals S_{AB} in scenario 2; and S_{AC} in scenario 1 equals S_{AC} in scenario 2. However, S_{BC} in scenario 1 is different from S_{BC} in scenario 2.

Figure A2. Trends in mortality stratification over five income quintiles, 1996–2017. *Source:* Authors' own calculation based on Finnish registry data.

Figure A3. Trends in the life-expectancy ratio between the lowest and the highest income quintiles, 1996–2017. *Source:* Authors' own calculation based on Finnish registry data.

Figure A4. Trends in the life-variability ratio between the lowest and the highest income quintiles, 1996–2017. *Source:* Authors' own calculation based on Finnish registry data.

Figure A5. Age-specific contributions to changes in the three indices between 1996–2000 and 2013–2017, by sex. *Source:* Authors' own calculation based on Finnish registry data. *Note:* Red bars refer to age-specific contributions due to changes in the lowest income quintile group; green bars refer to age-specific contributions due to changes in the highest income quintile group; blue bars refer to the sum of the red and green bars; i.e. total age-specific contributions. The decomposition is an application of the method proposed by Horiuchi et al. (2008).

Figure A6. Age-specific contributions to differences in the three indices between males and females, by period. *Source:* Authors' own calculation based on Finnish registry data. *Note:* Red bars refer to age-specific contributions due to changes in the lowest income quintile group; green bars refer to age-specific contributions due to changes in the highest income quintile group; blue bars refer to the sum of the red and green bars; i.e. total age-specific contributions. The decomposition is an application of the method proposed by Horiuchi et al. (2008).

Appendix B: A brief description of the dataset used in the empirical example

The dataset is constructed by linking the death registry with income data from the Finnish Tax Administration and the National Social Insurance Institution at the individual level. The sample is restricted to individuals aged 25 and above. We use yearly household disposable income per capita as our income measure and classify individuals into groups based on their age-specific income percentiles (Martikainen et al. 2009). We account for the household composition by dividing the total household disposable income by the sum of the consumption units in the household using the OECD modified equivalence scale, in which the first adult is assigned a value of one, each additional adult member is assigned a value of 0.5, and each child is assigned a value of 0.3 (OECD n.d.).

In the next step, we construct period life tables for each year, sex, age, and income quintile (obtained from the previous year) using death counts and exposures for the corresponding group. The original datasets cover the years between 1996 and 2017, and age is formatted as single-year groups (25–26, 26–27, ..., 94–95, 95+). We extend the last age group to 110+ using a penalized composite link model, which incorporates P -splines to smooth the death counts with exposures as offsets across age (Rizzi et al. 2015) because it is common for deaths and exposures to have very small counts for certain age groups (e.g. deaths at very young or old ages and exposures at very old ages). The age extension and smoothing method is implemented with the R package ‘ungroup’ (Pascariu et al. 2018).

References

- Horiuchi, Shiro, John R. Wilmoth, and Scott D. Pletcher. 2008. A decomposition method based on a model of continuous change, *Demography* 45(4): 785–801.
- Martikainen, Pekka, Tapani Valkonen, and Heta Moustgaard. 2009. The effects of individual taxable income, household taxable income, and household disposable income on mortality in Finland, 1998–2004, *Population Studies* 63(2): 147–162.
- OECD. N.d. *What are equivalence scales?* Available: <http://www.oecd.org/els/soc/OECD-Note-EquivalenceScales.pdf> (accessed: 12 May 2020).
- Pascariu, Marius D., Maciej J. Dańko, Jonas Schöley, and Silvia Rizzi. 2018. ungroup: An R package for efficient estimation of smooth distributions from coarsely binned data, *Journal of Open Source Software* 3(29): 937.
- Rizzi, Silvia, Jutta Gampe, and Paul HC Eilers. 2015. Efficient estimation of smooth distributions from coarsely grouped data, *American Journal of Epidemiology* 182(2): 138–147.