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**Statistical Characterization of a Confined Submarine Fan System: the Pennsylvanian
Lower Atoka Formation, Ouachita Mountains, USA**

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9 **ABSTRACT**

10 Our knowledge of submarine fan deposits has historically relied heavily on qualitative field
 11 and subsurface observations and interpretations, but recent studies using statistical analyses
 12 have enhanced the understanding of submarine fan sub-environments, including the degree of
 13 confinement, stratigraphic patterns, and potential control factors. The purpose of this study is
 14 to improve the quantitative understanding of synorogenic submarine fan deposition at
 15 foreland basin settings with a statistical approach. A suite of statistical methods is integrated
 16 and developed (Hurst Statistics/ rescaled range analysis, bed thickness frequency distribution
 17 analysis, Markov Chains, and time-series analysis), and applied to the well understood
 18 Pennsylvanian lower Atoka submarine fan system in the Ouachita Mountains, United States
 19 for this purpose. The results of the Hurst Statistics and bed thickness analyses corroborate
 20 qualitative interpretations that (1) the lower Atoka is lobe-dominated, and (2) the
 21 southeastern (wedge-top) portion of the system is more strongly confined than the northern
 22 (foredeep) portion. The Markov Chains and time-series analyses reveal the prevalence (56%)
 23 of stratigraphic orderliness and cyclicity; these results are used to discuss potential intrinsic
 24 and extrinsic controls on the turbidite sandstone recurrence cycles, which are otherwise
 25 difficult to distinguish qualitatively. The results of this study demonstrate that these
 26 integrated statistical methods can be utilized to quantify uncertainty in depositional
 27 interpretations of outcrops with limited exposures or 1D subsurface (e.g., well-log, core)
 28 datasets.

29 **INTRODUCTION**

30 Submarine fans deposited at active continental margins preserve valuable information of the
 31 tectonic-sedimentation relationships and regional paleogeography (Ricci-Lucchi, 2003;
 32 Morley *et al.*, 2011; Ingersoll, 2012; Salles *et al.*, 2014). However, such depositional systems

are often difficult to fully comprehend because of structural complexity and low preservation potential (Ingersoll, 2012; Pantopoulos *et al.*, 2013; Pinter *et al.*, 2018). How to extract the maximum amount of depositional information remains a challenge, particularly when most stratigraphic studies remain qualitative. Statistical methods based on bed-by-bed measurements have become increasingly insightful in understanding several key aspects of submarine fan systems. These aspects include the discrimination of the fan subenvironments (Carlson & Grotzinger, 2001; Chen & Hiscott, 1999a; Clark & Steel, 2006; Felletti & Bersezio, 2010b; Fryer & Jobe, 2019; MacDonald, 1986; Malinverno, 1997; Pantopoulos *et al.*, 2018; Sylvester, 2007), quantification or semi-quantification of the degree of confinement (Felletti & Bersezio, 2010a; Fryer & Jobe, 2019; Liu *et al.*, 2018; Marini *et al.*, 2016; Sinclair & Cowie, 2003; Tóké & Patacci, 2018), objective detection of stratigraphic patterns (Chen & Hiscott, 1999a; Amy *et al.*, 2000; Wilkinson *et al.*, 2003; Sumner *et al.*, 2012; Burgess, 2016a; b), and separation of the influence of potential control mechanisms (Weltje & Boer, 1993; Stow *et al.*, 2002; Dennielou *et al.*, 2006; Starek & Fuksi, 2017; Burgess *et al.*, 2019). Currently, studies with integrated statistical analyses either utilize only a small number of outcrops (Starek & Fuksi, 2017; Palozzi *et al.*, 2018; Bourli *et al.*, 2019) or only partially address these aspects of depositional systems (Pantopoulos *et al.*, 2013, 2018)

The purpose of this study is to integrate the established statistical methods, improve them by testing against a well-understood submarine fan system, and enhance the understanding of the depositional system and control mechanisms. This study uses a large (> 2,500 m measured sections) outcrop dataset from the Pennsylvanian, synorogenic lower Atoka submarine fan succession of the Ouachita Mountains in the United States, where the structural-depositional framework is well-documented (Houseknecht, 1986; Coleman, 2000; Arbenz, 2008; Hou *et al.*, 2020). This study utilizes integrated statistical methods to discuss potential sub-environments, the degree of structural confinement, stacking patterns, and intrinsic/extrinsic

controls on sedimentation. The results of this study have implications for the interpretation and prediction of deepwater systems with limited exposures, subsurface datasets (Hernandez-Martinez *et al.*, 2013), and possibly stratigraphy on other planets (Limaye *et al.*, 2012; Stack *et al.*, 2013).

GEOLOGIC SETTING

Tectonic history of the Ouachita Mountains

The Ouachita Mountains in the United States are the outcrop expression of the Paleozoic Ouachita Fold and Thrust Belt (Fig. 1a-b) bounded by the Laurentia Craton to the north and peri-Gondwana terranes to the south (Mickus & Keller, 1992; Keller & Hatcher, 1999). The stratigraphy of the Paleozoic Ouachita Basin is divided into pre-orogenic and syn-orogenic strata (Babaei & Viele, 1992). The pre-orogenic strata comprise Cambrian-Middle Mississippian deep marine shale, chert, and turbidites, with a maximum thickness of ~4,000 m and an average sedimentation rate of 30 m/Myr (Morris, 1974a). The syn-orogenic strata comprise middle Mississippian-middle Pennsylvanian sediments of the Stanley Group, Jackfork Group, Johns Valley Formation, and the Atoka Formation (Fig. 1d); the total thickness is >10,000 m, with an average sedimentation rate of 300 m/Myr (Morris, 1974a). The estimated paleolatitude of the basin is 15°-0° S during the Carboniferous (Coleman, 2000). The estimated basin size is 550 by 450 km² at the beginning of Carboniferous, and the amount of shortening is ~50% by mid-late Pennsylvanian due to north-to-south subduction (Mickus & Keller, 1992; Keller & Hatcher, 1999; Coleman, 2000). The mid-late Pennsylvanian marks the end of the transition from the remnant ocean basin to a continental foreland basin (Houseknecht, 1986; Arbenz, 2008). The Ouachita Mountains are divided into several structural-depositional zones (Stone & Haley, 1984; Arbenz, 2008). The focus of this study is the comparison between the orogenic foredeep and the wedge-top (Fig. 1c), the

definitions of which are adopted and modified from the most recent work on the regional structural geology (Arbenz, 2008).

Depositional system of the lower Atoka formation

The Atoka Formation (Pennsylvanian Atokan, 316.5-311.5 Ma) represents the final phase of the remnant ocean-foreland transition (Houseknecht, 1986). It is a shoaling upward succession from deep-marine turbidites to shallow marine-fluvial deposits (Houseknecht, 1986; Haley & Stone, 2006) that crops out extensively in the foredeep and wedge-top of the Ouachita Mountains (Fig. 1). The lower member of the Atoka Formation is a submarine fan succession with a maximum thickness of ~3000m in the foredeep and ~2000m in the wedge-top (Zachry & Sutherland, 1984; Haley *et al.*, 1993; Arbenz, 2008). The lower Atoka fan system (Fig. 2c) is laterally-confined and primarily fed by large fluvial-deltaic systems to the east (Coleman, 2000; Graham *et al.*, 1975, 1976; Houseknecht, 1986; Lowe, 1985; Sprague, 1985). Near the western margin of the foredeep, there is a small fan system fed by local sediment sources derived from the Arbuckle Mountains to the west (Ferguson & Suneson, 1988; Sharrah, 2006). Basin confinement has been interpreted from regional lithofacies trends, paleocurrent directions, tectonic reconstructions, and paleogeography (Fig. 2c)(Briggs & Cline, 1967; Graham *et al.*, 1975; Lillie *et al.*, 1983; Morris, 1974b; Nelson *et al.*, 1982; Sprague, 1985; Viele & Thomas, 1989), and the confinement appears to be stronger in the wedge-top during the Atokan (Arbenz, 2008; Hou *et al.*, 2020; Johnson, 2011; Thomas, 2011). The regional facies distribution is consistent with the structural framework (Arbenz, 2008), which shows a classical proximal-distal decrease in sand-rich components (Bouma *et al.*, 1985; Mutti & Ricci-Lucchi, 1978; Normark, 1978; Walker, 1978) in the foredeep but stable facies composition in the wedge-top (Hou *et al.*, 2020).

DATA AND METHODS

Dataset

The data analyzed by this study consists of 54 outcrops, 2,515 m of measured sections, and 11,117 individual bed-by-bed measurements (Fig. 1 & Fig. 2); context is provided by Hou et al (2020), who collected the data, analyzed the facies, and interpreted the depositional environments from measured sections and photo panels. The measured sections captured the changes in lithology, thickness, sedimentary structures, grain-size, and the nature of contacts at 2 cm resolution. Correlation of the measured sections is difficult due to the lack of datum and structural complexity (Fig. 1). Instead, this study uses composite sections and quadrants to show broad spatial variability, where each cluster of outcrops within the same thrust sheet is lumped into a composite section (Fig. 1c, Supporting Information), and the quadrants are divided by the Y City-Ti Valley faults and the Arkansas-Oklahoma state line (Fig. 2). The foredeep comprises the northeast and northwest quadrants, and the wedge-top the southeast and west-central (Fig. 2a). The southwest is not used because it is covered by the Mesozoic coastal plain deposits. Hou et al (2020) defined three levels of facies hierarchy: bed, lithofacies, and facies associations. Beds represent the deposits of individual sediment gravity flow events, lithofacies are groups of beds, and facies associations groups of lithofacies. The beds comprise four types of sandstones (B1-B4), one type of mudstone (B5), and one type of chaotic bed (B6). The lithofacies comprise three sandstone lithofacies (F1-F3), one mudstone lithofacies (F4), and one disturbed lithofacies (F5). The facies associations comprise channels (FA1), lobes (FA2), mudstone sheets (FA3), and mass transport deposits (FA4). Please refer to Table 1 and Fig. 3 for descriptions and Hou et al. (2020) for details. All statistical analyses are performed in MATLAB 2019a, with codes available in Supporting Information.

Statistical methods

To quantify lithologic features and stacking patterns, four statistical methods are selected: Hurst Statistics, bed thickness frequency distribution analysis, Markov Chains, and time series analysis. The Markov Chains and time series analyses are only used on continuous sections but not composite sections because of the intervening covered intervals (Fig. 1). The composite sections are used for the Hurst Statistics and bed thickness frequency analysis because they do not require continuous measurement.

The Hurst Statistics (or rescaled range analysis) can detect long-term persistence of facies clustering and indicate submarine fan subenvironments (Chen & Hiscott, 1999b; Felletti & Bersezio, 2010a; Kötelešová, 2012). This method is considered superior to other methods such as bed thickness distribution fitting (Pantopoulos *et al.*, 2018) in distinguishing submarine fan environments, because it utilizes more than thickness data and has been proved useful in various submarine fan successions (Chen & Hiscott, 1999b; Felletti & Bersezio, 2010a). For a long section ($N > 100$), the degree of clustering is measured by two variables, Hurst K and D. The Hurst $K = \log_{10}(R/S) / \log_{10}(N/2)$, where R is the range of cumulative departures from the mean, S the standard deviation, and N the sample size. The Hurst D is named in this study; $D = (K - \text{mean}(k)) / S(k)$, where k are the Hurst K values of bootstrap-resampled series, and S(k) is the standard deviation of k. Hurst $K > 0.5$ suggests the presence of long-term persistence (Hurst phenomenon), i.e. high values tend to cluster with high values or low with low values (Mandelbrot, 1982). The sandstone bed thicknesses and grain sizes are used to calculate the Hurst K of each composite section. A sensitivity test is performed to examine the influence of resampling on Hurst D. There are four broad groups of empirically defined subenvironments: channel-levee, lobe-interlobe, basin floor sheet sand (Chen & Hiscott, 1999b), and onlap termination (Felletti, 2004; Felletti & Bersezio, 2010a). The degree of long-term persistence indicates the subenvironment of the section.

Bed thickness frequency analysis is used to compare the degree of confinement. This method is suitable for the study area because it does not require laterally continuous outcrops as other methods so, e.g. bed tabularity method (Fryer & Jobe, 2019; Liu et al., 2018; Tóké & Patacci, 2018). The frequency distributions of turbidite sandstone, mudstone, and event beds are expressed by exceedance probability plots, the thick tail of which is a proxy for the degree of confinement (Malinverno, 1997; Marini et al., 2016; Sinclair & Cowie, 2003), and boxplots which compare the thickness quartiles by quadrant. The Q-Q plots are used to compare the thickness frequency distributions of lobe sandstones and lobes (FA2, Table 1) by quadrant, which can reveal potential hierarchical variations.

Markov Chains method (Burgess, 2016a, 2016b) is used to detect potential stratal order which means non-random arrangement of facies (Burgess, 2016b). This method calculates the transition probabilities between facies class and compares the results to those of random permutations; the degree of orderliness is expressed by Markov Metric and p-value (Burgess, 2016b). The Markov Metric is a summary of the transition probabilities (Fig. 4 b & e) and ranges 0-1 for strata with asymmetric cycles and 0-0.5 for strata with symmetric cycles. Higher Markov Metric values indicate higher degrees of order. The p-value is the probability that the order could have occurred by chance (Fig. 4 c & f). The evidence of order is considered strong, weak, or nonexistent, if the p-value is <0.01, 0.01-0.1, or >0.1, respectively (Burgess, 2016a; b). Specifically, this study uses the optimized Markov Chains method of Burgess (2016a) which involves rearranging the facies codes and minimizes the associated bias. The optimized method is superior in the objective identification of stratigraphic patterns, compared to other methods that only use part of the facies information, such as using thickening and thinning upward trends (Chakraborty et al., 2002; Chen & Hiscott, 1999b; Drummond & Wilkinson, 1996; Mukhopadhyay et al., 2004; Murray et al., 1996). To examine the influence of facies classification and details of observation, this study

uses two levels of facies hierarchy, beds and lithofacies (Table 1; cf. Hou et al., 2020). Sections with less than three types of facies class are excluded. Examples of the Markov Chains results of ordered and disordered sections are shown in Fig. 4.

The time series analysis consists of basic spectral density estimation and autocorrelation (Frigo & Johnson, 1998) based on sandstone thickness and grain size data of each continuous section. Assuming each bed is deposited at a linearly-averaged time-step, the nonuniformly sampled and distributed thickness- and grain size-domains can be converted to uniformly sampled time-domain (cf. Heard et al., 2008; Maurer et al., 2004). It is acknowledged that this practice likely oversimplifies the timing and recurrence of sediment-gravity flow deposits (Clare *et al.*, 2014; Allin *et al.*, 2016; Jobe *et al.*, 2018), but there is no high-resolution age data to for a more realistic age-model. Sections with and without Markov stratal order are analyzed separately. The method includes four steps (Fig. 5; after Li et al., 2019): (a) detrending and correction of non-stationarity, (b) spectral density estimation and conversion of the frequencies to periodicities (beds per cycle, or bpc), (c) autocorrelation and partial autocorrelation, and (d) filtering of periodicities <15 because the signals with the shortest periodicities are likely dominated by background noises (Martini *et al.*, 1978) and bed-scale compensational stacking (Stow *et al.*, 2002). The periodicities are expressed in probability distributions by section, quadrant, or formation (Fig. 5).

RESULTS

Hurst Statistics

All composite sections yield Hurst K values of 0.6-0.95 (Fig. 6), exhibiting Hurst Phenomenon. The sample sizes of most composite sections are 100-1000 and satisfy the requirement (>100) of Hurst K estimator (Chen & Hiscott, 1999b), except S12 which is 65. Both thickness-based and grain size-based results exhibit similar linear trends, but grain size-

based results yield overall greater values (Fig. 6a-b). The results are compared to four groups of previously recognized submarine fan subenvironments (Chen & Hiscott, 1999b; Felletti, 2004; Felletti & Bersezio, 2010a). Using bed thickness data, one composite section falls in Group I, 12 in II, and four in III, and none in Group IV; using grain size, five fall in Group I, nine in Group II, one in Group III, and two in Group IV. The Hurst results are compared to the depositional environments and positions interpreted by Hou et al (2020) and find that not all composite sections at proximal settings yield high Hurst K and D (e.g. S14). Overall, the results concentrate on the lobe-interlobe subenvironment (Fig. 6 a & b). Regionally, the quadrant centroids show a well-defined decrease in Hurst K and D from the southeast to west-central but not from the northeast to the northwest (Fig. 6b).

The influence of resampling on Hurst D has not been investigated in previous work (Chen & Hiscott, 1999b; Felletti, 2004; Felletti & Bersezio, 2010a; Palozzi *et al.*, 2018; Pantopoulos *et al.*, 2018). It is found that Hurst D is irreproducible using the standard 300 runs of resampling. The ranges and standard deviations of the D values for a particular section can be ~1 and ~0.2 respectively, while the range of D for sections from all subenvironments is typically 1-9. To investigate resampling sensitivity on Hurst D, this study uses the longest composite section, S4, and calculates the Hurst D 1000 times; each time, using 200, 400, 800... up to 102400 runs of bootstrap resampling, respectively. The range and standard deviation of Hurst D decrease markedly with the increasing numbers of resampling (Fig. 6c).

Bed thickness frequency distributions

The sandstone beds show a decrease in thickness and the convexity of the exceedance probability plots in the order of the northeast, southeast, west-central, and northwest (Fig. 7). The mudstones show minor differences among the northeast, northwest, and west-central, but are significantly thinner in the southeast. The mudstones are thicker and more convex in

exceedance probability plots in the western quadrants (Fig. 7a). The event bed thickness (sandstone + mudstone) is greater and more convex in exceedance probability plots in the foredeep than the wedge-top (Fig. 7a). When looking only at the beds of lobe deposits interpreted by Hou et al. (2020) (Fig. 7b), the event bed thicknesses decrease from the northeast to the northwest but remain constant from southeast to west-central. The sandstone and mudstone beds from lobe deposits have similar trends comparing to the entire dataset (Fig. 7b).

The Q-Q plots (Fig. 8) compare the thickness quantiles of lobe sandstone beds and lobe facies associations by quadrant and demonstrate longitudinal (proximal-to-distal) and lateral (foredeep vs wedge-top) trends. The quantiles of lobe sandstone beds and lobe facies associations are greater in the eastern quadrants. In the wedge-top, the proximal-distal contrast in quantiles is more pronounced at the facies association level than that at the bed level, while in the foredeep, such contrast is similar in both hierarchical levels (Fig. 8). The lateral trend is not consistent across hierarchical levels. For example, the quantiles of the lobe deposits in the northeast are greater at the bed level but slightly lower at the facies association level comparing to the southeast (Fig. 8). The quantiles at the bed level in the west-central are greater for the thick-bed population comparing to the northwest but are identical at the facies association level.

Markov Chains stratal order analysis

Using bed types (Table 1), 14 continuous sections show strong stratal order, 11 weak order, and 28 disorder (Fig. 9a); using lithofacies types (Table 1, Fig. 3), six show strong stratal order, seven weak order, and 38 disorder (Fig. 9b). Collectively, 56% of the sections exhibit stratal order using either bed types or lithofacies. The bed type-based results show that the wedge-top (orange and green crosses in Fig. 9c) are overall more ordered than foredeep (red,

blue crosses in Fig. 9c), whereas lithofacies-based results show all quadrants appear to be overall disordered (Fig. 9d). The p-values appear to associate with facies classes and the hierarchical level. With the increasing number of facies class, there is an increase in the mean optimized p-values and a decrease in the Mean Markov Metrics (Table 2; Fig. 9). For sections with four facies classes, either bed types or lithofacies, it is more likely to detect order using bed types; for those with five, it is more likely to detect order using lithofacies.

Time Series Analysis on sandstone periodicities

The sandstone recurrence periodicity profiles (Fig. 10) resemble lognormal or mixed-Gaussian distributions. Although all profiles show high amplitudes in the short periodicities and low amplitudes in the long periodicities, the profiles of individual sections appear erratic (thin lines in Fig. 10). However, the profiles by quadrant share some common peaks, particularly those from Markov-ordered sections (Fig. 10). The results of ordered sections (Fig. 10a, c, e, g) differ from those of disordered ones (Fig. 10b, d, f, h) by stronger lognormality, more abundant long periodicities, and a higher degree of similarity among quadrants. The results based on bed thickness and grain size are overall consistent. For ordered sections (Fig. 10a, c, e, g), all quadrants show the highest amplitude in the short periodicities (<30 bpc), while the west-central has stronger signal peaks in the long periodicities (>55 bpc) than others. For example, the signal peak between 90-100 bpc is persistent in all four profiles of the west-central (green in Fig. 10a, c, e, g).

DISCUSSION

Hurst Statistics: discrimination of submarine fan subenvironments

The Hurst Statistics may be influenced by two factors other than depositional environments: sensitivity of the Hurst method and basin confinement. The sensitivity test (Fig. 6c) shows that insufficient bootstrap resampling may lead to irreproducible results. It is considered that

an ideal balance between precision and computational efficiency is 3200 runs where the variability of Hurst D begins to plateau (Fig. 6c). Strong structural confinement can skew the Hurst Statistics which results in discrepancies with facies analyses (Felletti, 2004; Kötelešová, 2012; Pantopoulos *et al.*, 2013, 2018). Therefore, basin geometry and structural history require careful investigation.

Several composite sections of the lower Atoka (e.g. S11) show large discrepancies between bed thickness-based and grain size-based results (Fig. 6a). Similar discrepancies are also found in previous studies (Chen & Hiscott, 1999b; Felletti & Bersezio, 2010a; Kötelešová, 2012) but unexplained. It is interpreted that this discrepancy originates from two main factors. First, bed thickness and grain size data reflect different aspects of the properties of the sediment gravity flows during deposition. Bed thickness is a proxy of the magnitude (i.e., capacity) of the flow, while grain size a proxy of the flow dynamics (i.e., competence) and sediment supply (Sadler, 1982; Paola *et al.*, 1992; Hiscott, 1994; Malinverno, 1997; Basu, 2017). Thicker beds do not necessarily coincide with larger grain sizes and vice versa. Therefore, both aspects are important for interpretation. Second, the grain size cards used in fieldwork have a relatively large error range when the categorical values are converted to numerical values during digitization. The fine-grained nature of the lower Atoka sandstones may have also contributed to the difficulty in assigning accurate grain sizes to some beds (Morris, 1974b; Sprague, 1985).

The Hurst results concentrate on lobe-interlobe domain (Fig. 6a), which is consistent with field observations (Hou *et al.*, 2020), while inconsistencies exist in several composite sections, i.e. S7, S9, S18. For example, S7 and S9 are interpreted as lobe-dominated successions with channelized intervals based on field observations (Hou *et al.*, 2020) but are classified as channel-levee by Hurst Statistics (Fig. 6a), which is likely related to the way individual outcrops are combined into composite sections. S7 consists of 6 outcrops, 219 m

of measured sections, and 483 sandstone beds (Supporting Information), the Hurst result of which may be an amalgamated effect of different subenvironments. Regionally, the quadrant centroids of the northeast and southeast show similar Hurst values (Fig. 6b), consistent with their similar facies compositions (Fig. 2b). The centroid of the west-central yields lower Hurst values than the southeast, consistent with the proximal-distal progression. The abnormally high Hurst values of the northwest centroid are likely influenced by local sediment sources and structural regime. The northwest is influenced by both the main west-prograding fan and the local east-prograding fan which are likely separated by fault blocks (Ferguson & Suneson, 1988; Dickinson *et al.*, 2003; Sharrah, 2006; Hou *et al.*, 2020). In summary, despite some discrepancies discussed above, the results of Hurst Statistics largely (>75%) agree with previous field observations of Hou et al (2020), which supports the validity of Hurst Statistics in discriminating broad submarine fan subenvironments and regional variability of the depositional system.

Bed thickness frequency distribution: degree of confinement

The shape of bed thickness frequency distribution is influenced by depositional and structural controls (Malinverno, 1997; Sinclair & Cowie, 2003; Sylvester, 2007). For unconfined submarine fans, there is typically a proximal-distal decrease in the convexity and the thick tail population of exceedance probability distribution and a decrease in the median sandstone bed thickness (Carlson & Grotzinger, 2001; Fryer & Jobe, 2019; Marini et al., 2016; Pantopoulos et al., 2018; Sinclair & Cowie, 2003). For partially confined fans, the proximal-distal trend may be less pronounced or reversed due to increased sediment bypass and erosion in proximal areas (Tőkés & Patacci, 2018; Fryer & Jobe, 2019).

Both the foredeep and the wedge-top show an east-to-west decrease in the convexity of exceedance probability plots and median sandstone thickness (Fig. 7), consistent with

regional proximal-distal facies trends (Hou *et al.*, 2020). This decrease is less pronounced in the wedge-top despite the contribution of local sources in the distal foredeep (northwest, Fig. 2c), which indicates stronger confinement in the wedge-top and is consistent with the structural history (Fig. 1e). Moreover, the lateral contrast of the thickness quantiles (Fig. 8) suggests that the lobe deposits in the southeast consist of more sandstone beds on average, which indicates that a lower degree of compensational stacking (*sensu* Mutti & Normark, 1987; Straub & Pyles, 2012). Considering the similar facies composition of the northeast and southeast and their distance to the source (Fig. 2), this contrast in lobe stacking pattern may support stronger confinement in the southeast (Fig. 1e). Furthermore, the east-to-west increase in mudstone thicknesses is also consistent with increasing distance to the source. The unusually thin mudstones in the southeast (Fig. 7) can be explained by significant mud bypass during deposition that results in strong sand-mud partition. This partition is common in submarine fans at partially-confined intra-slope (Jobe *et al.*, 2017; Prather *et al.*, 2017) and foreland basin settings (Sinclair & Tomasso, 2002; Marini *et al.*, 2015). Thus, the bed thickness frequency distributions support that the regional confinement in the southeast is strong enough to confine sand deposition but still allows mud to bypass. Alternative interpretations of the bed thickness results may relate to potential contrast in the sediment supply of the foredeep and wedge-top but lack observable evidence. While other studies have interpreted the wedge-top to be more confined (Arbenz, 2008; Hou *et al.*, 2020; Johnson, 2011) than the foredeep, this analysis provides more granularity to the degree and style of confinement. Lastly, this study demonstrates that using multiple bed types (sand, mud, event) for statistical analyses is crucial to understand how seafloor topography can partition sediments by partially confining sediment gravity flows, which would otherwise be missed using only event or sand (coarse-division) beds.

Markov Chains: stratigraphic orderliness

The Markov Chains method (Burgess, 2016a; b) is robust for sections with at least four facies classes and two cycles in length (Fig. 4). Submarine fans are event-dominated environments, and the depositional styles may appear stochastic (Drummond, 1999). The prevalence of orderliness in the lower Atoka sections (56%) suggests that order may be more common than expected in submarine fan environments. Some workers deduce that stratal order is more likely detected at larger and longer spatiotemporal scales, e.g. parasequence vs bedset (Burgess, 2006, 2016b; Groenenberg *et al.*, 2010). Our data show that the percentage of orderliness using lithofacies is not necessarily higher than that using bed types (Table 2, Fig. 9). Therefore, at least from bed to lithofacies (bedset) scales, the relationship between details of observation and stratal order is not well-defined, and further investigations are needed using datasets with more spatiotemporal scales, diversified settings, chronological controls, and more dimensions (2D or 3D).

The origins of stratal order may be equally intrinsic, extrinsic, or convoluted by both intrinsic and extrinsic forcing (Burgess, 2006; Wang *et al.*, 2011; Harris *et al.*, 2020). Furthermore, the order observed in one section (e.g. in a lobe setting) is not necessarily the same with the order in another section (e.g. in a channel setting), as event frequency evolves downslope (Allin *et al.*, 2016; Jobe *et al.*, 2018). This study has explored potential factors including sample location, submarine fan subenvironment, and stacking pattern (Table 3). For bed type-based results, the wedge-top shows overall stronger order than the foredeep (Fig. 9), which may support a structural (i.e., extrinsic) control on stratal order. Architecturally, the ordered sections are more lobe-rich and the lobes are more likely to thicken upwards (Table 3) compared to disordered sections. Thickening-upwards trend in lobe deposits is most likely due to progradation or lateral migration, although significant variability can exist in 1D vertical sections (Kneller, 1995; MacDonald *et al.*, 2011; Prlat & Hodgson, 2013).

Therefore, the origins of the stratal order of the lower Atoka may include structural

confinement in the wedge-top and progradation or lateral migration of lobes. While it is difficult to discriminate between these or other unrecognized signals, these thickening-upward cycles are envisioned to be produced by convoluted extrinsic and intrinsic signals (Burgess, 2006; Groenenberg *et al.*, 2010; Prélat & Hodgson, 2013; Paola, 2016).

Time Series analysis: deciphering environmental signals

The results of the time series analysis are time-averaged characteristics. In this study, the signals are considered as the extrinsic (allogenic, allocyclic) and intrinsic (autogenic, autocyclic) controls (Burgess, 2006; Kneller *et al.*, 2009; Prélat *et al.*, 2010; Jerrett *et al.*, 2016; Hajek & Straub, 2017) that may result in statistically-robust stratigraphic patterns, and noises as the localized factors that lead to stochastic stratigraphic patterns (Drummond, 1999; Burgess, 2006). The periodicity profiles by section appear noisy but those grouped by quadrant are smoother and exhibit several dominant signals (Fig. 10). The similarity in the periodicity profiles of ordered sections suggests the presence of several common signals, whereas the contrast among disordered sections suggests the lack of such signals (Fig. 10). The spectrum of periodicities can be divided into short and long domains (Fig. Fig. 10). To interpret the origins of the dominant signals, investigations are made on other independent studies on potential extrinsic and intrinsic factors that can operate at similar spatiotemporal scales (Dickinson *et al.*, 1994; Hajek & Straub, 2017; Matenco & Haq, 2020).

The duration of the long periodicities (55-100 bpc, Fig. 10) in the lower Atoka is equivalent to 4th-5th order or 10s-100s kyr cycles (Vail *et al.*, 1977), estimated from the total thickness (3000m) and mean sedimentation rate (~1000m/Myr). Such cycles resonate with regional cyclicity in sea level and sediment supply. First, the sea-level cycles are evidenced by transgressive-regressive cycles in the Arkoma shelf during Early Atokan, which are interpreted to be jointly controlled by eustasy and tectonism (Saleh, 2004; Denham, 2018).

Second, cyclical sediment supply is supported by cyclothem deposition of the Upper Pottsville Formation in the Black Warrior Basin (Pashin, 1994, 2004) which is the updip time-equivalent unit of the Atoka Formation. Additionally, some intrinsic processes may also operate at similar timescales (Paola, 2016; Hajek & Straub, 2017) but there is a lack of data and absolute age control in the study area.

The short periodicities (20-30 bpc) are equivalent to 4.6-6.9 m thick packages, using the median event bed thickness of 0.23 m; if the mean thickness (0.44 m) is used, the packages would be 8-13 m. These thickness ranges are typical for partially-confined lobes (elements) worldwide (Prélat *et al.*, 2010; Pettinga *et al.*, 2018; Fryer & Jobe, 2019). Considering the lobe-dominated nature of the lower Atoka, it is interpreted that the short periodicities originate mainly from cyclic lobe avulsions and potentially also high-frequency extrinsic controls. However, the lack of high-resolution chronological constraints inhibits further distilling of these signals. Collectively, despite the stochastic nature of submarine fan settings and non-linear filtering during the propagation of environmental signals (Jerolmack & Paola, 2010; Li & Plink-Björklund, 2019; Romans *et al.*, 2016; Straub *et al.*, 2020), it is possible to distill the dominant extrinsic and intrinsic signals with sufficient knowledge of potential control factors (Burgess *et al.*, 2019), although chronological constraints serve as an important validation.

Advocacy for quantitative data collection and application of statistical methods

There are some caveats in data compilation during the analog-to-digital transition in this study; for example, the lumping of thin sandstones and mudstones can obscure the bed thickness frequency distributions (Sylvester, 2007; Fryer & Jobe, 2019). Similar caveats can be minimized by quantitative, detailed, and well-formatted data collection. It is the hope of the geoscience community that natively-digital collections of graphic logs enable more rapid

and accurate data analysis. The statistical methods in this study are useful for outcrops in structurally complex areas and those with limited lateral exposures (Pantopoulos *et al.*, 2013, 2018; Pinter *et al.*, 2018), because these methods are primarily based on bed-by-bed measurements and can be used to cross-validate architectural interpretations. The methods are also readily applicable to subsurface datasets (e.g. Hernandez-Martinez *et al.*, 2013). Meanwhile, statistical analyses in stratigraphic studies are rapidly evolving, and further testing is recommended in other areas and depositional systems.

CONCLUSION

This study demonstrates the statistical characteristics of the depositional environments, degree of confinement, stacking patterns, and potential control factors of the laterally-confined lower Atoka submarine fan system with a high-resolution, basin-scale outcrop dataset. The Hurst Statistics validates the lobe-interlobe dominated nature of the lower Atoka interpreted by previous work. The bed thickness frequency analyses show significant sand-mud partitioning and mud bypass in the southeast, which supports stronger lateral confinement in this region. The Markov Chains method reveals non-random stratigraphic order in 56% of the measured sections, which is likely related to systematic lobe progradation or lateral migration. Time series analysis demonstrates a potential method to separate the signals that control turbidite sandstone recurrence cycles. This study has implications for submarine fan systems in the genetically-related foreland basins along the Appalachian-Ouachita Orogen and other confined submarine fan systems of the world. The integrated statistical methods can be used to cross-validate the results of conventional (qualitative) facies analysis and extract hidden depositional information from outcrops with limited lateral extent and subsurface datasets.

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FIGURES

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812

813 **Fig. 1.** (a) Ouachita Fold and Thrust Belt relative to the continental US. (b) Paleozoic
 814 structural elements around the Gulf Coast (after Golonka et al., 2007). The foredeep and
 815 wedge-top are separated by the Y City-Ti Valley faults. (c) Structural framework and
 816 geologic map of the Ouachita Mountains showing the outcrop localities and composite
 817 sections of the lower Atoka formation (after Arbenz, 2008). For detailed localities see
 818 Supporting Information. (d) Carboniferous stratigraphy, relative sea level, precipitation, and
 819 tectonic intensity of the Ouachita Basin (after Coleman, 2000; Heckel and Clayton, 2006;
 820 Suneson, 2012). Hart: Hartshorne Formation. (e) Tectonic evolution of the Ouachita
 821 Mountains from Late Morrowan to Middle Atokan, showing structural contrast between the
 822 foredeep and the wedge-top (after Arbenz, 2008).

Fig. 2. (a) Boundaries of quadrants and representative measured sections with paleocurrent rose diagrams (from flute casts) of the lower Atoka formation. (b) Lithofacies and facies association thickness proportions by quadrant showing broad regional variations. (c) A conceptual model of the regional paleogeography (Coleman, 2000) and an updated structural-depositional model showing contrasting confinement in the foredeep and wedge-top (Hou *et al.*, 2020).

Fig. 3. Outcrop examples of the facies scheme in this study (Hou *et al.*, 2020). The lithofacies (a) include F1 – massive, amalgamated sandstone, F2 – thick-bedded sandstone with minor mudstone, F3 – thin-bedded sandstone and mudstone, F4 – mudstone with minor sandstone, F5 – disturbed mudstone and sandstone. The facies associations (b) include FA1 – channel, FA2 – lobe, FA3 – mudstone sheet, and FA4 – mass transport deposit (MTD). Descriptions are listed in Table 1.

Fig. 4. Examples of Markov Chains stratal order analysis of an ordered section (a) and a disordered section (d) of the lower Atoka using the MATLAB program of Burgess (2016a). (b) and (c) are transition probability matrices. The evidence of order is visualized by the degree of overlap between the observed Markov Metric and those of random permutations (histogram) and expressed as p-values (c & f).

Fig. 5. Time series analysis in this work using sandstone bed thickness data of lower Atoka Sec.4 as an example. (a) Detrending. (b) Spectral density estimation. The frequencies are then converted to the periodicities. (c) Sample autocorrelation. The peaks in the autocorrelation function indicate significant periodicities. Confidence bounds are 2 standard errors. (d) the Kernel Density Estimations (KDE) of the periodicities by different methods show consistent results, based on all ordered sections of the lower Atoka. Inspired by Li et al (2019) and Starek & Fuksi (2017).

Fig. 6. Scatter plots of Hurst K and D values of the lower Atoka labeled by composite section (a) and by quadrant (b), respectively. The boundaries of the subenvironments are based on Chen and Hiscott (1999), Felletti (2004), Felletti and Bersezio (2010). (c) Testing the sensitivity of bootstrap resampling on Hurst D.

Fig. 7. Turbidite bed thickness frequency distributions of the entire lower Atoka dataset (a), and the lobe facies associations only (b). The turbidite sandstones, mudstones, and event (sand-mud couplet) beds are shown separately.

Fig. 8. Quantile-Quantile plots of lobe sandstone bed thicknesses and lobe facies association thicknesses showing the longitudinal, lateral, and hierarchical variations. Note the consistent decrease in quantiles from proximal to distal locales for both bed and facies associations levels.

Fig. 9. Results of Markov Chains stratal order analysis (Burgess, 2016a) of the continuous sections of the lower Atoka based on bed types (a) and lithofacies (b). (c) and (d) show the results plotted by quadrants and marked with centroids, based on bed types and lithofacies types, respectively. Source data are in the Supporting Information.

Fig. 10. Turbidite sandstone recurrence periodicity distributions of the lower Atoka by continuous section (thin lines) and quadrant (thick lines). The periodicities are calculated using thickness (a, b, c, d) and grain size (e, f, g, h) respectively. The Markov ordered (a, c, e, g) and disordered (b, d, f, h) sections are shown separately.

TABLES

Table 1. Descriptions of the facies scheme in this study including beds, lithofacies, and facies associations

BED	Code	Name	Thickness (cm)	Descriptions	
	B1	very thick	>100	massive/planar-stratified; basal scour and loading structures	
	B2	thick sand	30-100	graded/planar-stratified; local basal scour and loading	
	B3	thin sand	10-30	planar-/ripple-laminated or graded; flat base and rippled-top	
	B4	very thin sand	2-10	ripple-/planar-laminated; rippled or flat top	
	B5	mud	>2	fissile, finely-laminated, or massive	
	B6	chaotic	5-1300	slump fold, fault; matrix-supported, rip-ups; mud-rich	
LITHOFACIES	Code	Name	Descriptions	Inferred Process	
	F1	massive, amalgamated sandstone	structureless, graded, or ripple-/planar-stratified; loading structures, internal scour, mud clasts, plant debris common; trace fossils absent	rapid deposition from large magnitude, high-density turbidity currents	
	F2	thick-bedded sandstone with minor mudstone	graded, planar-/cross-stratified, or weakly ripple-laminated; sole marks common; occasional loading, dewatering, and mud clasts; trace fossils uncommon	rapid deposition from high-density turbidity currents	
	F3	thin-bedded sandstone and mudstone	planar/ripple laminated; sole marks, trace fossils common; ripples occasionally convoluted; mudstones silty or heterolithic	deposition from low-density turbidity currents	
	F4	mudstone with minor sandstone	massive, parallel- or ripple-laminated; terrestrial plant debris, trace fossils common on bedding planes; sandstones thin or very thin	deposition from dilute turbidity currents and hemipelagic fallout	
	F5	disturbed mudstone and sandstone	ductile, plastic, or brittle soft-sediment-deformations; matrix-supported sand-mud clasts	deposition from slump, slide, or debris flow	
FACIES ASSOCIATION	Code	Name	Compositions	Descriptions	Inferred Sub-environment
	FA1	channel	major F1, F2; minor F5, F3	decimeter-meters of basal erosion; rapid changes in bed thickness/dip; concave-upward, wedge, or lens-shaped; occasionally filled with F5	submarine channels
	FA2	lobe	major F2, F3, some F1	flat basal contact with minor or no erosion; little or no change in bed thickness/dip; sandstones graded with Bouma Sequence; mudstone interval $\leq 0.4\text{m}$.	lobe or basin floor fan
	FA3	mudstone sheet	major F4, minor F3	flat, non-erosive basal and top contacts; isolated thin-bedded sandstones; occasionally interrupted by FA4; $>0.4\text{m}$	interlobe/levee/basinal mud drape
	FA4	mass transport deposit (MTD)	major F5, minor F4	single or multiple disturbed deposits not within channel, including thin muds in between ($\leq 0.4\text{m}$)	submarine mass failure

Table 2. Comparisons of the detail of observation, facies class, and stratigraphic orderliness of the lower Atoka formation

Bed Types	Number of Outcrops	Mean Markov Order Metric	Mean P-Value	Ordered Percent	Lithofacies Types	Number of Outcrops	Mean Markov Order Metric	Mean P-Value	Ordered Percent
6	Total: 11 Ordered: 8	0.3621 0.3902	0.1432 0.0129	69.6%			(not applicable)		
5	Total: 23 Ordered: 13	0.3402 0.4048	0.1694 0.0201	56.5%	5	Total: 18 Ordered: 11	0.3289 0.3674	0.1940 0.0409	61.1%
4	Total: 15 Ordered: 4	0.2410 0.3218	0.5558 0.0396	26.7%	4	Total: 19 Ordered: 2	0.2235 0.3847	0.4703 0.0411	10.5%
3	Total: 4 Ordered: 0	0.3783	0.8530	0.0%	3	Total: 14 Ordered: 0	0.2269	0.7928	0.0%

Table 3. Depositional comparisons of ordered and disordered sections of the lower Atoka formation

Mean values	Ordered	Disordered	Remarks
Section Length (m)	56.27	39.43	ordered sections are longer
Number of Bed Class	5.30	4.40	ordered sections have more bed class
Percent Sand	68%	59%	ordered sections are more sand-rich
Amalgamation Ratio	36%	36%	no difference
StdDev of Sand Bed	0.25	0.24	similar
Markov Order Metric	0.40	0.26	ordered sections are higher
Channel Thickness	12%	12%	no difference
Channel Frequency	8%	10%	similar
Lobe (FA2) Thickness	56%	45%	ordered sections have more FA2 by thickness
Lobe Frequency	46%	41%	ordered sections have more FA2 by frequency
Mud Sheet (FA3) Thickness	27%	37%	disordered sections have more FA3 by thickness
Mudstone Sheet Frequency	41%	42%	similar
MTD (FA4) Thickness	3%	4%	similar
MTD Frequency	2%	4%	disordered sections have more FA4 by frequency
Lobe: Thickening-Upward	55%	39%	lobes in ordered sections more often thicken
Lobe: Thinning-Upward	33%	36%	similar
Lobe: Constant	13%	26%	disordered sections have more lobes with constant vertical thickness trend