Tangent Vectors on Tangent Euclidean Spaces

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Abstract

This is an article on differential geometry that connects tangent vectors and tangent Euclidean spaces [1].

keywords: differential geometry, Euclidean space, tangent vector, tangent space

The most updated version of this white paper is available at
https://osf.io/vpz9x/download

The tangent space in each point of $\mathbb{R}^3$ is $\mathbb{R}^3$

1. In the following, for the sake of simplicity, sometimes we will omit the words "let", "consider", etc, in which is implicitly assumed hereafter.

Tangent vector

2. The set of all *ordered triples* of real numbers $\mathbf{p} = (p_1, p_2, p_3)$, called *points*, is a *Euclidean 3-space* $\mathbb{R}^3$.

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3. **p** and **v** are points of $\mathbb{R}^3$.

4. $\mathbb{R}^3$ is a vector space over the real numbers.

5. So, **p** and **v** are also vectors.

6. Let **p** be the starting point and **p** + **v** the end point.

7. **v** is a *vector* called the *change* of **p**.

8. Let's call **p** its *point of application* and **v** the *vector part.*

9. $\mathbf{v}_p$ is a **tangent vector** to $\mathbb{R}^3$ if it consists of two points (vectors), **p** and **v**.

10. An example of (9) is: $\mathbf{p} = (\mathbf{0}, \mathbf{0}, \mathbf{1})$, $\mathbf{v} = (\mathbf{0}, \mathbf{1}, \mathbf{1})$ and $\mathbf{v_p} = (\mathbf{0}, \mathbf{1}, \mathbf{2})$.

11. In (10), $\mathbf{v_p}$ consists of **p** and **v** because $\mathbf{v_p} = \mathbf{p} + \mathbf{v}$.

Tangent space

12. Consider T_p as the set of all tangent vectors having **p** as a point of application.

13. Then T_p is the **tangent space** of $\mathbb{R}^3$ at **p**.

14. **The tangent space in each point of $\mathbb{R}^3$ is $\mathbb{R}^3$**.

Final Remarks

15. This article connected the concept of a **tangent vector** with the concept of a **tangent Euclidean space**.

16. It has a *pure mathematical application* here [2].

Open Invitation

Review, add content, and `co-author` this white paper [3, 4].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

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The **latex file** for this *white paper* together with other *supplementary files* are available in [5].

How to cite this paper?

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Agreement

All authors agree with [4].

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