[microreview]

Diamond Open Access

The power set of power sets

Open Mathematics Collaboration^{*†}

March 10, 2020

Abstract

We compute $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$, where $\mathcal{P}(\emptyset)$ is the power set of the empty set.

keywords: power set, empty set

Definition of the Power Set

1. *If S is a set, then the power set of S is the set that consists of all the subsets of S [1, 2].*

^{*}All authors with their affiliations appear at the end of this paper.

[†]Corresponding author: mplobo@uft.edu.br | Join the Open Mathematics Collaboration

The power set of the power set

2. $\emptyset = \{ \}$
3. Let $\mathcal{P}^n(\emptyset) = \mathcal{P}(\mathcal{P}(\dots\mathcal{P}(\emptyset)))$ have n power sets.
4. $\mathcal{P}^1(\emptyset) = \mathcal{P}(\emptyset) = \{\emptyset\}$
5. $\mathcal{P}^2(\emptyset) = \mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$
6. $\mathcal{P}^3(\emptyset) = \mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

Final Remarks

7. The *power set* of the *power set* of the *power set* of the *empty set* is

$$\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}.$$

Note

- 8.

$$\begin{aligned}\mathcal{P}^4(\emptyset) &= \mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))) = \\ &= \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}, \\ &\quad \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\{\emptyset\}\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}, \{\{\emptyset\}, \{\{\emptyset\}\}\}, \\ &\quad \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}, \\ &\quad \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \\ &\quad \{\{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}, \\ &\quad \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}\end{aligned}$$

9.

$$\mathcal{P}^n = ?$$

10.

$$\mathcal{P}^\infty = \lim_{n \rightarrow \infty} \mathcal{P}^n = ?$$

11.

$$\mathcal{P}^\omega = ?$$

12.

$$\aleph_{\mathcal{P}^\infty} = ?$$

13.

$$\aleph_{\mathcal{P}^\omega} = ?$$

14.

$$\aleph_{\mathcal{P}^\omega} > \aleph_{\mathcal{P}^\infty} ?$$

Open Invitation

*Review, add content, and **co-author** this paper [3, 4].*

*Join the **Open Mathematics Collaboration**.*

Send your contribution to mplobo@uft.edu.br.

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [5], please cite it accordingly [6]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

- [1] Simovici, Dan, and Richard L. Tenney. *Theory of formal languages with applications*. World Scientific Publishing Company, 1999.

- [2] Velleman, Daniel J. *How to prove it: A structured approach*. Cambridge University Press, 2019.
- [3] Lobo, Matheus P. “Microarticles.” *OSF Preprints*, 28 Oct. 2019. <https://doi.org/10.31219/osf.io/ejrct>
- [4] Lobo, Matheus P. “Simple Guidelines for Authors: Open Journal of Mathematics and Physics.” *OSF Preprints*, 15 Nov. 2019. <https://doi.org/10.31219/osf.io/fk836>
- [5] COS. *Open Science Framework*. <https://osf.io>
- [6] Lobo, Matheus P. “The Power Set of Power Sets.” *OSF Preprints*, 8 Mar. 2020. <https://doi.org/10.31219/osf.io/v5zyb>

The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br)^{1,2}

¹Federal University of Tocantins (Brazil); ²Universidade Aberta (UAb, Portugal)