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[white paper: pedagogical]
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The terms of a language with one constant, one binary function, and one 4-ary function have an odd number of symbols

Open Mathematics Collaboration*†

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Abstract

We show using induction on complexity that all terms of a language with one constant, one binary function, and one 4-ary function have an odd number of symbols.

keywords: language, induction on complexity, first-order logic

The most updated version of this white paper is available at
https://osf.io/ue32a/download
https://zenodo.org/record/5457880

Introduction

1. This is a `pedagogical white paper` on *first-order logic*.
2. Our purpose is to discuss a result in [1] which is licensed under [2].
3. We use minimal notation but preserving all relevant mathematical information.

*All *authors* with their *affiliations* appear at the end of this white paper.
†Corresponding author: mplobo@uft.edu.br | **Open Mathematics Collaboration**

Meta-linguistic symbols

4. $:=$ means that what is on the left is defined by what is on the right.
5. $:\equiv$ means that the strings on both sides are identical.
6. `a,b` $\vdash$ `c` means deduction of `c` from `a,b`.

Proposition

7. $$\mathscr{L} = \{0, f, g\} \rightarrow \text{every } \mathscr{L}\text{-term has an odd number of symbols}$$
8. $\mathscr{L}$:= language
9. 0 := constant symbol
10. f := binary function symbol
11. g := 4-ary function symbol

Proof of (7) by induction on complexity

12. t := $\mathscr{L}$-term
13. We need to prove that t *has an odd number of symbols*.

Case 1

14. $t :\equiv 0$
15. t has 1 symbol.

Case 2

16. t is a variable.
17. t has 1 symbol.

Case 3

18. $t :\equiv f t_1 t_2$

19. $n :=$ number of symbols in t

20. Inductive hypothesis: *a term has an odd number of symbols.*

21. Let n_1 and n_2 be the number of symbols in t_1 and t_2, respectively.

22. (20) $\vdash$ n_1 and n_2 are odd numbers.

23. $n = n_1 + n_2 + 1$

24. (22), (23) $\vdash$ n is odd.

Case 4

25. $t :\equiv g t_1 t_2 t_3 t_4$

26. This case is similar to Case 3. □

Open Invitation

Review, *add* content, and `co-author` this white paper [3, 4].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [5, 6].

How to cite this paper?

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Agreement

All authors agree with [4].

References

[1] Leary, Christopher C., and Lars Kristiansen. *A friendly introduction to mathematical logic*, 2nd edition, 2015.
https://knightscholar.geneseo.edu/geneseo-authors/6

[2] CC. *Creative Commons*. https://creativecommons.org

[3] Lobo, Matheus P. "Microarticles." *OSF Preprints*, 28 Oct. 2019.
https://doi.org/10.31219/osf.io/ejrct

[4] Lobo, Matheus P. "Simple Guidelines for Authors: Open Journal of Mathematics and Physics." *OSF Preprints*, 15 Nov. 2019.
https://doi.org/10.31219/osf.io/fk836

[5] Lobo, Matheus P. "Open Journal of Mathematics and Physics (OJMP)." *OSF*, 21 Apr. 2020. https://osf.io/6hzyp/files

[6] https://zenodo.org/record/5457880

The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br)[1,2]
https://orcid.org/0000-0003-4554-1372

[1]Federal University of Tocantins (Brazil)
[2]Universidade Aberta (UAb, Portugal)