[white paper: pedagogical]

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Goldbach Conjecture, Twin Primes Conjecture, and Bounded Gap Theorem in the Language of Number Theory

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Abstract

We write the formulas of the theorem and the conjectures highlighted in the title of this white paper in the language of number theory for pedagogical purpose in first-order logic.

keywords: Goldbach conjecture, bounded gap theorem, number theory language, first-order logic

The most updated version of this white paper is available at

<https://osf.io/u45y3/download>

<https://zenodo.org/record/5428990>

Introduction

1. This is a pedagogical white paper on *first-order logic*.
2. Our purpose is to discuss a result in [1] which is licensed under [2].
3. We use minimal notation but preserving all relevant mathematical information.

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Meta-linguistic symbols

- 4. $:=$ means that what is on the left is defined by what is on the right.
- 5. $\equiv$ means that the strings on both sides are identical.

The language of number theory

- 6.
$$\mathcal{L}_{NT} = \{0, S, +, \cdot, E, <\}$$
- 7. $0 :=$ number zero
- 8. $S(x) = x + 1 :=$ successor function
- 9. $+, \cdot, < :=$ usual binary operations
- 10. $E :=$ exponentiation (e.g. $E(3, 2) = 9$)
- 11. $\mathcal{L}_{NT} :=$ the language of number theory assumed to be interpreted with respect to $\mathbb{N} = \{0, 1, 2, 3, \dots\}$

Prime number

- 12. $n \in \mathbb{N}$
- 13. $\bar{n} := \underbrace{SS \dots SS}_n 0$
 n times
- 14. $\text{prime}(x) \equiv \bar{1} < x \wedge \neg(\exists y)(\exists z)[(y + \bar{2}) \cdot (z + \bar{2}) = x]$
- 15. Note that $+\bar{2}$ appears summing y and z in (14) due to (11).
- 16. $\text{prime}(n)$ states that n is a prime number.

There is no largest prime

17.

$$(\forall x)(\exists y)[x < y \wedge \text{prime}(y)]$$

Twin Primes Conjecture

18. *There are infinitely many pairs (x, y) such that x and y are both prime and $y = x + 2$.*

19.

$$(\forall x)(\exists y)[x < y \wedge \text{prime}(y) \wedge \text{prime}(y + \bar{2})]$$

20.

$$(\forall x)(\exists y)(\exists z)[x < y \wedge y < z \wedge \text{prime}(y) \wedge \text{prime}(z) \wedge z < y + \bar{3}]$$

21. (19) and (20) are equivalent.

Goldbach Conjecture

22. *Any even number, except 0 and 2, can be written as a sum of two primes.*

23.

$$(\forall x)[(\exists y)[(y + \bar{2}) \cdot \bar{2} = x] \rightarrow (\exists u)(\exists v)[\text{prime}(u) \wedge \text{prime}(v) \wedge u + v = x]]$$

Bounded Gap Theorem

24. *There are infinitely many pairs of prime numbers that differ by 70,000,000 or less.*

25.

$$(\forall x)(\exists y)(\exists z)[x < y \wedge y < z \wedge \text{prime}(y) \wedge \text{prime}(z) \wedge z < y + \overline{70\,000\,000}]$$

26. This theorem was proved by Yitang Zhang in 2013 [3–5].

Final Remarks

27.

$$\mathcal{L}_{NT} = \{0, S, +, \cdot, E, <\}$$

28.

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

29.

$$\overline{1} < x \wedge \neg(\exists y)(\exists z)[(y + \overline{2}) \cdot (z + \overline{2}) = x]$$

30.

$$(\forall x)(\exists y)[x < y \wedge \text{prime}(y)]$$

31.

$$(\forall x)(\exists y)(\exists z)[x < y \wedge y < z \wedge \text{prime}(y) \wedge \text{prime}(z) \wedge z < y + \overline{3}]$$

32.

$$(\forall x)(\exists y)[x < y \wedge \text{prime}(y) \wedge \text{prime}(y + \overline{2})]$$

33.

$$(\forall x)[(\exists y)[(y + \overline{2}) \cdot \overline{2} = x] \rightarrow (\exists u)(\exists v)[\text{prime}(u) \wedge \text{prime}(v) \wedge u + v = x]]$$

34.

$$(\forall x)(\exists y)(\exists z)[x < y \wedge y < z \wedge \text{prime}(y) \wedge \text{prime}(z) \wedge z < y + \overline{70\,000\,000}]$$

Open Invitation

Review, add content, and co-author this white paper [6, 7].

*Join the **Open Mathematics Collaboration**.*

Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [8, 9].

How to cite this paper?

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Agreement

All authors agree with [7].

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