Prediction of Critical Factor of Safety in Soil Slopes Using Adaptive Network-Based Fuzzy Inference System (ANFIS)

Khadijeh Shirzad

PhD Student, Department of Computational Science and Engineering

North Carolina A&T State University, Greensboro, NC

Email: nrpugh@aggies.ncat.edu

Babak Vafaei

PhD Student, Department of Civil, Architectural and Environmental

University of Miami, Miami, FL

Email: bxv158@miami.edu

Hyoshin Park, Ph.D.

Visiting Professor, Robotics Mobility Group 347,

NASA Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA

Assistant Professor, Department of Computational Science and Engineering

North Carolina A&T State University, Greensboro, NC

Email: hpark1@ncat.edu

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ABSTRACT

Slope stability analysis is among the uncertain problems in geotechnical engineering; This is mostly because of the presence of the complex nature of the soil and rock mass and uncertain parameters affecting the problem. To Determine the structure of the model and deal with the complexity of the geotechnical behavior of soil a data approach is required. Previous studies in the literature do not predict slope stability with high accuracy. In this study, to increase the accuracy of prediction of slope stability, the Adaptive Network-Based Fuzzy Inference System (ANFIS) was used. Three clustering methods are used in combination with ANFIS, namely the Grid Partitioning (GP) method, Subtractive Clustering Method (SCM), and Fuzzy C-means Method (FCM) for predicting the critical Factor of Safety (FS). Since the coefficient of determination R^2 in the SCM model is closer to 1, the prediction performance of the SCM model outperforms the GP and FCM. The results of the study indicate that ANFIS can provide more accurate predictions compared to other conventional slope stability analysis approaches. ANFIS method efficiently reduced the uncertainties involved in the estimation and analysis of slope stability as compared to previous approaches in the literature.

Keywords: Soil Slope Stability, Artificial Intelligence, Factor of Safety, Adaptive Network-Based Fuzzy Inference System

1 INTRODUCTION

2 Slope failures are natural phenomena causing significant natural disasters in many
 3 countries. The irreparable consequence imposes fatal and financial damages to both private and
 4 public sectors. Hence, a reliable analysis of slope failures is essential in order to prevent and
 5 minimize the damages. Slope stability analysis is not a simple practice due to the mutual
 6 interactions of the variables affecting the slope stability. Deriving a mathematical description for
 7 slope stability is not a trivial task. The objective of the conventional methods of slope analysis is
 8 to provide an estimation of the stability by evaluating the critical Factor of Safety (FS), which is
 9 considered one of the most challenging problems in soil and rock mechanics (1). Geometrical data,
 10 physical information on the geologic materials, and their shear-strength parameters (cohesion and
 11 angle of internal friction), data on pore-water pressures needed for the calculation of the FS (2-3).
 12 Based on the analytical approaches, one can identify the most critical sliding surface and estimate
 13 the FS. However, analytical methods require input factors for every single point along the slope.
 14 Hence, in practice, analytical methods are only appropriate for predicting the stability in a small
 15 area (4).

16 Although the nature of the problem is statistically uncertain, some simplifying assumptions are
 17 essential to obtain a critical FS (5). Considering different assumptions, various techniques have
 18 been proposed by (6-11). The use of numerical methods has recently grown in simulation of the
 19 behavior of the slope. Recent attempts were mostly done by finite element method (FEM) (12-
 20 16) and finite difference method (FDM) (17-18). The prediction and analysis of slope stability
 21 using probabilistic methods are also considered as a powerful tool (19-20). However, in fact, the
 22 amount of the available data is limited, making the use of probabilistic methods demanding.
 23 Generally, machine learning models are build up based on historical databases (21-22). The
 24 slope stability assessment can be defined as a classification problem in which prediction is either
 25 “stable” or “unstable” by utilizing artificial intelligence (AI) models. Machine learning
 26 techniques can learn events of slope failure in the past and produce predictive results for
 27 unlabeled patterns (23).

28 In recent years, Many researches have utilized the Artificial Neural Network (ANN) to predict
 29 slope stability(24-29). Samui and Kothari(30) applied the Least Squares Support Vector Machine.
 30 Zhao et al. (31) utilized the Relevance Vector Machine (RVM) to investigate the nonlinear correlation
 31 between slope stability and its critical parameter.

32 Support Vector Machine (SVM) has been developed to predict slope stability. It was identified
 33 that SVM prediction models have additional benefits to slope stability evaluation over the ANN model
 34 under the condition of restricted data (32-34). Gordan et al. (35) have proposed a combination of
 35 particle swarm optimization and neural network(PSO-ANN) for prediction of slope stability.
 36 They use critical factors that affect slope stability as the input of their model and could predict
 37 FS with better accuracy compared to ANN.

38 Adaptive Network-Based Fuzzy Inference System (ANFIS) is today considered as an
 39 efficient prediction method, used in different scientific applications. By utilizing the learning
 40 ability of the neural networks along with the linguistic variables and expressions used in fuzzy
 41 systems, the systems employing this method take advantage of two models for analyzing complex
 42 processes. In ANFIS, the parameters of the membership perform are obtained from the dataset
 43 showing the system behavior. Then, ANFIS trains the characters present in the dataset and
 44 accommodates the system parameters according to a given error criterion (36).

In this study, three models, namely the Grid Partitioning (GP) method, Subtractive Clustering Method (SCM), and Fuzzy C-means Method (FCM) were used for predicting the factor of safety. To evaluate the prediction obtain from these three methods, they are compared with the prediction obtained by neural network approach.

DATASET

In this study the analyses of factor of safety were conducted on a dataset, including 103 data associated with a circular failure mechanism. The dataset was collected in the literature review from Sah et al. (37) and Zhi-Qiang(38). Six samples of which used in the ANFIS model for training are given in **Table 1**. The studies conducted on slope stability indicate that the slope is affected by six parameters, namely the soil density(γ), cohesion (C), angle of internal friction(ϕ), slope angle (β), slope height (H), and pore water pressure (r_u), (Manouchehrian et al.(39). These parameters were used as the input to our ANFIS model, and the FS was expected as the output.

TABLE 1 Sample data used in ANFIS

NO.	γ (kN/m ³)	c (kPa)	ϕ -	β -	H (m)	r_u	FS -	$Stability$
1	18.68	26.34	15	35	8.23	0.00	1.11	F
2	16.50	11.49	0	30	3.66	0.00	1.00	F
3	18.84	14.36	25	20	30.50	0.00	1.88	F
4	28.44	29.42	35	35	100.00	0.00	1.78	F
5	28.44	39.23	38	35	100.0	0.00	1.99	F
6	20.60	16.28	27	30	40.00	0.00	1.25	F

FUZZY INFERENCE SYSTEM(FIS)

Classical logic only permits binary conclusions, i.e., True or False, 0 or 1, and Black or White. On the contrary, the fuzzy logic assigns a value ranging from 0 to 1 to express the degrees of truth for a statement (40). Fuzzy reasoning is required to infer results from the rules, the form of which is described as IF-THEN statements and is known as “FUZZY IF-THEN” rules. Their general form is the following:

IF x is A THEN y is B

Where the ‘if’ a part of the rule is that the antecedent, whereas the ‘then’ half is named the consequence. x and y are known as the linguistic variables, and A and B the linguistic values, that are outlined as a set of the universes of x and y , respectively. The membership function is the fundamental concept of fuzzy logic, numerically representing the degree to that a given component belongs to a fuzzy set. The number of membership functions, their location, and their form (e.g., triangular or trapezoidal) may be introduced by expert judgment or by data analysis (41). Mamdani is known as the most common fuzzy inference technique involving the following four stages:

1. Fuzzification of the input variables,
2. Rule evaluation,
3. Aggregation of the rule outputs, and
4. Defuzzification

FIS is a well-known computational system based on fuzzy theory. FIS includes a fuzzification interface and a defuzzification interface. Fuzzification interface transforms the input data to fuzzy values, a rule base containing a set of IF-THEN rules, a database defining the membership functions to be used with the fuzzy rules, a decision-making unit responsible for inferring from the rules and the inputs, while a defuzzifier transforms fuzzy results into a output (42).

ADAPTIVE NETWORK-BASED FUZZY INFERENCE SYSTEM

In recent years, ANFIS have been employed in different scientific applications. By utilizing the learning ability of the neural networks along with the linguistic variables and expressions used in fuzzy systems, these systems have been able to employ the advantages of FIS and ANN methods for the powerful analysis of complex processes. Both FIS and ANN have their own advantages and weaknesses. The fuzzy system is capable of utilizing the human language as well as the experience of expert individuals but it is incapable of learning. In other words, the data collected from observation cannot be used to train the fuzzy system. On the other hand, neural networks are capable of self-learning when provided with the appropriate datasets but are incapable of using human language as they are implicit neural networks. In 1993, Jang successfully combined the linguistic power of FIS with the learning ability of the neural networks for the first time, presenting an ANFIS to overcome the weaknesses of the both systems. ANFIS is an adaptive, trainable network that is very similar to FIS in terms of operating mechanism. To simplify the calculation procedure, without loss of generality, we may assume the input variables x and y , and the output variable z for our system. Given the following rules (Equation 1) (43):

Rule1: if x is A_1 and y is B_1 then $f_1 = p_1x + q_1y + r_1$

Rule2: if x is A_2 and y is B_2 then $f_2 = p_2x + q_2y + r_2$ (1)

Using the center of area as the defuzzification method, the output is obtained as (Equation 2) (44):

$$f = \frac{w_1f_1 + w_2f_2}{w_1 + w_2} = \bar{w}_1f_1 + \bar{w}_2f_2 \text{ st } \bar{w}_1 = \frac{w_1}{w_1 + w_2}, \bar{w}_2 = \frac{w_2}{w_1 + w_2} \quad (2)$$

The equivalent ANFIS structure is as **Figure 1**:

Figure 1 The structure of the ANFIS network (36)

Output of the layers: In our experiment, a neural fuzzy model consists of five layer(45).

Layer 1: Inputs in this layer use the membership functions (Equation 3) (44).

$$\begin{aligned} O_{1,i} &= \mu A_i(x), & \text{for } i = 1,2 \\ O_{1,i} &= \mu B_i(x), & \text{for } i = 3,4 \end{aligned} \quad (3)$$

The membership functions can be selected as any appropriate parametric function. Mostly, Gaussian functions are used, the general relation for which is defined as follows (Equation 4):

$$\mu A(x) = \frac{1}{1 + \left| \frac{x - c_i}{a_i} \right|^{2b_i}} \quad (4)$$

where $\{a_i, b_i, c_i\}$ is the parameters set. The parameters in this layer are known as the premise parameters.

Layer 2: The output of this layer is obtained by multiplying the input signals, which in fact is the equivalent for the antecedents proceeding the IF conjunction (Equation 5).

$$O_{2,i} = w_i = \mu A_i(x) \mu B_i(y), \quad i = 1,2 \quad (5)$$

Layer 3: The output of this layer is obtained by normalizing that of the previous layer (Equation 6):

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1,2 \quad (6)$$

Layer 4 (Equation 7):

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \quad (7)$$

Layer 5: The output of the layer is the overall output of the system (Equation 8):

$$O_{5,i} = \sum_i \bar{w}_i f_i = \frac{\sum_i \bar{w}_i f_i}{\sum_i \bar{w}_i} \quad (8)$$

ANFIS MODELS

Various ANFIS models employing different identification methods can be applied to a given dataset. The three methods, namely GP method, SCM, and FCM method were used in this study in order to identify the output membership functions. Since the identified structure by each of these methods is different and different membership functions can be considered for the input, each method is individually investigated, and its membership function is demonstrated in this section.

Grid Partitioning Method

Fuzzy networks based on the input and output systems of the training data can generate the fuzzy rules. Hence, creating such a network is a time-consuming process. Moreover, since the membership functions represent the significance of the linguistic terms in a particular context, the adaptive fuzzy grid partitioning can be used to modify and optimize the parameters of membership function, simply by partitioning the domains of the antecedent variables into many equally shaped and spaced membership functions. In this method, the grid partitioning is uniform and is used with

an initial value. The main objective of this method is to describe the behavior of the system using the input and output data based on the physical behavior and the practical knowledge about the unknown systems (46).

Subtractive Clustering Method

SCM was developed by Chiu in 1994(47). In this algorithm, it is assumed that each data point may be a potential cluster center and calculates a degree of the probability that each data point would define the cluster center based on the density of surrounding data points. Assuming m is the dimension of the algorithm, and the n data points are $(x_1, x_2, \dots, x_n)$, the density function D_i of the data points x_i are using the following Relations (Equation 9):

$$D_i = \sum_{j=1}^n \exp \left(- \left\| \frac{x_i - x_j}{\left(\frac{r_a}{2} \right)^2} \right\|^2 \right) \quad (9)$$

$$D_i - D_{ci} = \exp \left(- \left\| \frac{x_i - x_j}{\left(\frac{r_a}{2} \right)^2} \right\|^2 \right) \quad (10)$$

where r_a is a positive constant, each data point can be better generalized using Equation 10.

The density function of the data points is decreased compared to the first cluster; hence, they are not considered in determining the cluster center. r_b is selected as the closest surface on which the density function of the data points is decreased and usually assumes a constant value such that $r_b > r_a$. In order to prevent the close cluster centers from interfering, the following Relation can be used (Equation 11) (48):

$$r_b = \eta \cdot r_a \quad (11)$$

Fuzzy C-Means Method

This method was first introduced in 1973 by James Bezdek(49). In this algorithm, the samples are divided into c clusters, the value of which is already given. FCM is among the best methods for minimizing the mean squared error of the weights using the following Relation (Equation 12):

$$J(U, v_1, \dots, v_K) = \sum_{i=1}^n \sum_{k=1}^K (u_{ik})^m d^2(x_i, v_k) \quad (12)$$

where $v_k = (v_{ka})$, $k = 1, \dots, K$, $a = 1, \dots, p$ indicates the center of the k^{th} cluster, and $X_i = (X_{ia})$, $i = 1, \dots, n$, $a = 1, \dots, p$ indicates the i^{th} object according to the variables p as well as the Euclidian distance $d^2(x_i, v_k)$ between x_i and v_k . The exponent m determines the degree of fuzziness, the value of which is already defined in the interval $(1, \infty)$.

The objective of this algorithm is to obtain the solution U and $v_1, \dots, v_K$ as well as minimizing Equation 13 and Equation 14:

$$u_{ik} = \frac{1}{\sum_{l=1}^K \{d(x_i, v_l) / d(x_i, v_k)\}^{2/m-1}} \quad (13)$$

$$v_k = \frac{\sum_{i=1}^n (u_{ik})^m x_i}{\sum_{i=1}^n (u_{ik})^m} \quad (14)$$

Then, the Minimizer of the above Relations are specified as the objective function of these equations according to Equation 15 (50).

$$J(U) = \sum_{k=1}^K \left(\sum_{i=1}^n \sum_{j=1}^n ((u_{ik})^m (u_{jk})^m d_{ij}) / (2 \sum_{s=1}^n (u_{sk})^m) \right) \quad (15)$$

SLOPE STABILITY ANALYSIS USING ANFIS MODEL

As already mentioned, three different ANFIS models were investigated in this study to analyze and estimate slope stability. ANFIS was used based on the GP, SCM, and FCM to model the factor of safety and predicting slope stability. The structure of the ANFIS model is demonstrated in **Figure 2** the input parameters to the ANFIS model (for all of the methods mentioned above) were the specific weight or density γ , cohesion C , angle of internal friction φ , slope angle β , slope height H , and pore water pressure r_u , and FS was considered as the only output. 70% of the data were considered as the training data, and the remaining 30% were used as the test data. The error of the network was calculated in the last step. The six input variables, and one output variable, were prepared and provided to the network in a single matrix as the input. After the first run of the network, overfitting was observed in the output. Measures had to be taken to overcome the problem and to achieve accurate results. Hence, bootstrapping was used to prevent overfitting. The results of training the membership functions using the input parameters and based on the GP, SCM and, FCM methods are demonstrated in **Figures 3.a**, **Figure3.b** and **Figure3.c**.

Figure 2 The ANFIS architecture based on the GP, SCM, and FCM

1

2 **Figure 3.a** Membership functions obtained using GP

3 **Figure 3.b** Membership functions obtained using SCM

1

2 **Figure 3.c** Membership functions obtained using FCM method

3 The collected data of 103 epochs were divided into a 70-epoch for training and a 30-epoch
 4 for evaluating the accuracy of the ANFIS models in estimating FS and slope stability. The error
 5 was considered as the criteria for the comparison of different ANFIS structures, lower error values
 6 indicated more appropriate networks for modeling purposes. The remaining control parameters of
 7 the process, such as experiment and comparison were performed. In order to evaluate the
 8 performance of the ANFIS models, mean squared error (MSE) (Equations 16) and root-mean-
 9 square error (RMSE) (Equations 17) was used according to the following Relations:

$$MSE = \frac{1}{N} \sum_{i=1}^n (y - y')^2 \quad (16)$$

$$RMSE = \sqrt{MSE} \quad (17)$$

10

11 Where y and y' represent the measured values and the estimated values, respectively. N indicates
 12 the number of samples, Lower MSE and RMSE values, as well as the coefficient of
 13 determination R^2 closer to 1, indicate a better performance of the respective model. As can be
 14 seen in **Table2**, the error of the SCM is lower than those of other ANFIS models, and its
 15 regression value is closer to 1, which indicates its better performance and a closer factor of safety
 16 values to the real-world scenarios. The results of the three aforementioned ANFIS models based
 17 on the test data are depicted in **Figures 4**.

18

19

TABLE 2 The values of MSE and RMSE for the ANFIS models

Performance indexes	RMSE	MSE	R^2
ANFIS (FCM)	0.1419	0.0201	0.5552
ANFIS (SCM)	0.0455	0.002	0.7222

Figure 4 A comparison between the measured value and the estimated value of FS, a) grid partitioning, b) subtractive clustering, and c) fuzzy c-means

Moreover, the error of the FS prediction obtained from the proposed method and the neural network method for the training and test data are presented in the diagrams of **Figure 5** and **Figure 6**.

1 **Figure 5** Error comparison between FS prediction of the proposed method and the neural
2 network(for the training data).

3 **Figure 6** Error comparison between FS prediction of the proposed method and the neural
4 network (for the test data).

As demonstrated by the figures, the SCM generally outperformed other ANFIS methods and the neural network. The results reveal that the proposed fuzzy-neural inference system performs well in decreasing the uncertainties involved in analysis and estimation of slope stability, and hence, the method may be developed by geotechnical engineers as a novel approach according to their needs.

CONCLUSION

A novel model for the estimation and analysis of slope stability was proposed in this study. The objective of the model was to estimate the factor of safety and ultimately slope stability. Three different ANFIS models namely GP method, SCM, and FCM were investigated. Adaptive Network-Based Fuzzy Inference Systems is used in different scientific applications. By utilizing the learning ability of the neural networks along with the linguistic variables and expressions used in fuzzy systems, these systems were able to employ the advantages of FIS and ANN methods for the robust analysis of complex processes. ANFIS is today considered an efficient prediction method. The factor of safety was also predicted using neural networks to compare the performance of the three models. The performance comparison of the three models and the neural network revealed the superior performance of the ANFIS model based on the SCM. In addition to closer regression values to 1, the prediction accuracy of the SCM outperformed the other two ANFIS models, i.e., the GP and FCM methods. We may conclude that SCM can predict the factor of safety more accurately and closer to real-world scenarios with smaller errors. The proposed method in this study efficiently reduced the uncertainties involved in the estimation and analysis of slope stability as compared to previous approaches in the literature. Hence, it is suggested that the ANFIS model based on the SCM is used for the estimation and analysis of slope stability.

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