



[white paper]

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# The positional argument and the continuum hypothesis

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## Abstract

We present a discussion on the definition of the positional argument and the continuum hypothesis.

keywords: positional argument, continuum hypothesis, set theory, infinity

*The most updated version of this white paper is available at*

<https://osf.io/tvg64/download>

## Introduction

1. [1]

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# Real numbers

2. Let  $x \in (0, 1)$  be a real number.

## Positions

3.  $i, j \in \mathbb{N} = \{0, 1, 2, 3, \dots\}$

4. (2) can be written in the following **positional form**

$$x = 0 . n_1 n_2 n_3 \dots$$

where  $n_i \in \mathbb{N}$ .

5. For example, let

$$x_1 = 0 . 0 1 2 3 4 5 6 7 8 9 10 11 12 13 \dots$$

6. In (5), one *choice* is  $n_0 = 0$ ,  $n_1 = 1$ ,  $n_2 = 2$ , ...,  $n_{12} = 12$ , ..., i.e.,  
 $n_i = i$ .

## Redundancy

7. Let  $x_2 = 0.12$ .

8. In (7), there are infinite possibilities for  $n_i$ , namely,

(a)  $n_1 = 1$ ,  $n_2 = 2$ ,  $n_3 = n_4 = n_5 = \dots = 0$ ,

(b)  $n_1 = 12$ ,  $n_2 = n_3 = n_4 = n_5 = \dots = 0$ ,

(c)  $n_1 = 120$ ,  $n_2 = n_3 = n_4 = n_5 = \dots = 0$ ,

(d) :

# Partition

9. Let  $\mathcal{P}$  be the partition

$$\mathcal{P} = \bigcup_i P_i.$$

10.  $P_i$  is the set of all real numbers  $x \in (0, 1)$  with  $n_{j \neq i}$  fixed, and  $n_i \in \mathbb{N}$ .

11. Since  $\mathcal{P}$  is a partition, there are no redundancies in  $P_i$ ; in other words, if a number is in  $P_{j \leq i}$ , it will not be in  $P_{j \geq i+1}$ .

12. In the construction of  $\mathcal{P}$ , the **axiom of choice** is used implicitly.

13. Note that there is a *choice* of  $\mathcal{P}$  such that  $|P_i| = \aleph_0$ .

14. Let  $\aleph_{\mathcal{P}}$  be the cardinality of  $\mathcal{P}$ .

15. From *Cantor's diagonal argument*,

$$\aleph_0 < \aleph_{\mathcal{P}}.$$

# Positional argument

16. Proposition: There is a **bijection**  $f : \mathcal{P} \rightarrow \mathbb{R}$ .

17. From (16),

$$\aleph_{\mathcal{P}} = \aleph_1.$$

18. The crucial point here is that, in the construction of  $\mathcal{P}$  (under the *classical logic*), **either** it is a finite union of  $P_i$  (then  $|\mathcal{P}| = \aleph_0$ ), or an infinite union of  $P_i$  (then  $\aleph_{\mathcal{P}} = \aleph_1$ ), meaning there is no  $\aleph_x$  such that  $\aleph_0 < \aleph_x < \aleph_1$ .

# Final Remarks

19. L := positional argument

- 20.  $D :=$  diagonal argument
- 21.  $C :=$  axiom of choice
- 22.  $CH :=$  continuum hypothesis
- 23.

$L, D, CH \vdash CH ?$

## Open Invitation

*Review, add content, and **co-author** this *white paper* [2, 3].*

*Join the **Open Mathematics Collaboration**.*

*Send your contribution to `mplobo@uft.edu.br`.*

## Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [4].

## Ethical conduct of research

This original work was pre-registered under the OSF Preprints [5], please cite it accordingly [6]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

## Acknowledgement

+ **Center for Open Science**

<https://cos.io>

+ **Open Science Framework**

<https://osf.io>

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