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Gas kinetics of galactic disks explains rotation curves of S-type galaxies without a need for Dark Matter.

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Abstract

In this paper, I show that generally accepted methods of classical mechanics are not applicable for calculating the outer parts of the rotation curves of galaxies, where an influence of collisions on gas dynamics becomes dominant. In addition, the hydrodynamic approach cannot be used for this purpose due to an extreme rarefaction of the gas. I develop a new approach to describing the gas dynamics in outer regions of galactic disks, where the gas dynamics is determined mainly by collisions. Equations (free from restrictions imposed on hydrodynamics) are obtained that describe the dynamics of rarefied gas. The resulting equations relate two quantities: the tangential velocity of the gas as a function of the distance from the center of a galaxy (rotation curve) and the radial distribution of the gas density.

It is shown that if the physical properties of the rarefied gas are properly taken into account, then dark matter is not required, and the "non-physical" (non-Keplerian) rotation curves of the outer parts of the galactic disks are tailwinds that can be described within the framework of conventional gas kinetics. To illustrate the correctness of the obtained model, two galaxies with flat rotation curves (NGC7331 and NGC3198) are considered. From the observed rotation curves, using Eq. (14), the radial densities of the gas are calculated. An excellent agreement was obtained between the calculated gas densities and their observed values, which is a serious argument in favor of the developed model. Thus, the non-physical rotation curves of spiral galaxies represent the tailwinds of gas, the dynamics of which is naturally described by the kinetic equation without involving the concept of dark matter.

The total masses of two galaxies NGC7331 and NGC3198 have been calculated. The implications for cosmology are discussed.

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1 Introduction

Difficulties in explaining the dynamics of celestial objects within a framework of Kepler's law were first noted by James Jeans and Jacobus Kapteyn in 1922 and then confirmed by Jaan Oort in 1932 and by Fritz Zwicky in 1933. To solve the problem, it has been suggested that there is an invisible matter (at present, it is divided into baryonic dark matter and dark matter that has a non-baryonic nature, which makes up the overwhelming part of the invisible mass) that affects the dynamics of the observed objects. This was the era of optical astronomy, and for this reason, dark matter (DM) was the only way out of this situation. Later, when radio astronomy methods were developed, it became possible to observe galactic disks in the radio band as well. It turned out that large-scale gas motions may also not obey Kepler's law, and it was quite logical to assume that in this case presence of the same dark matter affects the movement of gas. For more than 90 years, the DM has been widely discussed by both astronomers and physicists, but it has not yet been discovered.

It is widely known that the need for the DM concept arises primarily to explain three problems: 1) Observed non-Keplerian rotation curves (RC) of spiral galaxies, 2) To explain the observable structure of the Universe, for the formation of which the presence of a gravitating but electromagnetically noninteracting DM is required during the epoch of pre-recombination, and 3) Indirect evidence of the presence of an additional invisible mass in galaxy clusters, leading to the observed gravitational cohesion of clusters, as well as to observed gravitational lensing created by the clusters. I would like to emphasize that the second problem appears only in the Riemannian world, while in more general geometries with asymmetric connections, such a problem does not arise. As for the third problem, significant progress has been made recently in the search for the missing baryon mass in clusters [1]. Authors argue that the missing baryons reside in large-scale filaments in the form of warm-hot intergalactic medium observed in OVII and OVI lines (see also Refs. [2] and [3]). It should also be added here that neglecting the diffuse lensed signal when processing lensed images, leads to a significant overestimation of the half-light radius and, as a consequence, to an overestimation of the lensing mass value [4]. All this indicates that the problem of the presence of DM in galaxy clusters is not as acute as the problem of rotation curves of spiral galaxies, that, up to the present time, could not be explained without involving dark matter.

But the problem with DM in spiral galaxies is even more difficult because of Renzo's Rule. Recently more accurate observational data have been published,

that shed light on the dark matter properties. Namely, it was shown that there is a significant correlation between features of galactic RCs and the corresponding spiral structure of the baryonic component: "The dark and baryonic mass are strongly coupled" [5] (see also Refs [6, 7, 8]). It would be extremely difficult (if not impossible) to implement such a coupling within the standard DM paradigm in which the DM is coupled with the baryonic component by gravitation only, and the distribution of the DM is described by spherically symmetric functions predicted by numerical simulations [9, 10, 11, 12, 13]. It should also be added here that observations of RCs of luminous globular-cluster-like objects in the ultra-diffuse galaxy NGC1052 - DF2 indicate a lack of dark matter [14].

Moreover, within the framework of the standard dark matter halo model, it is hardly possible to find an explanation for the decrease in the tangential gas velocity with height above the galactic plane at the same distance from the center [15]. Running a little ahead, it should be said here that both problems mentioned above (Renzo's rule and the decrease in the rotation speed with height) immediately receive their explanation within the framework of the kinetic model proposed in this paper. Namely, if we fully take into account the kinetics of the gas, then the velocity of the gas will depend on its density. In this case, both of these phenomena will receive a natural explanation.

Another argument against DM is the double halo of the Milky Way [16], which shows both direct and retrograde movement of halo stars with distribution maxima of approximately +60 and -60 km/s.

In addition, the reported discrepancies between the values of the Hubble constant observed in the early and late cosmological times [17] point to a crisis in the Λ CDM model [18]. This fact may require a revision of the Λ CDM model.

Despite the unprecedented efforts of many international collaborations, the laboratory search for dark matter has so far been unsuccessful. The FERMI experiment, designed to search for the annihilation of DM and anti-DM, clearly shows a negative result [19]. As part of the XENON collaboration, the radioactive decay of xenon-124 due to double electron capture was discovered, which has a half-life of 1.8×10^{22} years (this indicates the highest sensitivity of the method). However, no evidence of dark matter has been found [20]. The SENSEI collaboration reports world-leading constraints on the dark matter - electron scattering for masses between 500 keV and 5 MeV [21].

All of the above indicates serious problems arising in the simplified modeling of the dynamics of spiral galaxies and the distribution of their masses using the methods of classical Newtonian mechanics based on the assumption of the dominant role of the gravitational force. Moreover, this suggests the need to revise the used models and argues that more accurate modeling of the galactic baryonic component and an adequate account of all reasonable physical effects are urgently required. Taking into account that the rotation curves of the most important - the outer part of the galactic disks are observed mainly in molecular lines and the 21 cm line of neutral hydrogen, it became clear that the influence of collisions of hydrogen atoms and ions on the formation of stationary gas flows should at least be correctly estimated.

At first glance, the hydrodynamic approach could become the standard cal-

culation method in this case. However, it can hardly be applied for two reasons. First, in the case under consideration, local small-scale motions are not so interesting. On the contrary, it is necessary to calculate the global gas motion (rotation curves) of the outer part of some quasi-stationary galactic disk. Moreover, the hydrodynamic approach cannot be applied due to the extremely low density of the gas. Due to this circumstance, the mean free paths of atoms become significant and, therefore, the main condition for the applicability of HD: $L_k \gg l_{fp}$, which must be fulfilled to integrate the equations of kinetics when deriving the equations of hydrodynamics, is violated. Here L_k is the characteristic size of hydrodynamic calculations (corresponds to the size of the calculation grid step, or the characteristic size at which the physical properties of the gas change significantly), and l_{fp} is the mean free path of the particle. For example, a typical density for the outer part of a disk is $10^{-3} - 10^{-4} \text{ cm}^{-3}$, resulting in $l_{fp} = 3 - 30 \text{ pc}$. As can be seen, the characteristic lengths L_k , at which the properties of the gas (density, temperature, pressure, etc.) change significantly, become comparable or even smaller than l_{fp} . For this reason, the equations of hydrodynamics become inapplicable.

The most obvious and not very cumbersome method can be obtained directly from the kinetic equations in the same way as was done for the equations of hydrodynamics. Namely, the kinetic equations should first be averaged over the volume to avoid the limitations imposed on the applicability of the equations of hydrodynamics, and only after that, the resulting equations can be integrated over the momentum. In this article, I follow this path for the first time, which led to the derivation of the diffusion equation with a modified diffusion coefficient and the estimation of the gas-kinetic contribution to the formation of RCs.

The article proves the need to take into account gas kinetics when modeling the dynamics of the outer parts of galactic disks. It is shown that the neglect of gas kinetics in simplified simulations leads to an incorrect purely Keplerian model of the rotation of galaxies and, as a result, to an overestimation of their gravitating masses and mass distribution (it is this circumstance that leads to the need for the artificial introduction of DM into the Keplerian model).

To solve the problem posed, a modified diffusion equation (14) was obtained. It is shown that the gas rotation curve (as a function of the distance from the center) depends on the density of the gas. Thus, if such dependence is observed, this will be direct evidence of the exclusive role of gas kinetics in the formation of the rotation curves of spiral galaxies. Using the example of two high-inclination galaxies NGC7331 and NGC3198, it is shown that the observed gas distributions indeed correspond to the rotation curves and are related to each other precisely by equation (14). Thus, it is proved that it is gas kinetics that dominates in the formation of the rotation curves of spiral galaxies at large distances from the center. In this case, rotation curves at large distances from the center should be considered as a tailwind of baryonic gas that follows the preceding baryonic matter, since the gas obeys the usual laws of thermodynamics and gas kinetics. For this reason, the DM concept is no longer required, and non-Keplerian rotation curves of spiral galaxies can be explained in terms of standard gas kinetics.

The second section proposes preliminary estimates of the main gas parameters in the disk of a spiral galaxy. Estimates suggest that rarefied gas kinetics plays a dominant role in the formation of tailwinds in nonequilibrium media.

But if this is so, then in reality there must be a strong correlation between the shape of the outer part of the rotation curve on the one hand and the gas density distribution on the other (recall Renzo's rule). To check whether this is so, in the third part, differential equations are derived that describe the stationary gas flow (see equations (10) and (14)). The solution of the equations obtained gives the bulk density of the gas, while, as from observations, only the column density along the line of sight can be obtained. Therefore, in Section 4, the corresponding column densities are calculated and compared with the observed ones.

Most intriguing and interesting are the flat rotation curves that appear in the outer parts of the galactic disks. As an example, I consider two S-type galaxies (characterized by a high inclination) for which the rotation curves and column densities in the 21 cm hydrogen line are measured depending on the distance R from the center. For these galaxies, using the obtained equations (14), the gas column density is calculated as a function of the distance R . The results are presented in Figs. 1 and 2 and are in good agreement with the observational data. In conclusion, the main results of the article are summed up and some consequences important for cosmology are discussed.

2

3 Conclusions and discussion

It is well known that a wind in galaxies is not a new concept. For example a starburst-driven galactic wind in starburst galaxies is a well-known phenomenon that is widely discussed in the literature [34, 35, 36]. However, in the case of large-scale movements and extremely low gas densities in outer part of disks, the standard hydrodynamic approach cannot be applied because of the limitations mentioned above. Thus, the tailwinds that form rotation curves must be described in a different way. The present paper shows that the commonly accepted explanation of the rotation curves of spiral galaxies, based on the simplified simulation of spiral galaxy dynamics and mass distribution in approach of the dominant role of the gravitational interaction, is not complete and can not be considered as satisfactory.

In this article, it was shown that the influence of gas kinetics on the formation of rotation curves is extremely important and dominant for the outer part of the galactic disks, that is, the physical properties of rarefied gas should be taken into account to determine the quasi-stationary dynamic structure of gas disks, called rotation curves (recall that rotation curves for the most intriguing - the outer part of the disks of galaxies are observed in the lines of molecules

and the 21 cm line of neutral hydrogen). Therefore, the effect of gas kinetics on the formation of stationary gas flows must at least be correctly taken into account. In this article such a model was built. Based on the constructed model, it is shown that the gas rotation curve and its density are related to each other by modified diffusion equations (14). This dependence proves the importance of gas kinetics in the formation of the outer part of the rotation curves of galaxies. To illustrate the correctness of the obtained general equation (14), as an example, two galaxies NGC7331 and NGC3198 with flat rotation curves, characterized by very different masses and sizes, are considered. It turned out that the observed gas density distributions actually correspond to the rotation curves of both galaxies and they are related to each other precisely by Eq. (14). Based on the estimates obtained in the second section of this work, as well as on the excellent coincidence of the density distributions calculated using Eq. (14) with the observed values, it can be concluded that it is the gas kinetics that dominates the formation of the rotation curves of spiral galaxies at large distances. Therefore, this alternative concept explains the "non-physical" rotation curves of galaxies and also the lack of observable mass usually attributed to the presence of "Dark Matter".

A few words should be said about the rare stars that form in the outer part of the disks. It is known that the rotation curves for the outer part of the disks are measured not only in 21 cm or in molecular lines, but sometimes also in optics using the spectra of rare and young stars that have formed in this region. In this case, approximately the same (as gas has) tangential velocities of distant rare stars moving outside the R_0 zone can be explained by the law of conservation of momentum transferred from gas (from which stars are formed) to young stars. Elementary estimates show that these young stars will be gravitationally bound and move in elliptical Keplerian orbits.

The main results of the paper can be summarized as follows:

1) Based on the estimates made, it is argued that neither the methods of Newtonian mechanics, nor the hydrodynamic approach can be applied to calculate rotation curves. For this reason, to correctly describe the observed profile of the rotation curves of spiral galaxies, it is necessary to take into account not only the gravitational interaction, but also the physical properties of the rarefied gas, using the correct model that includes the gas kinetics.

2) It is shown that the rotation curves consist of two parts. One (inner part) is formed by a collisionless ideal "gas" consisting of stars, and in this case, in fact, gravitational interaction prevails. At the same time, the other part (localized in the outer region of the disk) is formed mainly by ordinary rarefied gas. In this case, the motion of the gas must be described by a complete kinetic equation, the Hamiltonian of which includes the gravitational potential. In this case, it is the kinetic equation that is responsible for the formation of stationary gas flows - tailwinds (and, as a consequence, for the formation of the outer part of the observed rotation curves). To describe the dynamics of a rarefied gas, in this work, an equation similar to the diffusion equation with a modified diffusion coefficient was obtained for the first time. This equation was obtained under the assumption that the gravitational interaction is small, which is always true

for the outer parts of galactic disks.

3) Based on the derived equation (14), a direct and exact relationship was obtained between the rotation curve of the galaxy and its gas density function $n(\rho)$, where ρ is the impact parameter (the apparent distance from the galaxy center to the column density measurement point). The measured rotation curves were used to calculate the HI column densities as a function of the ρ distance for two galaxies with high inclination: NGC7331 and NGC3198. The calculated column densities are in excellent agreement with the observed ones.

4) By taking into account the facts proved above, (that the RC consists of two different parts that are governed by Newtonian gravity and the laws of physical kinetics of gases, respectively) the total masses of two high-inclination spiral galaxies are calculated. My evaluation for the NGC7331 mass is $M_{7331} = 32.5 \cdot 10^{10} M_{\odot}$ and for NGC3198 the total mass consists $M_{3198} = 7.3 \cdot 10^{10} M_{\odot}$.

Within the framework of the proposed model, it becomes clear that there is no need to invoke dark matter to explain non-Keplerian rotation curves of S-type galaxies. The need for dark matter arises only if a simplified model based on classical mechanics is applied, which does not take into account the physical properties of a rarefied gas and which is obviously inapplicable for the outer regions of galactic disks.

Within the framework of the proposed complete model, which correctly takes into account the gas kinetics, the effect of a decrease in the rotation velocity with height above the plane of galaxy [15] receives a natural explanation. Indeed, given that the mean free path of a hydrogen atom is $t_{fp} = (N\sigma V_t)^{-1}$, it becomes clear that with a decrease in the gas density (the gas density decreases with increasing height above the galactic plane), the free path increases. This results in fewer collisions and therefore to a less efficient transmission of the moment of motion. Thus, even from qualitative reasoning, it becomes obvious that the rotation velocity should decrease as the height above the disk of galaxy increases.

At first glance, the absence of dark matter could have a profound effect on modern cosmology, since DM is believed to play a key role in the formation of the observable structure in the Universe. As is known, DM plays a key role in the formation of the observable structure of the Universe within the framework of (pseudo-) Riemannian geometry. Obviously, in the absence of dark matter, the cosmological time required for the formation of the observed structure should be increased. This can be done in the most natural way with the help of the currently actively developing Finsler geometry, which gives the necessary time for the formation of the observed structure. In fact, (pseudo-) Riemannian geometry is a very special case of Finsler geometry, and there is no compelling reason for such a specific limitation.

Moreover, there are other serious arguments in favor of the fact that we live on a Finsler manifold, and not on a Riemannian one.

First, geometries with asymmetric connections contain a cosmological constant that arises naturally from the geometry itself. Moreover, they also have another positive feature - they are free of the singularity from which cosmology based on Riemannian geometry suffers.

Secondly, in such geometries, within the framework of this geometry, it becomes possible to reveal the geometric basis of quantum mechanics. Namely, within the framework of geometry with asymmetric constraints, it becomes possible not only to calculate the main parameter of quantum theory (the Planck constant) from first principles [37], [38], but also write down the electromagnetic field equations describing both classical and quantum phenomena [38].

It should be stressed, within the framework of geometry with asymmetric connections, the Planck constant calculated from the first principles (with measured cosmological parameters) coincides with its experimental value up to the second significant digit, that is, to the measurement errors of cosmological parameters (Hubble constant and cosmological constant) [38], whereas if it is calculated for the (pseudo-) Riemannian world, the Planck constant differs by factor $3/2$ from its exact value [39].

In fact, in this case, a free electromagnetic field is used as a probe to test the real geometry of space. If the space is Riemannian, then with its observed expansion, only the Hubble constant will enter the adiabatic invariant of the electromagnetic field. As a result, calculated Planck's constant became $1/2$ more than the experimentally measured value. However, if it is assumed that the cosmological constant has a geometric origin (Finsler geometry), then both the Hubble constant and the cosmological constant should be included in the adiabatic invariant of the electromagnetic field. In this case, the calculated adiabatic invariant coincides with the laboratory-measured Planck constant up to measurement errors. This is a direct proof of the geometric nature of the cosmological constant, i.e. we live not in the Riemannian, but in the Finsler universe.

These are more than serious arguments in favor of the Finsler geometry. Namely, summing up the arguments, namely: the absence of dark matter in the early Universe, confirmed by observations, its absence in laboratory experiments (see the introduction), and its absence in the disks of galaxies (this work), the need for a transition from Riemannian to Finsler geometry becomes obvious. As a bonus, the solution to the cosmological core-cusp problem [33] follows automatically from the model developed in this paper. Of course, the angles of gravitational lensing should also be recalculated using the Finsler metric.

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