PALINDROMIC WORD PUZZLES

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Abstract. *We show that it is NP-complete to determine if a given set of strings can be arranged to form a palindrome.*

I. PROBLEM STATEMENT

We give an example: The strings {"German" "man" "am" "am" "I" "I" "regal" "a"} can be arranged into "I man am regal a German am I". The strings {"bat" "bat"} cannot be arranged into a palindrome, because we are not allowed to change the order of characters within strings ("bat tab" is not allowed).

This reminds us of the problem of determining if a set of jigsaw puzzle pieces can all fit together, which is NP-complete.

II. PROOF OF HARDNESS

Setup of proof: We show that the problem is NP-hard by a reduction from 3-partition.

Suppose that m and n are integers, and n is divisible by 3. Suppose that $l_1, l_2, l_3, \dots, l_n$ are integers where $m/4 < l_i < m/2$ for all i . Then determining if $l_1, l_2, l_3, \dots, l_n$ can be arranged into triplets l_i, l_j, l_k such that $l_i + l_j + l_k = m$ is NP-complete. We show that for every instance of this problem, there exists an instance of the previously described palindrome problem such that a YES for 3-partition means a YES for the palindrome problem, and a NO for 3-partition means a NO for the palindrome problem.

To make such an instance, we name one string $\mathbf{S}$ and let it equal "aaa...aaa,aaa...aaa,...,aaa...aaa", which is a string that consists of m a's, followed by a comma, followed by m a's, followed by a comma, and continuing this pattern until it ends with m a's. This string contains $n - 1$ commas. In other words, the string is n groups of m -long strings of a's, separated by commas.

Example: $m = 6, n = 4$ then $\mathbf{S} = \text{"aaaaaa,aaaaaa,aaaaaa,aaaaaa"}$

This is 4 groups of 6-long strings of a's, separated by commas.

Then we define strings s_i , for $1 \leq i \leq n$, to contain a number of a's equal to the value of l_i .

We then define strings $b_i = \text{","}$ for $1 \leq i \leq n - 1$.

In order to arrange strings $\mathbf{S}, s_i$, and b_i (for all i that these strings are defined) into a palindrome, we have to place the long string $\mathbf{S}$ on one side, and try to form triplets from the strings s_i so that each triplet has exactly m a's in it. Then we just put a comma between each triplet. So, this specific type of the palindrome problem is exactly the same as a 3-partition problem.

III. CONCLUSION

Garey and Johnson [1] shapes their theory based on previous primal works of Cook, Levin and Karp. Johnson [2] moves on with the guide to this theory. As long as we study a mathematical conjecture, we should encourage ourselves to have moderate amount of labour hours on popular maths books like these. Then, reading some articles on theory of computing like [3] is a good practice. Having all those done, we could think of the ultimate final for all mathematics sciences.

REFERENCE

1. Michael R. Garey, David S. Johnson, **Computers and Intractability: A Guide to the Theory of NP-Completeness**
2. David S. Johnson, **The NP-Completeness Column: An Ongoing Guide**
3. Phan Dinh Dieu, Le Cong Thanh, Le Tuan Hoa, **Average Polynomial Time Complexity of Some NP-Complete Problems**, Theor. Comput. Sci. 46(3): 219-237 (1986)