# Black hole information paradox as an instance of the paradox of reversible processes

Blind review

It is noted that the black hole information paradox is an instance of the paradox of reversible processes, in case late-stage black hole states are approximated by thermodynamic equilibrium states, as conventionally expected. Some purported classical solutions to the paradox of reversible processes, such as the quasi-process interpretation, are shown to be insufficient in light of black holes. Three leading solutions to the information paradox - holography of information, replica wormholes and fuzzballs - are cast in the language of a reversible process, which then suggest different solutions to the paradox of reversible processes.

## 1 Introduction

The paradox of reversible processes is about the difficulty in providing a physical process that can approximate a reversible process [15]. In response, a foundation based on a non-physical process may be suggested [22], but the original difficulty is only evaded and still remains. Furthermore, it turns out that black hole thermodynamics makes such foundations implausible - a key point of this paper, along with construction of a reversible process from late-stage equilibrium states in an evaporating black hole.

The main issue in the paradox of reversible processes is that they are about thermodynamic equilibrium states, which must have $dY/dt = 0$ for some thermodynamic variable $Y$. Yet a reversible process requires $dY/dt \neq 0$. Therefore, a reversible process is to be naturally understood as a close approximation to a physical process. The question is whether such a physical process can be found.

The paradox may alternatively be resolved by dividing a reversible process into non-equilibrium sub-processes, with initial and final states of each sub-process being thermodynamic equilibrium states despite intermediate states being non-equilibrium states. That is, each sub-process is an equilibriation process that moves a system from one equilibrium to another equilibrium. But connecting these sub-

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processes is not physical (or at least in sense of requiring observer interventions), since at the end of each sub-process, one must have $d^kY/dt^k \approx 0$ for every $k \geq 1$, where as the start of each sub process has $d^kY/dt^k \neq 0$ for some $k \geq 1$. These sub-processes (often referred to as quasi-static processes) may nevertheless be connected as a quasi-process as to form a foundation for a reversible process, as suggested in [22]. It could be said that this quasi-process view is a short-cut for representing non-equilibrium subsystem states that, if not for interactions with other subsystems, will internally equilibriate to stated equilibrium states.

As aforementioned, the quasi-process view is to be shown unsatisfactory in light of black holes. According to black hole thermodynamics, they are (at least eventually) described by equilibrium states. In the conventional quantum picture [16], late-stage evaporating black hole states, until close to completion of black hole evaporation, are approximated by thermodynamic equilibrium states. This creates a reversible process of equilibrium states that cannot be explained away via equilibrium states actually being non-equilibrium states - the quasi-process view cannot work. The very difficulty of providing a physical process that approximates such a reversible process is the black hole information paradox, with purported solutions to the information paradox suggesting different approaches to the paradox of reversible processes.

## 2 Black hole thermodynamics, combined with ordinary thermodynamics

### 2.1 Basics

For simplicity, without loss of generality, it is assumed that a black hole is uncharged and non-rotating. It is expected in classical general relativity that any such black hole stabilizes to a Schwarzschild black hole, when looked from the outside. A Schwarzschild black hole respects the following laws and properties of black hole thermodynamics, simplified for this class of black holes with $c = G = \hbar = k_B = 1$, where $c$ is speed of light, $G$ is Newton's gravitational constant, $k_B$ is Boltzmann constant and $\hbar$ is reduced Planck constant:

- First Law: $dE = \frac{1}{(8\pi)(4M)}dA$, where $E$ refers to the energy, $\kappa = 1/(4M)$ refers to the surface gravity, $M$ refers to the mass enclosed and $A$ refers to the horizon area. Given the simplification, heat differential $dQ = dE$.

- With analogy to the first law of ordinary thermodynamics, it is typically asserted that entropy $S = A/4$, which allows the first law to be restated as $8\pi M dE = dS$.
- Black holes have temperature $T = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}$. This allows us to restate the first law as $dE = TdS$.
- $M$ relates to Schwarzschild radius $r_s$ by $2M = r_s$.
- $A$ relates to $r_s$ by $A = 4\pi(r_s)^2$.

Now suppose that a black hole with temperature $T_{BH}$ interacts with the outside which can 'coarsely' be described as a thermodynamic system with temperature $T_O$.

- $T_O > T_{BH}$: We expect heat to flow from the outside to the black hole. This raises black hole entropy $S_{BH}$, which is proportional to horizon area $A$, increasing Schwarzschild radius $r_s$ and black hole mass $M$ ($2M = r_s$). Since $T_{BH} = \frac{1}{8\pi M}$, this decreases $T_{BH}$. Then until complete equilibriation between the outside and the inside (of the black hole), $T_{BH}$ continues to drop monotonically. This is the standard classical gravity scenario along with $T_O = T_{BH}$, where black holes are non-decreasing in their size - the second law of black hole thermodynamics: $dA \geq 0$.
- $T_O < T_{BH}$: We expect heat to flow from the black hole to the outside. Hawking radiations provide a mechanism of how this is to be done. This heat outflow (from the black hole perspective) decreases $S_{BH}$ via the first law, which decreases $r_s$, which in turns lowers mass $M$, increasing $T_{BH}$. Until equilibriation with the outside, $T_{BH}$ increases monotonically.
- $T_O = T_{BH}$: In classical gravity, a stable equilibrium emerges, if not for other factors not related to the black hole. This is expected to change in quantum gravity, to be discussed separately.

## 2.2 Quantum scenario and a reversible process

For simplicity, start with the case with initial exterior temperature $T_{O,i} = 0$ and initial black hole temperature $T_{BH,i}$. Since $T_{O,i} < T_{BH,i}$, heat ('Hawking radiations') flows from the inside to the outside of the black hole. The black hole continues to evaporate until approximate completion of equilibriation: $T_{O,p} \approx T_{BH,p}$ ($p$ refers to Don Page, in reference to Page curve [16]). Classically, this leads to a stable equilibrium for the universe at temperature $T_{BH,p}$, which can then be translated as a quantum state.

The issue, however, is that Hawking radiations are observed at far infinity - they do not fall into the black hole. And since $T_{O,i} = 0$, there is nothing from the outside that gets absorbed into the black hole. Therefore, the black hole can only monotonically decrease. Instead of the stable equilibrium at $T_{BH,p}$, we get a reversible process of late-stage equilibrium states with temperatures $T_{BH}$, which monotonically rises until complete evaporation.

Having stated the key point of a late-stage black hole forming a reversible process, two additional observations can be made.

- First, naive final temperature $T_{BH,f} \approx \infty$, since the black hole mass becomes close to zero. The issue is that this temperature applies for the universe as whole. This is resolved by a quantum argument (as in section 2.4 of [19]) that when the Hilbert space of some system becomes really small, pure states of the universe can no longer be close to thermal states. Therefore, at very late stages of the black hole, a thermodynamic description cannot be used as an approximate tool.
- Second, the temperature of the entire universe still rises significantly during evaporation (the temperature of the outside started as zero and the final temperature is greater than the initial black hole temperature), and the universe as a whole cannot do this without something being done in the background - deviations from classical spacetime even outside the black hole should be expected, with deviations propagating across space over time as subsystems re-equilibriate.

The points made so far are nothing new - while not often explicitly stated, they have been well-understood, especially so since the only quantum thing that enters the picture is the nature of Hawking radiations at infinity. In fact, this is simply the Page curve story [16]. Now we connect the late-stage reversible process of a black hole with the paradox of reversible processes.

# 3 Information paradox as an instance of the paradox of reversible processes

## 3.1 Dynamics paradox

The black hole information paradox can be cast as follows: is there a physical process that can well-approximate the reversible process generated by the trajectory of Bekenstein-Hawking entropy $S_{BH}$ (or temperature $T_{BH}$)? This is an instance of the paradox of reversible processes. And because we expect the universe to remain close to equilibrium states at late stages of black hole lifetime, attempts to solve the paradox with equilibrium states actually being non-equilibrium states are bound to fail.

Note the following caution: the quantum state of a black hole is not exactly a thermodynamic equilibrium state. In fact, many expect that there is no analog to the no-hair theorem in quantum gravity such that the quantum state of a black hole does not need to stabilize to some equilibrium state. However, it is known that in a large enough Hilbert space, most pure states are very close to thermal states (see section 2.4 of [19]). It implies that unless dynamics (in other words, the question of unitary evolution) suggests otherwise, small corrections to equilibrium states are sufficient to be consistent with states of the entire universe remaining pure states. This view is often considered to resolve most parts of the original information paradox posed by Stephen Hawking, though definitely not a 'complete' resolution, since this does not provide whether dynamics are linear and unitary evolution, as quantum physics requires.

Indeed, it is the question of 'dynamics' that both the 'full' information paradox and the paradox of reversible processes are concerned with. For former, it is whether the universe evolves unitarily. For latter, the question is about finding a physical process (or 'dynamics') that approximates a reversible process. The conclusion that [15] makes is that there is no physical process that can approximate a reversible process - and the natural interpretation in black hole contexts is that despite each state of a black hole sharing equilibrium properties, the laws of black hole thermodynamics as transitions between equilibria and governing a reversible process do not actually hold. And this relates to approximate (non-)validity of the smooth event horizon since black hole thermodynamics can be derived from classical general relativity and be converted back to spacetime solutions.

Stated in this form, we now see that the information paradox has much deeper scope than conventionally considered. Resolving the information paradox, at least in a way consistent with the Page curve, means providing solutions to both its original black hole context and classical thermodynamics. From the inverse direction, providing a solution to the paradox of reversible processes may highlight how a physical (classical, semiclassical or quantum) process resembling black hole evaporation is carried out.

For sure, there are unconventional alternatives - for example, the laws of black hole thermodynamics may break down as relics of classical gravity, with the laws ordinary thermodynamics remaining valid.

In such a case, black holes provide zero clue about the paradox of reversible processes.

## 3.2 From quantum gravity

Since this paper is not a review of quantum gravity, most discussions are to be centered around a reversible process, though some quantum gravity details are to be briefly discussed.

### 3.2.1 Fuzzballs

We have just considered the view of [15] for the paradox of reversible processes that argues impossibility of a physical approximation to a reversible process. The natural question is whether there is a similar view in quantum gravity. The answer is yes - fuzzballs [14], though not exclusively so.

Fuzzball proponents argue that smooth black hole horizons postulated by the conventional view cannot be consistent. These views have recently been summarized in [7] as the small corrections 'theorem' (though its initial form can already be found in [13]) - no small correction to Hawking's results can ever restore unitary dynamics of the universe. (Note that while this is a mathematical result, it has not yet been universally accepted as valid.) They argue that while black hole microstates as fuzzballs respect Bekenstein-Hawking entropy, there is no smooth horizon as argued by conventional black hole thermodynamics. Limiting equilibrium properties are there, but no reversible process of equilibrium states, as argued in [15].

### 3.2.2 Replica wormhole view

The possibility of a semiclassical solution to the information paradox was briefly mentioned in reference to a semiclassical solution to the paradox of reversible processes. And such a view in quantum gravity exists - replica wormholes [3, 4, 17]. While it is not a classical solution, it does not involve details of quantum gravity - thus semiclassical (classical gravity combined with quantum physics). This view asserts that in path integral calculations of values like entanglement entropy utilizing the replica trick, previously discarded wormhole saddles need to be incorporated, and such inclusions restore pure state evolution of the universe and respects the Page curve. (Again, since this is not a review paper on quantum gravity, we will not go into the details too deeply.)

In terms of the subregion duality ('quantum error-correcting code') view of AdS/CFT [2, 9, 1], replica wormholes provide semiclassical path integral means to properly eliminate redundancy ('same information in different places') in the conformal field theory (CFT) side of AdS/CFT, thereby restoring monogamy of information ('information can only be in one place'). In some cases, CFT is replaced by effective field theory (EFT) [1], though the basic idea remains the same - for example, EFT leads to the Hawking result (involving a black hole) that information is in two different places, which is eliminated by a state-dependent holographic map that eliminates redundancy - this process being equivalent to incorporating replica wormhole saddles to semiclassical path integrals [1].

Most importantly, in terms of the paradox of reversible processes, this solution purports to provide a semiclassical mechanism that can approximate a given reversible process (in this case, black hole evaporation) consistently.

### 3.2.3 Holography of information

One issue with a reversible process is that arriving at each equilibrium state in the process should take finite-duration equilibriation even in approximate sense - this point was discussed along with each quasi-static process in a quasi-process. The issue may be avoided by assuming that a system is really small such that equilibriation is extremely quick. Or better, if we utilize language of holography of information, some boundary or horizon captures all information about what it encloses such that no equilibriation over the entire system is required. (For recent expositions, see [8, 10, 18]. For their historical origin, see [6, 12].)

In this view, the information paradox is resolved in the following way: there is no information actually entering the event horizon and what we see as the black hole interior is actually an illusion that can be reconstructed via state-dependent reconstructions [10, 18]. Therefore, there is nothing with chance of non-unitarity in the first place - the universe always evolves unitarily and that is all there is. For the paradox of reversible processes, all there is 'is' information about some system of finite volume holographically captured in a small horizon surface such that equilibriation to reach each equilibrium state in a reversible process may safely be ignored.

That said, all of these views - fuzzball, replica wormhole, holography of information - are heavily debated and controversial, with no universal acceptance at this stage. Fuzzball proponents criticize replica wormholes to be the misunderstanding of replica tricks and argue that the small corrections theorem denies unitary evolution for replica wormholes and possibly holography of information as well [7]. Replica wormhole and holography of information views argue that smooth black hole horizons should be expected [20], though the way this happens is very different (conventional horizon and interior versus reconstructed horizon and interior, though they share use of state-dependent operators) - in fact, depending on how one phrases it, holography of information suggests the flat Page curve, since nothing really enters a black hole. There are still many issues and questions for each view that await clarifications.

#### 3.2.4 Black hole complementarity

In the era between emergence of the AdS/CFT correspondence [11] and the AMPS firewall paradox [5], black hole complementarity [21] was thought to resolve the information paradox. The main idea was that it is impossible to verify same information being both in and outside a black hole. This solution suggests that the information paradox is irrelevant for the paradox of reversible processes, since complementarity is not applicable for other thermodynamic systems.

Due to the AMPS firewall paradox ([5], but [13] works as a historical and prophetic precursor as well), the spirit of complementarity had to be revised. In holography of information, original black hole complementarity was superseded with the idea that the black hole interior is a state-dependent reconstruction of information outside the black hole, and in this sense information is both inside and outside the black hole - but there is only one set of information in the Hilbert space, not two sets. In replica wormholes, semiclassical effective field theory calculations that lead to same information being both inside and outside a black hole are corrected by sub-leading wormhole saddles - while such corrections are small, they eventually build up such that monotonic increase in entanglement entropy (von Neumann entropy of a black hole) is curbed. In any case, naive black hole complementarity has fallen out of favor and cannot constitute as an example where a solution to the information paradox does not provide one for the paradox of reversible processes.

### 3.3 Classical resolutions?

As summarized in [15], purported classical resolutions to the paradox of reversible processes go as follows:

- Existential supposition.
- Driving non-equilibrium forces differing insensibly from zero.
- Processes reversed by very small changes of driving force.
- Processes that are infinitesimally removed from equilibrium.
- Infinitely slow processes.
- Processes in which the initial state can be restored. Similar to existential supposition.
- Processes that are mechanically reversible. Not meaningful in modern parlance of thermodynamics.
- Quasi-static processes.
- Additionally, quasi-processes as discussed in [22].

For the second and the fourth cases ('driving non-equilibrium forces differing insensibly from zero' and 'processes that are infinitesimally removed from equilibrium'), we have seen in quantum gravity takes that pure states can be approximately close to thermal states, which provides reasons why each non-equilibrium state of a physical process may remain approximately equilibrium states. However, they were considered insufficient for explaining the dynamics side. We cannot consider infinitely slow processes in black hole thermodynamics, given that almost all late-stage states of a black hole remain close to equilibrium states. This was also why quasi-processes of [22] were considered insufficient - we do need to provide dynamics that can approximately replicate evolution of equilibrium states. Quasi-processes require separating a reversible process into many sub-processes that each undergo finite-duration equilibriation (and potentially infinite duration, as charged by [15]), which cannot be afforded.

If this is all there is to possible classical solutions, what only remains is the claim in [15] - there is no classical physical process that can approximate a reversible process, and reversible processes cannot and should not seek a foundation on non-physical processes (especially in light of black holes).

# 4 Conclusion

In this paper, the black hole information paradox was shown to be an instance of the paradox of reversible processes, with a solution to either side providing clues about the other side. Black hole considerations suggest that the quasi-process view of thermodynamics as in [22] cannot be universally applicable, as long as black holes are approximately thermodynamic systems. This so, since late-stage black hole states are expected to remain close to equilibrium states, such that a physical process approximating evolution of equilibrium states is directly required. No classical solution to the paradox of reversible processes was deemed satisfactory.

Three purported and leading solutions to the information paradox - holography of information, replica wormholes and fuzzballs - were cast in language of a classical reversible process. Holography of information speaks for information of some significant volume captured fully in a holographic surface (horizon or boundary), which then avoids significant equilibriation required for a system to reach a new equilibrium. Replica wormholes suggest a semiclassical process that can approximate a reversible process. Fuzzball proponents argue that the laws of thermodynamics with smooth horizons break down in quantum gravity, despite states sharing some equilibrium properties, as analogously argued in [15]. Despite the solutions resolving the paradox of reversible processes in one form or the other, there is no universal acceptance on validity of these approaches.

But regardless of a solution, it must be reminded that resolving one paradox very likely leads to resolving another, suggesting why these paradoxes are so difficult to resolve. We await for the final solution that kills two birds in one stone.

# Declaration of interests

The author(s) have no funding source to declare. Furthermore, there is no conflict of interests.

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