Quantum Mathematical Integrated Information Theory: From the Classical Version

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Abstract

This paper is an attempt to give a Quantum theory of Mathematical integrated information theory which is mathematical version of integrated information theory by Masafumi Oizumi, Larissa Albantakis, Giulio Tononi [2]. Using the definitions of Classical Mathematical Integrated Information Theory [1]. And considering that the Quantum theory is given by the functor which maps from a category whose objects is topology to a linear category whose objects are Hilbert spaces indexed with the objects from previous category. Also, we will be using the definition of conditional density matrix to define repertoire. We will be also discussing the relationship between consciousness and entanglement

1. Introduction

In the following sections we will develop a theory of consciousness of a quantum system, consider definitions from Classical Mathematical Integrated Information Theory [1] and try to Quantise it by considering a functor, $\mathcal{F}$ which maps from classical system to the quantum system. And there by using classical definition and generating their quantum analog.

The parts in italic are from Classical Mathematical Integrated Information Theory and parts in normal font is the explanation for Quantum Mathematical Integrated Information Theory

2. Postulates

“..... if we consider a set of elements Λ then system is the topology, $\mathcal{T}$ of the set Λ and mechanisms, M is open subset of set Λ i.e. M belongs $\mathcal{T}$; $M \in \mathcal{T}$”

A set of elements Λ then system is the topology, $\mathcal{T}$ of the set Λ and mechanisms, M is open subset of set Λ i.e. M belongs $\mathcal{T}$; $M \in \mathcal{T}$. Let $\mathcal{F}$ be a functor from topological category $\mathcal{L}$ to linear category $\mathcal{H}$. With $\text{Obj}(\mathcal{L}) := \mathcal{T}$ and $\text{Obj}(\mathcal{H}) := \{\otimes_{\alpha \in M} \mathbb{H}_{\alpha}\}_{M \in \mathcal{T}}$. Considering state as endomorphism if s is the state of mechanism M , $\rho_M := \mathcal{F}(s)$ is endomorphism of $\mathbb{H}_M$, where $\mathbb{H}_M := \otimes_{\alpha \in M} \mathbb{H}_{\alpha}$. ρ_M can be written as $\frac{1}{N} \sum_{\alpha} \otimes_{\alpha_i \in \Lambda} \sigma(M)_{\alpha_i}$ where $\sigma(M)_{\alpha} := \begin{cases} \rho_{\alpha_i} \text{ if } \alpha_i \in M \\ \frac{id_{\alpha_i}}{\dim \mathbb{H}_{\alpha_i}} \text{ if } \alpha_i \notin M \end{cases}$ and $\alpha := (\alpha_i)_i$. For some $\rho_{\alpha} \in \text{End } \mathbb{H}_{\alpha}$ with trace equals to one and are Hermitian. Here N is such that trace of ρ_M is equal to 1.

3. Mechanisms

- “**INFORMATION:**, a mechanism in a state generates information only if it constrains the states of a system that can be its possible causes and effects – its **cause-effect repertoire**. The more selective the possible causes and effects, the higher the **cause-effect information (cei)** specified by the mechanism”
- **Definition: Cause-effect repertoire:** “Itis set of all the probability distributions of states of mechanisms called Perviews, P knowing that mechanism, M is in state, s mathematically speaking $\{p^x(P|M_s)\}_{P \in \mathcal{T}}^{P \neq M}$ where $P \in \mathcal{T}$ and $P \neq M$, p represents probability distribution function, $x \in \{e, c\}$ where e represents effect and c represents cause.”

Here $p^x(P|M_s)$ is conditional probability density so the quantum analog of it is conditional density matrix,

$$q^x(\rho_P|\rho_M(t_0)) := \frac{\text{tr}_M(\rho_P(t)\rho_M(t_0))}{\text{tr}(\rho_P(t)\rho_M(t_0))}$$

Where,

$$\text{tr}_M: \text{End}(\mathbb{H}_\Lambda) \rightarrow \text{End}(\mathbb{H}_\Lambda)$$

such that tr_M of ρ_N is $\frac{1}{N} \sum_\alpha \otimes_{\alpha_i \in \Lambda} \psi(M, N)_{\alpha_i}$ for some $N \in \mathcal{T}$. Here

$$\psi(M, N)_{\alpha_i} := \begin{cases} \sigma(N)_{\alpha_i} & \text{if } \alpha_i \notin M \\ \text{tr}_{\alpha_i}(\sigma(N)_{\alpha_i}) \text{id}_{\alpha_i} & \text{if } \alpha_i \in M \end{cases}$$

And $\text{tr}: \text{End}(\mathbb{H}_\Lambda) \rightarrow \mathbb{C}$, is trace in denominator is over Λ . Also $x := e$ for $t > t_0$ and $x := c$ for $t < t_0$. Note that $\text{tr}_{\alpha_i}: \text{End}(\mathbb{H}_{\alpha_i}) \rightarrow \mathbb{C}$, is the trace over α_i . $\alpha := (\alpha_i)_i$

In the rest of the sections and subsections we need to replace, $q^x(\rho_P|\rho_M(t_0))$ with $p^x(P|M_s)$ and the term generated by $q^x(\rho_P|\rho_M(t_0))$ with term generated by $p^x(P|M_s)$ to develop the quantum analog for the classical one.

- **Definition: Cause information (ci) and effect information (ei):** Let $\mathcal{D}$ be a distance function over probability distribution space. Then the information is defined as follows

$$xi(P|M_s) := \mathcal{D}(p^x(P|M_s), p^x(P|\emptyset))$$

Where unconstrained cause repertoire (same on the effect side), $p^x(P|M_s)$ and $x \in \{e, c\}$.

It is straight forward to see that the analog to **ci** and **ei** is gained by replacing

- $p^x(P|M_s)$ with $q^x(\rho_P|\rho_M(t_0))$

- $p^x(P|\emptyset)$ with $q^x(\rho_P|\rho_\emptyset(t_0))$
- $\mathcal{D}$ with $\mathcal{D}_Q$
- xi with Qxi

$$Qxi(\rho_P|\rho_M(t_0)) := \mathcal{D}_Q \left(q^x(\rho_P|\rho_M(t_0)), q^x(\rho_P|\rho_\emptyset(t_0)) \right)$$

Where $\mathcal{D}_Q$ is quantum distance function unconstrained cause repertoire (same on the effect side), $q^x(\rho_P|\rho_\emptyset)$ and $x \in \{e, c\}$.

Definition: Cause-effect information (cei): Cause-effect information of a mechanism, M in state, s is given by:

$$cei(P|M_s) := \min\{ei(P|M_s), ci(P|M_s)\}$$

Quantum Cause-effect information of a mechanism can be given by:

$$Qcei(\rho_P|\rho_M(t_0)) := \min\{ei(\rho_P|\rho_M(t_0)), ci(\rho_P|\rho_M(t_0))\}$$

- “**INTEGRATION:** Let $p^x(P|\hat{M}_s, \tilde{M}_s)$ be probability distribution of states of states of system $\mathcal{T}$ knowing that mechanism $\hat{M}$ and $\tilde{M}$ are in state s . Where $\hat{M} \cup \tilde{M} := M$, $\hat{M}, \tilde{M} \neq M$ and $\hat{M}, \tilde{M} \in \mathcal{T}$ then,

$$\varphi^x(P|M_s) := \min \left\{ \mathcal{D} \left(p^x(P|M_s), p^x(P|\hat{M}_s, \tilde{M}_s) \right) \right\}_{\hat{M} \cup \tilde{M} := M}$$

Where $x \in \{e, c\}$ and integrated cause-effect information of mechanism, M and in state, s is:

$$\varphi(P|M_s) := \min\{\varphi^e(P|M_s), \varphi^c(P|M_s)\}$$

The quantum analog to $\varphi^x(P|M_s)$ can be given as.

$$\varphi^x(\rho_P|\rho_M(t_0)) := \min \left\{ \mathcal{D} \left(q^x(\rho_P|\rho_M(t_0)), q^x \left(q^x(\rho_P|\rho_{\hat{M}}(t_0)) | \rho_{\tilde{M}}(t_0) \right) \right) \right\}_{\hat{M} \cup \tilde{M} := M}$$

Where minimum is taken over $\hat{M} \cup \tilde{M} := M$, $\hat{M}, \tilde{M} \neq M$ and $\hat{M}, \tilde{M} \in \mathcal{T}$ then, and $x \in \{e, c\}$.

The quantum analog to $\varphi(\rho_P|\rho_M(t_0))$ can be given as

$$\varphi(P|M_s) := \min\{\varphi^e(\rho_P|\rho_M(t_0)), \varphi^c(\rho_P|\rho_M(t_0))\}$$

- “**EXCLUSION:** It is plain and simple to see MICE

$$\varphi^{\max}(M_s) := \max\{\varphi(P|M_s)\}_{\substack{P \in \mathcal{T} \\ P \neq M}}$$

That is maximum of all the integrated cause-effect information taken over all the preview not equals to mechanism, M . If $\varphi^{\max}(M_s) > 0$ then it constitutes a concept, which is defined as follows

$$\mathcal{C}(M_s) := (\varphi^{max}(M_s), \varphi^c(M_s), \varphi^e(M_s))"$$

The quantum analog to $\varphi^{max}(M_s)$ can be given by replacing $\varphi(P|M_s)$ with $\varphi(\rho_P|\rho_M(t_0))$

$$\varphi^{max}(\rho_M(t_0)) := \max\{\varphi(\rho_P|\rho_M(t_0))\}_{\substack{P \in T \\ P \neq M}}$$

The quantum analog to $\mathcal{C}(M_s)$ can be given by

$$\mathcal{QC}(\rho_M(t_0)) := (\varphi^{max}(\rho_M(t_0)), \rho_M^c(t_0), \rho_M^e(t_0))$$

4. Systems of mechanisms

“INFORMATION: Conceptual structure, $\mathcal{CS}(\mathcal{U})$ where $\mathcal{U} \subset \mathcal{T}$, is subsystem.

$$\mathcal{CS}(\mathcal{U}) := (\mathcal{C}(M_s))_{\substack{M \in \mathcal{U} \\ S \in \mathcal{S}}}$$

Where $\mathcal{S}$ is set of all possible state. The conceptual information, $\mathcal{CI}$ of conceptual structure, $\mathcal{CS}(\mathcal{U})$ is linear functional on space of conceptual structure space and is defined as:

$$\mathcal{CI}(\mathcal{CS}(\mathcal{U})) := \sum_{\substack{M \in \mathcal{U} \\ S \in \mathcal{S}}} \varphi^{max}(M_s) \mathcal{I}(\mathcal{U})$$

Where $\mathcal{I}(\mathcal{U}) := \max\{cei(P|M_s)\}_{\substack{P \in T \\ M \in \mathcal{U}}}$. This mathematical expression suites the definition as, it is increases with increase in number of points and also with φ^{max} .

The quantum analog to $\mathcal{CS}(\mathcal{U})$ can be given as, is given by replacing $\mathcal{C}(M_s)$ with $\mathcal{QC}(\rho_M(t_0))$

$$\mathcal{QCS}(\mathcal{U}) := (\mathcal{QC}(\rho_M(t_0)))_{\substack{M \in \mathcal{U} \\ S \in \mathcal{S}}}$$

The quantum analog to $\mathcal{CI}(\mathcal{CS}(\mathcal{U}))$ can be given as, is given by replacing $\varphi^{max}(M_s)$ with $\varphi^{max}(\rho_M(t_0))$ and replacing $\mathcal{I}(\mathcal{U})$ with $\mathcal{QI}(\mathcal{U})$.

$$\mathcal{QCI}(\mathcal{QCS}(\mathcal{U})) := \sum_{\substack{M \in \mathcal{U} \\ t_0 \in \mathcal{R}}} \varphi^{max}(\rho_M(t_0)) \mathcal{QI}(\mathcal{U})$$

where $\mathcal{QI}(\mathcal{U}) := \max\{Qcei(\rho_P|\rho_M(t_0))\}_{\substack{P \in T \\ M \in \mathcal{U}}}$

- *INTEGRATION: A set of elements can be conscious only if its mechanisms specify a conceptual structure that is irreducible to non-interdependent components (strong integration). Strong integration/irreducibility Φ is assessed by partitioning the set of elements into subsets with unidirectional cuts.*

$$\Phi(\mathcal{U}) := \min \left\{ \Delta(\mathcal{CS}(\mathcal{U}), \mathcal{CS}(\hat{\mathcal{U}}) + \mathcal{CS}(\tilde{\mathcal{U}})) \right\}_{\substack{\hat{\mathcal{U}} \cup \tilde{\mathcal{U}} = \mathcal{U} \\ \hat{\mathcal{U}}, \tilde{\mathcal{U}} \neq \mathcal{U} \\ \hat{\mathcal{U}}, \tilde{\mathcal{U}} \subset \mathcal{T}}}$$

Where minimum is taken over all $\hat{\mathcal{U}}, \tilde{\mathcal{U}}$, satisfying three conditions:

- $\hat{\mathcal{U}}, \tilde{\mathcal{U}} \subset \mathcal{T}$: i.e., $\hat{\mathcal{U}}, \tilde{\mathcal{U}}$ belongs to the system
- $\hat{\mathcal{U}} \cup \tilde{\mathcal{U}} = \mathcal{U}$: i.e., union of $\hat{\mathcal{U}}, \tilde{\mathcal{U}}$ is $\mathcal{U}$
- $\hat{\mathcal{U}}, \tilde{\mathcal{U}} \neq \mathcal{U}$: i.e. None of $\hat{\mathcal{U}}, \tilde{\mathcal{U}}$ equals to $\mathcal{U}$

where Δ is distance function over space of conceptional structure and could be defined as follows:

$$\Delta(\mathcal{CS}(\mathcal{U}), \mathcal{CS}(\mathcal{V})) := |\mathcal{CJ}(\mathcal{CS}(\mathcal{U})) - \mathcal{CJ}(\mathcal{CS}(\mathcal{V}))|$$

Where $\mathcal{U}, \mathcal{V} \in \mathcal{T}$.

The quantum analog to $\Phi(\mathcal{U})$ can be given as, is given by replacing $\mathcal{CS}$ with $\mathcal{QCS}$

$$\Phi_{\mathcal{Q}}(\mathcal{U}) := \min \left\{ \Delta(\mathcal{QCS}(\mathcal{U}), \mathcal{QCS}(\hat{\mathcal{U}}) + \mathcal{QCS}(\tilde{\mathcal{U}})) \right\}_{\substack{\hat{\mathcal{U}} \cup \tilde{\mathcal{U}} = \mathcal{U} \\ \hat{\mathcal{U}}, \tilde{\mathcal{U}} \neq \mathcal{U} \\ \hat{\mathcal{U}}, \tilde{\mathcal{U}} \subset \mathcal{T}}}$$

The quantum analog to $\Delta(\mathcal{CS}(\mathcal{U}), \mathcal{CS}(\mathcal{V}))$ can be given as, is given by $\mathcal{CS}$ with $\mathcal{QCS}$ and $\mathcal{CJ}$ with $\mathcal{QCJ}$.

$$\Delta(\mathcal{QCS}(\mathcal{U}), \mathcal{QCS}(\mathcal{V})) := |\mathcal{QCJ}(\mathcal{CS}(\mathcal{U})) - \mathcal{QCJ}(\mathcal{CS}(\mathcal{V}))|$$

- **EXCLUSION:** Of all overlapping sets of elements, only one set can be conscious – the one whose mechanisms specify a conceptual structure that is maximally irreducible (MICS) to independent components. A local maximum of integrated information $\Phi^{\max}$ (over elements, space, and time) is called a complex.

It is synonymous to MICE.

$$\Phi^{\max}(\mathcal{T}) := \max\{\Phi(\mathcal{U})\}_{\mathcal{U} \subset \mathcal{T}}$$

The maximum is taken over all the subsets, $\mathcal{U}$ of the system, $\mathcal{T}$. Where $\Phi^{\max}(\mathcal{T})$ is measure of consciousness of system $\mathcal{T}$.

The quantum analog to $\Phi^{\max}(\mathcal{T})$ can be given as, is given by

$$\Phi_{\mathcal{Q}}^{\max}(\mathcal{T}) := \max\{\Phi_{\mathcal{Q}}(\mathcal{U})\}_{\mathcal{U} \subset \mathcal{T}}$$

The maximum is taken over all the subsets, $\mathcal{U}$ of the system, $\mathcal{T}$. Where $\Phi_{\mathcal{Q}}^{\max}(\mathcal{T})$ is measure of consciousness of quantum system $\mathcal{T}$.

5. Entanglement and Consciousness

Let $\{\mathbb{H}_M\}_{M \in \mathcal{T}}$ be a Quantum system, then the set of repertoires is:

$$\left\{ \left\{ \varrho^x(\rho_P | \rho_M(t_0)) \right\}_{\substack{P \in \mathcal{T} \\ P \neq M}} \right\}_{M \in \mathcal{T}}$$

Where repertoire is:

$$\varrho^x(\rho_P | \rho_M(t_0)) := \frac{\text{tr}_M(\rho_P(t) \rho_M(t_0))}{\text{tr}(\rho_P(t) \rho_M(t_0))}$$

We know that QMICE, $\varphi^{max}(\rho_M(t_0))$ is $\min\{\varphi^e(\rho_P | \rho_M(t_0)), \varphi^c(\rho_P | \rho_M(t_0))\}$,

where

$$\varphi^x(\rho_P | \rho_M(t_0)) := \min \left\{ \mathcal{D} \left(\varrho^x(\rho_P | \rho_M(t_0)), \varrho^x \left(\varrho^x(\rho_P | \rho_{\tilde{M}}(t_0)) \middle| \rho_{\tilde{M}}(t_0) \right) \right) \right\}_{\substack{\tilde{M} \cup \tilde{M} := M \\ \tilde{M}, \tilde{M} \neq M \\ \tilde{M}, \tilde{M} \in \mathcal{T}}}$$

And

$$\varrho^x \left(\varrho^x(\rho_P | \rho_{\tilde{M}}(t_0)) \middle| \rho_{\tilde{M}}(t_0) \right) := \frac{\text{tr}_{\tilde{M}} \left(\text{tr}_{\tilde{M}}(\rho_P(t) \rho_{\tilde{M}}(t_0)) \rho_{\tilde{M}}(t_0) \right)}{\text{tr} \left(\text{tr}_{\tilde{M}}(\rho_P(t) \rho_{\tilde{M}}(t_0)) \rho_{\tilde{M}}(t_0) \right)}$$

Entanglement of the Quantum system is:

$$E_{RE} := \min\{E_{RE}(\rho_N)\}_{N \in \mathcal{T}}$$

Where

$$E_{RE}(\rho_N) := \min \left\{ S \left(\rho_N \parallel \left(\prod_{\alpha_i \in \Lambda} \dim \mathbb{H}_{\alpha_i} \right) \rho_{\tilde{N}} \rho_{\tilde{N}} \right) \right\}_{\substack{\tilde{N} \cup \tilde{N} := N \\ \tilde{N}, \tilde{N} \neq N \\ \tilde{N}, \tilde{N} \in \mathcal{T}}}$$

And $S(*\|*)$ is the relative entropy. Considering the distance function, $\mathcal{D}$ as S and now making the following observation. For each pair of $M, P \in \mathcal{T}$ we can find a $N \in \mathcal{T}$ such that $\rho_N := \varrho^x(\rho_P | \rho_M(t_0))$ and for each pair of $\tilde{M}, \tilde{M}$ we can find a pair of $\tilde{N}, \tilde{N}$ satisfying all the above conditions and $\rho_{\tilde{N}} \rho_{\tilde{N}} := \varrho^x \left(\varrho^x(\rho_P | \rho_{\tilde{M}}(t_0)) \middle| \rho_{\tilde{M}}(t_0) \right)$, due to way more constrains in the definition of QMICE the converse is not true that's why

$$E_{RE} \left(\varrho^x(\rho_P | \rho_M(t_0)) \right) \leq \varphi(P | M_S)$$

Hence

$$E_{RE} \leq \varphi^{max}(\rho_M(t_0))$$

Hence entanglement generate concept.

We define Entanglement information, $\mathcal{EI}$ by replacing QMICE with E_{RE} in $\mathcal{QCI}$ and it is not difficult to see that $\mathcal{EI} \leq \mathcal{QCI}$. Similarly, with the definition that we have of Δ ,

$$\Delta(\mathcal{QCS}(\mathcal{U}), \mathcal{QCS}(\mathcal{V})) := |\mathcal{QCI}(\mathcal{CS}(\mathcal{U})) - \mathcal{QCI}(\mathcal{CS}(\mathcal{V}))|$$

We define $\Phi_{\mathcal{E}}(\mathcal{U})$ for the subsystem $\mathcal{U}$ by replacing $\mathcal{QCI}$ with $\mathcal{EI}$ in the definition of $\Phi_{\mathcal{Q}}(\mathcal{U})$. And we get $\Phi_{\mathcal{E}}(\mathcal{U}) \leq \Phi_{\mathcal{Q}}(\mathcal{U})$ for all $\mathcal{U}$. $\max\{\Phi_{\mathcal{E}}(\mathcal{U})\}_{\mathcal{U} \subset \mathcal{T}} \leq \max\{\Phi_{\mathcal{Q}}(\mathcal{U})\}_{\mathcal{U} \subset \mathcal{T}}$. Which implies $\Phi_{\mathcal{E}}^{max}(\mathcal{T}) \leq \Phi_{\mathcal{Q}}^{max}(\mathcal{T})$, where $\Phi_{\mathcal{E}}^{max}(\mathcal{T}) := \max\{\Phi_{\mathcal{E}}(\mathcal{U})\}_{\mathcal{U} \subset \mathcal{T}}$. Which means if entanglement occurs it creates consciousness, which would explain the breakdown of wavefunction due to it.

6. Discussion and Conclusion

For a set of elements Λ (e.g. degrees freedom), its open subset is a mechanism, M (in case of discrete topology all the subsets are open) and set of open sets i.e. topology, $\mathcal{T}$ is system. The QIIT is the functor, $\mathcal{F}$ from the topological category i.e. category whose set of object is the topology $\mathcal{T}$ to the linear category, i.e. the category whose objects are the set Hilbert spaces. The state of endomorphism between on the Hilbert spaces. Cause- effect repertoire of a given mechanism, M is the its conational density matrix, $\varrho^x(\rho_P|\rho_M(t_0))$ of other mechanisms, P knowing a given mechanism, M in a given particular state. Given a distance function on density matrix space, $\mathcal{D}_{\mathcal{Q}}$. The minimum distance between the repertoire of a mechanism, $\varrho^x(\rho_P|\rho_M(t_0))$ and repertoire of its sub-mechanism and then considering that repertoire as perview for calculating the repertoire for its complement sub-mechanism, $\varrho^x(\varrho^x(\rho_P|\rho_{\bar{M}}(t_0))|\rho_{\bar{M}}(t_0))$. The maximum of all quantum cause effect integrated information in QMICE and if it is non zero then it creates concept. The concept is 3-tuple of QMICE, respective cause repertoire and effect repertoire. Conceptual structure is a subsystem (subset of topology) is tuple over all states and mechanisms over that system. Conceptual information is measure of information in conceptual structure and increases with increase in QMICE of concept of conceptual structure and with increase in number of concepts of conceptual structure and could be defined as:

$$\mathcal{QCI}(\mathcal{CS}(\mathcal{U})) := \sum_{\substack{M \in \mathcal{U} \\ t_0 \in R}} \varphi^{max}(\rho_M(t_0)) \mathcal{QI}(\mathcal{U})$$

Where $\mathcal{QI}(\mathcal{U}) := \max\{\mathcal{QCI}(\rho_P|\rho_M(t_0))\}_{\substack{P \in \mathcal{T} \\ M \in \mathcal{U}}}$. Quantum Integrated conceptual information is the minimum of the distance between conceptual structure of a sub-system, $\mathcal{U}$ and the subtotal of the conceptual structure of its constituents, $\hat{\mathcal{U}}$, $\tilde{\mathcal{U}}$ and is synonymous to Quantum integrated cause-effect information and can be defined as,

$$\Phi_{\mathcal{Q}}(\mathcal{U}) := \min \left\{ \Delta \left(\mathcal{QCS}(\mathcal{U}), \mathcal{QCS}(\hat{\mathcal{U}}) + \mathcal{QCS}(\tilde{\mathcal{U}}) \right) \right\}_{\substack{\hat{\mathcal{U}} \cup \tilde{\mathcal{U}} = \mathcal{U} \\ \hat{\mathcal{U}}, \tilde{\mathcal{U}} \neq \mathcal{U} \\ \hat{\mathcal{U}}, \tilde{\mathcal{U}} \subset \mathcal{T}}}$$

Where Δ is distance function over space of conceptual structures and can be best defined as,

$$\Delta(QCS(u), QCS(v)) := |QCI(CS(u)) - QCI(CS(v))|$$

Where $u, v \in \mathcal{T}$. Quantum Maximum of integrated conceptual information is measure of consciousness.

7. Declarations

1. Ethics approval and consent to participate

Not applicable

2. Consent for publication

I give my consent to publication

3. Availability of supporting data

Not applicable. If your manuscript does not contain any data, please state 'Not applicable' in this section.

4. Competing interests

No competing interests

5. Funding

No funding

6. Authors' contributions

I solely contributed in every capacity

7. Acknowledgements

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