A simple extension to indefinite terms of the Classical Syllogistics and of the Rules of Valid Syllogism

To the memory of my sister Cristina Popa, 1948-2018.

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Abstract. One uses a minimal extension to indefinite terms of the Classical Syllogistics and of the Rules of Valid Syllogism. The valid syllogisms (VS) are replaced by valid categorical arguments (VCAs), made of conclusive pairs of categorical premises (PCPs) and their logical consequences (LCs), which can contain indefinite (both positive and negative) terms. The Classical Syllogistics is replaced by explicit formulas reproducing all 32 VCAs and their LCs. One notices that all the VCAs (and VS) follow one of the Barbara, Darapti or Darii “patterns of inclusion and intersection”, namely either $S \subseteq M \subseteq P$, or, $M \subseteq S$, $M \subseteq P$, or $M \cap S \neq \emptyset$, $M \subseteq P$. One shows that by using slight generalizations of the rules of valid syllogism (RofVS) one can transform the RofVS into a “theory” equivalent to Classical Syllogistics. By using appropriate rules of valid categorical argument (RofVCA), (which disregard the RofVS asserting that “two negative premises are not allowed” and “the middle term has to be distributed in at least one premise”), one can make the RofVCA into a “theory” “almost equivalent” with the formulas which describe all the VCAs. One also notices that each precise LC of the Barbara, Darapti and Darii patterns pinpoints a unique partitioning subset of the universe of discourse, U : $S = S \cap M \cap P$ and $P' = S' \cap M' \cap P'$ for Barbara’s pattern - which entails two LCs; $M = M \cap S \cap P$ for Darapti’s pattern; $S \cap M \cap P \neq \emptyset$ for Darii’s pattern. The middle term elimination from the LC, initiated by Aristotle, simplifies but weakens the LC.

Keywords: Categorical Premises, Valid Categorical Argument, Valid Syllogism, Term Relabelings, Rules of valid syllogism, Rules of valid categorical argument.

1 Preliminaries

One uses the following notations and abbreviations: U for the universe of discourse of a 3-term syllogism, made of 2^3 partitioning subsets of U , which will be simply called subsets. No other set will be called a subset except a partitioning subset of U . (Boole, [1], calls these partitioning subsets “constituents”.) Adjacency of set names will denote set intersections: for example, SM denotes the intersection, $S \cap M$, of the sets S and M . The union of sets will be denoted by a $+$ instead of $\cup$. PCP will stand for pair of categorical premises; LC for logical consequence or conclusion. Existential Import

will be shortened to ei. VCA stands for valid categorical argument, i.e., stands for any PCP which entails a logical consequence (LC). VS stands for valid syllogism(s), recognized as such by the Classical Syllogistics, and the RofVS stand for the rules of valid syllogism as accepted by Classical Syllogistics. The RofVCA stands for the rules of valid categorical argument. DofA stands for Domain of Applicability of the RofVCA or of the RofVS. ESC stands for Empty Set Constraints. A syllogism contains three categorical statements - two premises and their proposed logical consequence (LC) or conclusion. Each of the two premises contains the middle term, denoted by M, and two other terms, P and S which will appear again in the LC. The S, P, M terms are called positive terms and their complementary sets in U, S', P', M', are the negative terms; together they are the indefinite terms. For valid syllogisms, the Classical Syllogistics accepts, by definition, only these LC formats: A(S,P), E(S,P), I(S,P), O(S,P). By contrast, even after eliminating the middle term from it, the LC of a VCA can have any of the eight formats E(S*,P*), I(S*,P*), where $P^* \in \{P, P'\}$, $S^* \in \{S, S'\}$.

2 Introduction

One will distinguish, e.g., between the Barbara pattern, $S \subseteq M \subseteq P$, (one could have written it, using more “neutral” terms, as $M_1 \subseteq M_2 \subseteq M_3$), and the Barbara type VCAs, which one may denote by $S^* \subseteq M^* \subseteq P^*$, where $P^* \in \{P, P'\}$, $M^* \in \{M, M'\}$, $S^* \in \{S, S'\}$. The Barbara pattern is unique, but, once the middle term is decided upon, there are eight valid categorical arguments (VCAs) of Barbara’s type. Firstly one gives formulas which encompass all the VCAs and their conclusions (LCs) for each of the three patterns of VCAs. These formulas can replace the moods and figures of the Classical Syllogistics, and represent an extension to premises and conclusions (LCs) which contain negative terms. Next, one defines what distribution means, one describes the Classical Syllogistics and one gives a listing of the Rules of valid syllogism (RofVS). A generalization of the RofVS transforms them into a “theory” which gives the same results as the Classical Syllogistics. After one abandons two of the RofVS, which cannot be extended to indefinite terms, namely the rules “two negative premises are not allowed” and “the middle term has to be distributed in at least one premise”, the four remaining Rules of valid categorical argument (RofVCA) can predict, for each VCA or existential import (ei) VCA, its LC – out of which the middle term was eliminated. One recognizes that the whole purpose of the RofVS and the RofVCA - to be able to quickly find the LC of any PCP, can be accomplished with about the same ease by using the formulas for the three patterns of the VCAs. The last two sections examine empty set constraints (ESC) and how many simultaneously sound VCAs one may obtain out of three given terms.

3 The notions of distribution and of affirmative and negative categorical statements extended to indefinite terms

“A term is said to be distributed when reference is made to all the individuals denoted by it; it is said to be undistributed when they are only referred to partially, i.e., information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class.” Keynes [2]. Firstly, one may expand the definition of distribution, by agreeing that whatever distribution the two terms appearing in a categorical statement may have, then their complementary terms in U , are automatically assigned an opposite distribution. Thus since in $I(M,P)$, the terms M and P are undistributed, the terms M' , P' are distributed in the same $I(M,P)$ statement. This is in agreement with the obversion and contraposition rules and the standard definition of distribution: from $I(M,P)=O(M,P')=O(P,M')$ one realizes that M' and P' are distributed since M,P were not. The same definition of distribution, extended to indefinite terms, was used, e.g., by Alvarez and Correia [3] and was cited in Alvarez-Fontecilla and Lungenstrass [4] as having been used in 1932 by Wilkinson [5]. Note also that the arguments of a statement and of its contradictory one, have opposite distributions – in $E(M,P)$ both M and P are distributed, while in the contradictory statement, $I(M,P)$, both M and P are undistributed, (while M' , P' are distributed). Similarly, in $A(M,P)$, M is distributed and P is not, while in the contradictory statement, $O(M,P)$, the term distributions are reversed.

Now one defines, for both universal and particular premises, when they are considered as affirmative and when they are considered as negative statements. The universal negative premises are $E(M,P)$, $E(M,S)$, $E(M',P')$, $E(M',S')$, and the only particular negative premises are $O(P,M)$, $O(S,M)$, $O(M,P)$, and $O(M,S)$. Denoting $h \in \{S,P\}$, $h' \in \{S',P'\}$, the universal negative premises are $E(M,h)$, $E(M',h')$, the particular negative premises are $I(M',h)$, $I(M,h')$, the universal affirmative premises are $E(M',h)=A(h,M)$, $E(M,h')=A(M,h)$, the particular affirmative premises are $I(M,h)$, $I(M',h')$. One can see that the switch $E \leftrightarrow I$ while the arguments are left unchanged transforms universal negative premises into particular affirmative premises and vice versa, and transforms universal affirmative premises into particular negative premises and vice versa. The switch $M \leftrightarrow M'$, (resp. $h \leftrightarrow h'$), transforms affirmative premises into negative premises and vice versa. One can define the negativity or signature, s , of a statement symbol, $s(A)=s(I)=0$, $s(E)=s(O)=1$, the signature of a term, $s(M)=s(P)=s(S)=0$, $s(M')=s(P')=s(S')=1$, and, the signature of a whole statement as the sum of the signatures modulo 2 of the statement's symbol and all of its terms. Then a statement is affirmative, (or positive), if its signature is zero, and is negative if its signature is 1. Thus, e.g., $s(A(M,P))=s(E(M,P'))=s(A(P',M'))=0$, $s(A(M',P))=s(E(M',P'))=1$; therefore the first three statements are affirmative, and the last two are negative. These definitions will be used to prove by cases the RofVCA #4 – see Section 5.

4 Comments on the Classical Syllogistic and its minimal extension to indefinite terms

Given three concrete terms and their complementary ones, e.g., good - bad people, wise - foolish people, intelligent - dumb people, one can construct a valid syllogism of each of the Barbara, Darapti or Darii patterns in $3!2^3=48$ ways. But once one decides which term plays the role of the middle term, then the $3!$ permutations “of the role played by each term” are reduced to only $2!$ permutations of the remaining two “end terms” (S and P). Thus, for each pattern, and once the middle term is agreed upon, one can construct $2*8=16$ distinct PCPs out of the three given terms and their complementary ones. But, (in the usual, S,P,M term notation), there are only eight valid categorical arguments of Barbara’s type, (1) $E(M^*,P^*)E(M^*,S^*)$: $S^*=M^*P^*S^*$, $P^*=M^*P^*S^*$, where the precise, “one pinpointed subset” LCs, were written after the column sign, and where, from now on, $M^* \in \{M, M'\}$, $P^* \in \{P, P'\}$, $S^* \in \{S, S'\}$. For the simple proofs, based on decomposing S^* and P^* into subsets, see Radulescu [4]. ($S^*=S^*M^*+S^*M^*=S^*M^*=S^*M^*P^*+S^*M^*P^*=S^*M^*P^*$, etc.) There are only eight, not 16, VCAs of Barbara’s type, $S^* \subseteq M^* \subseteq P^*$, because, e.g., the two choices, $S \subseteq M \subseteq P$ and $P' \subseteq M' \subseteq S'$, correspond to the same pair of Barbara type premises. (Nevertheless, one can see that each Barbara type VCA entails two different LCs.) Also, there are only eight valid categorical arguments (VCAs) of Darapti’s pattern, $M \subseteq S$, $M \subseteq P$, because the Darapti type premises are invariant to the permutation $S \leftrightarrow P$. They can all be written as (2) $E(M^*,P^*)E(M^*,S^*)$: $M^*=M^*P^*S^*$. Finally, Darii’s pattern is not symmetric to the $S \leftrightarrow P$ permutation – thus it generates 16 distinct PCPs: One gets eight Darii type valid categorical arguments, (3i) $E(M^*,P^*)I(M^*,S^*)$: $S^*M^*P^* \neq \emptyset$, and another eight Disamis type valid categorical arguments, (3ii) $I(M^*,P^*)E(M^*,S^*)$: $P^*M^*S^* \neq \emptyset$, even if both sets follow the Darii pattern (one might have called it the Disamis pattern as well). By accepting both positive and negative terms in the premises and the conclusions, one thus gets 32 valid categorical arguments (VCAs) – only eight of them are the distinct, i.e., Boolean, valid syllogisms from the Classical Syllogistics. See Copi [7], Hurley [8]. One may re-write the precise LCs from the formulas (1)-(3ii) as LCs out of which the middle term was eliminated: (1') $E(M^*,P^*)E(M^*,S^*)$: $A(S^*,P^*)$, $A(P^*,S^*)$; $I(S^*,P^*)$ if $S^* \neq \emptyset$, $I(P^*,S^*)$ if $P^* \neq \emptyset$. (2') $E(M^*,P^*)E(M^*,S^*)$: $I(S^*,P^*)$ if $M^* \neq \emptyset$. (3i') $E(M^*,P^*)I(M^*,S^*)$: $O(S^*,P^*)$. (3ii') $I(M^*,P^*)E(M^*,S^*)$: $O(P^*,S^*)$ One can see by inspection, that the above formulas (1') - (3ii'), listing the LCs from which the middle term was eliminated, are in agreement with the four Rules of valid categorical arguments (RofVCA) listed below:

RofVCA #1: The distribution of the end terms is conserved in all non-ei and ei VCAs, except in type (1') ei VCAs, where ei on S^* , (resp. P^*) changes S^* , (resp. P^*), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P^* , (resp. S^*), remains the same as it was in the PCP. (One cannot impose ei on an undistributed term and expect an ei LC!)

RofVCA #2: From two universal premises follows a universal LC, except when an ei condition on M^* , S^* or P^* is imposed – then a particular LC follows.

RofVCA #3: If the PCP contains exactly one particular premise, then the LC is particular.

RofVCA #4: From two affirmative or two negative premises an affirmative LC follows; from one affirmative and one negative premises a negative LC results. This rule is valid even for LCs obtained after an ei condition was imposed. Note that the formulas (1') - (3ii') already prove, by inspection, the RofVCA #1, #2, and #3. But to prove the RofVCA #4 one has to show that RofVCA #4 is satisfied for each choice of S^*, M^*, P^* in each of the formulas (1') - (3ii') – see Section 5.

In the reverse direction, one can apply – to the PCPs listed in the formulas (1) - (3ii), (or to the same PCPs listed in the formulas (1') - (3ii')) – the RofVCA listed above, to deduce, the formulas (1') - (3ii') which contain only LCs from which the middle term was eliminated. By definition, the RofVCA, as well as the RofVS, deal only with LCs whose term variables are $P^* \in \{P, P'\}$ and $S^* \in \{S, S'\}$. Therefore the RofVCA cannot, and do not, predict the universal LCs from the formulas (1)-(3ii). In particular, the RofVCA do not predict the universal LCs of the type (2) non-ei Darapti VCAs, $A(M^*, S^*P^*)$ or $M^* = M^*S^*P^*$. Any universal LC of the type $E(S^*, P^*)$ which one might try for the type (2) PCPs, $E(M^*, P^*)E(M^*, S^*)$, will not satisfy all the RofVCA themselves since two negative premises entail an affirmative LC (rule #4) and the distributions of the terms S^* and P^* are conserved (rule #2). Thus the only LCs which satisfy all the RofVCA for type (2) PCPs, are the particular LCs which preserve in the LCs the distributions which the end terms already had in the PCPs: $I(S^*, P^*)$. Thus, for the Darapti type VCAs, the RofVCA automatically detect that, in the variables S^* and P^* , only a particular, existential import (ei) LC is possible. So, if the middle term is unwanted, the RofVCA “forces upon us” an ei LC.

One could have obtained the above results starting with an eight by eight PCP matrix obtained by pairing up the eight categorical P-premises, $E(M^*, P^*)$, $I(M^*, P^*)$, with the eight categorical S-premises, $E(M^*, S^*)$, $I(M^*, S^*)$. Besides the 32 VCAs' PCPs listed above, the 64 PCP matrix contains 32 PCPs which do not entail any LC. They are split into patterns and subtypes as follows. The two particular premises pattern: (4i) $I(M^*, P^*)I(M^*, S^*)$, (4ii) $I(M^*, P^*)I(M^*, S^*)$. The pattern of one universal premise plus one particular premise, one “acting” on M and the other on M': (5i) $E(M^*, P^*)I(M^*, S^*)$, (5ii) $I(M^*, P^*)E(M^*, S^*)$. Note that the lack of LCs for the PCPs of types (4i)-(5ii) is an obvious consequence of the “one subset LC” paradigm, (since these PCPs do not pinpoint a subset of U). For each such PCP, Classical Syllogistics offers the counterexample method or the Venn diagrams method to prove that the PCP does not entail any LC. The RofVS “theory” uses the RofVS #1 and #2, (which are not true outside of Classical Syllogistics), to prove that those PCPs of types (4i)-(5ii) which are formulable via only positive terms do not entail any LC. See next Section.

Classical Syllogistics uses premises formulable only via positive terms, uses syllogistic figures, accepts as generating valid syllogisms only those PCPs which entail a logical consequence (LC) of one of the formats $A(S, P)$, $E(S, P)$, $I(S, P)$, $O(S, P)$. One can stop considering redundant syllogistic figures if one cuts off, from the 64 PCP matrix mentioned above, two rows, (resp. columns), corresponding to the two P-premises, (resp. S-premises), containing negative terms, $E(M', P') = A(M', P) := A'$, and

$I(M',P') = O(M',P) := O'$, (resp. $E(M',S') = A(M',S) := A'$, and $I(M',S') = O(M',S) := O'$). Cutting off two rows and two columns eliminates $8 \times 4 - 4 = 28$ elements from the eight by eight PCP matrix, and one is left with a six by six, 36 PCP matrix made of premises formulable via only positive terms. (The subtracted 4 equals the number of matrix elements which, when subtracting 4×8 , were eliminated twice.) Out of these 36 PCPs, eight PCPs will generate the eight Boolean valid syllogisms (VS) which become 15 VS when redundant syllogistic figures give different names to the same content VS: e.g., the same VS, “No M is P, Some M is S, Therefore Some S is P”, is counted four times under four different names Ferio, Festino, Ferison, Fresison, according to the syllogistic figures corresponding to various permutations of the premises’ terms. Another three PCPs out of 36 will generate the existential import (ei) VS Bramantip, Darapti and Felapton/Fesapo. Thus, a total of only 11 distinct PCPs formulable via only positive terms generate all the 19, or even 24 VS and ei VS (if one adds Barbari, Celaront/Cesaro and Camestros/Camenos obtained via ei on S). About the other 25 PCPs from the 36 PCP matrix, Classical Syllogistics has to assert, either that they do not entail any LC at all, or that the LC is not having one of the correct formats, $A(S,P)$, $E(S,P)$, $I(S,P)$, $O(S,P)$, imposed by the requirement that P should be the predicate of the LC.

As part of the 36 PCP matrix of the PCPs formulable using only positive terms, one is left with five PCPs of type (4i), made of two particular premises both acting on either M or M': $I(M,P^*)I(M,S^*)$, $P^* \in \{P, P'\}$, $S^* \in \{S, S'\}$, and also $I(M',P)I(M',S)$ – since there exists only one PCP formulable via positive terms if both premises contain M'. There are only four PCPs of type (4ii): $I(M,P^*)I(M',S)$, $P^* \in \{P, P'\}$, and $I(M',P)I(M,S^*)$, $S^* \in \{S, S'\}$. There are four PCPs of type (5i), $E(M,P^*)I(M',S)$, $P^* \in \{P, P'\}$, and $E(M',P)I(M,S^*)$, $S^* \in \{S, S'\}$, and another four PCPs of type (5ii): $I(M,P^*)E(M',S)$, $P^* \in \{P, P'\}$, and $I(M',P)E(M,S^*)$, $S^* \in \{S, S'\}$. The total number of PCPs formulable via only positive terms which do not entail any LC is thus 17. As already mentioned, one arrives at this lack of LCs conclusion not by using the Venn diagram method of Classical Syllogistic, but rather the obvious and intuitive “one subset LC” paradigm: if the pair of categorical premises (PCP) pinpoint a unique subset of U one has an LC, otherwise there is no LC. This leaves eight PCPs about which the Classical Syllogistic has something interesting to say.

Four, out of these last eight PCPs, are dismissed by Classical Syllogistics since their LCs is $O(P,S)$: IE (Fireo?), AO (Bacordo?), I'E' (Boraco?), AE (Falepton?). Note that Classical Syllogistic uses the particular, ei LC, $I(S,P)$, of the Bramantip premises, E'A, but not the universal LC, $A(P,S)$, of the same premises, since $A(P,S)$ does not have one of the four LC formats required by the condition that the P term be the predicate of the LC. On the other side, the four PCPs entailing the $O(P,S)$ LC, and Bramantip's PCP, entailing the $A(P,S)$ universal LC, do not contradict any of the RofVS, and both Classical Syllogistics and the RofVS predict for these PCPs the same LCs: either $O(P,S)$ or $A(P,S)$. Since the RofVS do not apply to the ei VS, one can, for now, say that the domain of applicability (DofA) for the RofVS is made of a total of 13 PCPs: the eight Boolean VS plus the five PCPs whose LCs are $A(P,S)$ or $O(P,S)$. But in order to expand the RofVS into an (almost axiomatic) “theory” equivalent to the

Classical Syllogistics one has to extend the RofVS such that they can also predict the existential import (ei) LCs of the ei VS recognized by Classical Sylogistics: Bramantip,, Darapti, Felapton/Fesapo, Barbari, Celaront/Cesaro, Camestros/Camenos. Section 7 describes how the RofVS “theory” does that, and how the RofVS “theory” eliminates from the RofVS DofA the other 17 PCPs mentioned above, which do not entail any LC. The Classical Syllogistics’ reason for “explaining away” and “brushing over” the VCAs having A(P,S) and O(P,S) as LCs is that a premises’ transposition and a relabeling $S \leftrightarrow P$, (Quine [7] calls it relettering), would transform them into VS. [Another very good reason to dismiss the VCAs whose LCs are A(P,S) and O(P,S), would have been that their premises, “contradict pairwise” the premises of the VS whose LCs are A(S,P) and O(S,P). Indeed, the following pairs of PCPs have premises which either contradict each other, EI and IE, OA and AO, E’I’ and I’E’, or are contraries to each other, EA and AE – the latter two PCPs are both sound only if $M=P=S=0$. Barbara’s and Bramantip’s premises, AE’ and E’A, can be all sound only if $S=P=M$. Thus, when the terms take concrete values, whichever PCP generates a sound VCA, it can be written as a VS. See Section 9.]

Finally, there are four PCPs which are discarded by the Classical Syllogistics in accordance with the RofVS #1 and RofVS #2. The PCP, $E'E'=E(M',P)E(M',S)=A(P,M)A(S,M)$, is discarded because M is undistributed in both premises; then, the PCPs, EE, OE, EO, are discarded because in each PCP both premises are negative. The RofVS #1 and RofVS #2 postulate that such PCPs, i.e., the four PCPs above, E’E’, EE, OE, EO, cannot entail any of the LCs accepted by Classical Syllogistic – which can indeed be shown using Venn diagrams – see Quine [7]. But, as formula (2’) shows it for E’E’ and EE, and formulas (3i’) and (3ii’) show for the EO and OE PCPs, the above four PCPs generate ei VCAs or VCAs, all having $I(S',P')$ as their LC.

But since the RofVS do not refer at all to the ei VCA, as said, one has to slightly generalize them, using the RofVCA as a model, such that the RofVS DofA extends to the ei VS of Classical Syllogistics for which the RofVS should predict the ei LCs admitted by Classical Syllogistics. The Rules of VCA (RofVCA) are satisfied by all the 32 PCPs which entail LCs. On their above 32 PCPs DofA, the RofVCA can predict both existential import (ei) LCs and non-existential import (non-ei) LCs, out of which the middle term was eliminated. If one postulates that the PCPs of type (4i) do not entail any LCs, (which is true according to the “one subset LC” paradigm used above to show that the all the PCPs of types (4i)–(5ii) do not entail any LCs), then the RofVCA show that the PCPs of types (4ii), (5i) and (5ii) cannot entail any LCs, either. Thus RofVCA can replace the Classical Syllogistics extended to indefinite terms to decide if a PCP entails an LC, and to efficiently predict that LC. See Section 7. An analogous proof is given in Section 6, that if one postulates, in accord with the RofVS that “two negative premises and two premises in which the middle term is undistributed do not entail an LC”, then any proposed LCs being entailed by any of the 17 PCPs mentioned above, (as not entailing any LC according to the “one subset LC” paradigm), would contradict at least one of the RofVS. Note that one can simply ignore the RofVS, as Quine [9] does. For an axiomatic treatment of the RofVS and another (“sprawling” - since syllogistic figures are taken into account) extension to PCPs containing negative terms, see Alvarez and Correia [3].

5 A proof of RofVCA #4 - “if one premise is negative, the conclusion must be negative, if the premises are both affirmative or both negative, the conclusion is affirmative”

One has to show that RofVCA #4 is satisfied for each choice of S^* , M^* , P^* , in each of the formulas (1') - (3ii'). For each of the four formulas, one uses the definitions introduced in Section 3, and one does a proof by cases. For type (1') Barbara, with premises $E(M^*, P^*)E(M^*, S^*)$, and with the LCs $A(S^*, P^*)=A(P^*, S^*)=E(S^*, P^*)$; $I(S^*, P^*)$ if $S^* \neq \emptyset$, $I(P^*, S^*)$ if $P^* \neq \emptyset$, the premises' signatures are $(1+s(M^*)+s(P^*)) \bmod 2$, $(2+s(M^*)+s(S^*)) \bmod 2$, the universal LC, (after middle term elimination), has the signature $(1+s(P^*)+s(S^*)) \bmod 2$ and the particular, ei LCs have the very same signature: $(1+s(P^*)+s(S^*)) \bmod 2$. It results that when $s(S^*)=s(P^*)$ the LCs are negative statements, while the premises have different signatures, i.e., one premise is affirmative and one negative; thus RofVCA #4 holds. When $s(S^*)$ and $s(P^*)$ have different signatures then the LC is an affirmative statement, while the premises have the same signature, i.e., the premises are either both affirmative or both negative – and RofVCA #4 holds again. Instead of continuing to verify RofVCA #4 in the same way for the formulas (2') - (3ii'), one can slightly simplify the verification procedure by observing that the formulas (1') - (3ii') - from whose LCs the middle term was eliminated - were already written to formally satisfy all the RofVCA, including RofVCA #4: For types (1') and (2') – the two premises are negative, $E(M^*, P^*)E(M^*, S^*)$, (resp. $E(M^*, P^*) E(M^*, S^*)$), and, according to the RofVCA #4, the LCs are affirmative: $A(S^*, P^*)$; $I(S^*, P^*)$ if $S^* \neq \emptyset$, $I(P^*, S^*)$ if $P^* \neq \emptyset$, (resp. $I(S^*, P^*)$). In these affirmative LCs the distributions of S^* and P^* are the same as they were in the PCPs, except in type (1') ei VCAs, where ei on S^* , (resp. P^*) changes S^* , (resp. P^*), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P^* , (resp. S^*), remains the same as it was in the PCP – and this proves RofVCA #1 for PCPs of types (1') and (2').) Premises of types (3i') and (3ii'), (one negative and one affirmative premises), $E(M^*, P^*)I(M^*, S^*)$, (resp. $I(M^*, P^*)E(M^*, S^*)$), entail the negative LCs, $O(S^*, P^*)$, (resp. $O(P^*, S^*)$), in which the distributions of S^* and P^* are again the same as they were in the PCPs. To prove RofVCA #4 in a simpler way, it remains to show that if the terms signatures in the LCs change the LC statement's signature, those terms signatures change the premises signatures in such a way that the RofVCA #4 is still satisfied. Therefore, this slightly simpler proof of RofVCA #4 is based on dropping the statement symbol signatures, and considering only the statements' arguments signatures, via introducing the “argumental signature” of each premise, and of the LC. For type (1') Barbara, the argumental signature of each negative premise, $E(M^*, P^*)$, (resp. $E(M^*, S^*)$), is $[s(M^*)+s(P^*)] \bmod 2$, (resp. $[s(M^*)+1+s(S^*)] \bmod 2$), the argumental signature of the universal LC, $A(S^*, P^*)$, is $[1+s(P^*)+s(S^*)] \bmod 2$ and the particular, ei LCs have the very same signature: $(1+s(P^*)+s(S^*)) \bmod 2$. It follows that when $s(P^*)=s(S^*)$ the LC is in fact negative, but then the premises are one affirmative and one negative – as they should be according the RofVCA #4. When $s(P^*) \neq s(S^*)$, the LC remains affirmative, and the two premises have the same argumental signatures – thus the two premises will both remain negative, or both become affirmative. For the other formulas, (2') - (3ii'), the proof of the RofVCA #4 proceeds similarly: For type (2') Darapti, $E(M^*, P^*)E(M^*, S^*)$ with the LC

$I(P^*, S^*)$ if $M \neq \emptyset$, the premises' argumental signatures are $(s(M^*) + s(P^*)) \bmod 2$, $(s(M^*) + s(S^*)) \bmod 2$, and the LC has the argumental signature $(2 + s(P^*) + s(S^*)) \bmod 2$. It results that when $s(S^*) = s(P^*)$, the LC remains an affirmative statement, while the premises have the same signature, i.e., both premises are affirmative or both are negative. When $s(S^*) \neq s(P^*)$ the LC becomes a negative statement, while the premises will have different signatures, i.e., one premise is affirmative and the other is negative. For type (3i') Darii, $E(M^*, P^*)I(M^*, S^*)$ with the LC $O(S^*, P^*)$, the premises' argumental signatures are $(s(M^*) + s(P^*)) \bmod 2$, $(s(M^*) + s(S^*)) \bmod 2$, and the LC has the argumental signature $(s(P^*) + s(S^*)) \bmod 2$. It results that when $s(S^*) \neq s(P^*)$ the LC becomes an affirmative statement, while the premises will have different argumental signatures, i.e., both premises are either affirmative or both are negative. When $s(S^*) = s(P^*)$ then the LC remains a negative statement, while the premises have the same argumental signatures, i.e., the premises remain one affirmative and one negative. The proof that the type (3ii') VCAs also satisfy the RofVCA #4 is identical to the one for type (3i') VCAs.

6 The developing of the RofVS “theory”

First thing to note, is that the RofVS, like the Classical Syllogistics, suppose that the PCPs are formulable using only positive terms, and that, if a PCP entails an LC, then the LC is formulable using only positive terms, too. One lists the RofVS, Copi [7] Hurley [8] Stebbing [10] Keynes [2], in the following order: #1 - “the middle term has to be distributed in at least one premise”, #2 - “two negative premises are not allowed”, (these two syllogistic rules refer to the PCPs as not entailing an LC (of one of the “standard” four formats $A(S, P)$, $E(S, P)$, $I(S, P)$, $O(S, P)$ – the following four rules refer to the PCP and the entailed LC, and will be used to predict the LC of any PCP from the RofVS' DofA), #3 - “any term distributed in the LC must be distributed in the PCP”, (i.e., the distribution of the end terms, P and S cannot “increase” from undistributed in the premises to distributed in the LC, but can, conceivable, decrease from distributed in the premises to undistributed in the LC), #4 - “if either premise is negative, the LC must be negative”, #5 - “from two universal premises, no particular LC may be drawn”, #6 - “if one premise is particular, then the LC is particular”. The RofVS were developed into a more rounded “theory”, (or even into a set of axioms, see Alvarez and Correia [3]): firstly, one specifies a set of PCPs which is not part of the RofVS' DofA: according to RofVS #1, “PCPs in which the middle term is not distributed in any of the two premises, do not entail any LCs”, and, according to RofVS #2, “PCPs made of two negative premises do not entail any LCs”. Copi [7], Hurley [8], show that such PCPs, indeed, cannot entail LCs formulable via only positive terms, but Carroll [11] and the formulas (1') - (3ii')), show that such PCPs can entail LCs of the format $I(S', P')$, thus generating VCAs. After the “initial set” of PCPs to which the RofVS do not apply is postulated by RofVS #1 and #2, one can deduce that there are other PCP sets for which the RofVS will predict LCs which contradict the RofVS themselves: therefore, those PCP sets will not be part of the RofVS' DofA, either. What is left from the set of PCPs formulable via only positive terms, after the above two rounds of PCP removals, should be exactly those PCPs on which the Clas-

sical Syllogistics' and the RofVS' results coincide: the PCPs which generate VS, the PCPs whose entailed LCs are $A(P,S)$ and $O(P,S)$ – the latter PCPs also satisfy all the RofVS, as the VS do. Moreover one generalize the RofVS #5 to say “From two universal premises follows a universal LC, except when an ei condition on M, S or P is imposed – then a particular LC follows” and one generalize the RofVS #3 to say “The distribution of the end terms is conserved in all non-ei and ei VS, except in type Barbara ei VS, where ei on S, (resp. P) changes S, (resp. P), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P, (resp. S), remains the same as it was in the PCP”. Then the RofVS will also predict the ei LCs of Darapti, Felapton/Fesapo, Bramantip, Barbari, Celaront/Cesaro and Camestros/Camenos. As Stebbing [10] shows, no PCPs made of two particular premises, may produce LCs compatible with the RofVS. Indeed, PCPs made of two particular and affirmative premises – in which, therefore, no term is distributed, (resp. two particular and negative premises – the “not allowed” PCPs), are already excluded from the DofA, by RofVS #1, (resp. RofVS #2), and the PCPs made of one affirmative and one negative particular premises, should have an LC which is particular and negative, (according to RofVS #4 and #6). But this means that an end term will be distributed in the LC, without being distributed in the PCP – since, according to the distribution's definition, only the middle term will be distributed in the negative particular premise: contradiction. Thus, by postulating, via RofVS #1 and #2, two new classes of PCPs which do not entail LCs, one was able to prove that any PCP made of two particular premises does not entail an LC – therefore all PCPs of types (4i) and (4ii) will be excluded from the DofA of the RofVS. One may now prove that PCPs of types (5i) and (5ii) do not entail any LCs, either. Namely, one can check, that out of the four PCPs of type (5i) and four PCPs of type (5ii) which are formulable only via positive terms, two of them contain only negative premises, in four of the PCPs the middle term is not distributed at all, and in another two PCPs one of the premises is particular, one is negative, then in the PCP both end terms are undistributed, since the middle term has to be distributed at least once, and therefore, the LC should be negative and particular, thus one end term would be distributed in the LC, without being distributed in the premises - thus contradicting RofVS #3. [The complete details are as follows. The formula (5i), $E(M^*,P^*)I(M^*,S^*)$, leads, for $M^*=M$, $P^*=P$ and $S^*\in\{S, S'\}$ to two PCPs, (containing only positive terms), where M is nowhere distributed – thus, in accordance with RofVS #1 these PCPs do not entail an LC. Similarly, the formula (5ii), $I(M^*,P^*)E(M^*,S^*)$, leads, for $M^*=M$, $S^*=S$ and $P^*\in\{P, P'\}$ to two PCPs, (containing only positive terms), where M is nowhere distributed. According to RofVS #1, all the above four PCPs will not entail any LC. In $A(M,P)O(S,M)$ – which is formula (5i) for $M^*=M'$, $S^*=S$ and $P^*=P'$, and in $O(P,M)A(M,S)$ – which is formula (5ii) for $M^*=M'$, $S^*=S'$ and $P^*=P$, the middle term is distributed in both premises, but S and P are nowhere distributed; since the LC should be particular and negative, (due to one premise being negative and particular), the LC would distribute either S or P, thus contradicting RofVS #3. Finally two PCPs, $E(M,P)O(S,M)$ – formula (5i) for $M^*=M'$, $S^*=S$ and $P^*=P$, and $O(P,M)E(M,S)$ – formula (5ii) for $M^*=M'$, $S^*=S$ and $P^*=P$, are made only of negative premises, and therefore they are not contained in the RofVS' DofA, in accordance with RofVS #2.]

7 The development of the RofVCA “theory”

The RofVCA “theory” can be presented in a similar way to the RofVS “theory”. One defines the Domain of Applicability (DofA) for the RofVCA #1 to #4, as the maximal set on which the RofVCA will correctly predict the same LCs as the extension of Classical Syllogistics to indefinite terms. As one noticed this RofVCA DofA is made of all the 32 VCAs. To exclude, (in a “RofVCA fashion” – and thus develop a RofVCA “theory”), the other 32 PCPs which do not entail any LCs, one postulates, (and this is a true fact - which cannot be said in general about the postulates expressed by the RofVS #1 and #2), that PCPs of type (4i) are not part of DofA for the RofVCA. Then one can prove that PCPs of types (4ii), (5i) and (5ii) are not part of DofA for the RofVCA, either, since when applied to such PCPs, the RofVCA will predict LCs which contradict the RofVCA themselves. The PCPs left, are those of types (1) - (3ii), on which – as proved in previous Sections - the results of RofVCA and of the extension of Classical Syllogistics to indefinite terms, coincide. For example, applying the RofVCA to (5i) PCPs, $E(M^*, P^*)I(M^*, S^*)$, one obtains – since one premise is negative and one is affirmative and particular – that the LC is $O(S^*, P^*)$. Therefore if $s(S^*)=s(P^*)$, (resp. $s(S^*)\neq s(P^*)$), the LC remains a negative statement, (resp. becomes an affirmative statement), and the premises will have opposite, (resp. identical), argumental signatures – this way the premises become either both negative or both affirmative, (resp. the premises remain one negative and one affirmative), contrary to what the RofVCA #4 said – while explicitly being used to find the false LC. (As one already knows from the Introduction, the definition of a syllogism implies that PCPs of types (4i), (4ii), (5i) and (5ii) do not entail any LCs, since they do not pinpoint a unique subset of U.) If one also extends the RofVCA #3 to say that if at least one premise is particular then the LC is particular, then these “extended” RofVCA applied to PCPs of type (4ii), would predict LCs which are contradicted by the RofVCA themselves. The proof is similar to the one given above for PCPs of type (5i). The proof that for PCPs of type (5ii), the RofVCA will predict LCs which contradict the RofVCA themselves – is also similar to the proof given above that PCPs of type (5i) are not part of DofA for the RofVCA.

Consider the PCPs $E'E'$, EE , OE , EO - whose LCs, (or ei LCs), as one already knows, are all the same: $I(S', P')$. One can see that the RofVCA predict the correct LC for these PCPs: In the case of $E'E'=E(M', P)E(M', S)=A(P, M)A(S, M)$, one has two affirmative premises, (as one knows, of type (2) Darapti). According to the RofVCA, (which do not depend on the PCP type, provided that the PCP is contained in the RofVCA's DofA), the LC, (out of which the middle term was eliminated), should be affirmative and should conserve the fact that both S and P are distributed in the premises. But the only statement out of the eight possible LCs remaining after the middle term is eliminated, $E(S^*, P^*)$, $I(S^*, P^*)$, where $S^* \in \{S, S'\}$, $P^* \in \{P, P'\}$, which is affirmative and in which both S and P are distributed, is $I(S', P')$. If, instead one counts only the statements signatures, and one applies the RofVCA #4 “directly” to $E'E'=E(M', P)E(M', S)$, counting that the LC of two negative premises has to be an affirmative LC which conserves S and P being distributed in the premises, one again obtains that the unique LC which satisfies all the RofVCA #3 to #6, is $I(S', P')$. Indeed,

trying instead $A(P, S')$ as LC of the above premises does not work, since $A(P, S') = E(P, S)$ which contradicts RofVCA #4. Thus, for the Darapti's PCP, the RofVCA predict even that the only VCA from which M is eliminated is an ei VCA. One sees once more, that on the set of PCPs of types (1) - (3b), the RofVCA may effectively replace logic theory by "prediction rules". In cases, such as OE, EO, the RofVCA will similarly choose the same LC, $I(S', P')$ - based on the fact that two negative premises imply an affirmative LC, that both S and P are distributed in the premises, and that if one premise is particular, the LC will be particular. If both premises are universal - as in EE - the RofVCA, together, will still predict, as above, that the LC will be particular, (and thus, the ei condition - on M - is necessary). As already noticed the RofVS and the RofVCA refer only to VCAs out of which the middle term was eliminated. The RofVCA cannot, and do not, predict the universal LCs of the type (2) non-ei Darapti VCAs, $A(M^*, S^*P^*)$ or $M^* = M^*S^*P^*$. But any universal LC of the type $E(S^*, P^*)$, (i.e., out of which the middle term was eliminated), that one might try for the type (2) Darapti PCPs, $E(M^*, P^*)E(M^*, S^*)$, will not satisfy all the RofVCA themselves; the only LCs which satisfy all the RofVCA for type (2) PCPs, are the particular LCs, which preserve in the LCs, the distributions which the end terms already had in the PCPs: $I(S^*, P^*)$.

8 About empty sets

The three, (or four), types of VCAs may also be used to settle which VCAs are compatible with some of the sets S, P, M, S', P', M' being empty. In the modern square of opposition $A(M, P)$, $E(M, P)$ are not contraries anymore - unless one adds the condition $M \neq \emptyset$. Instead when both $A(M, P)$, $E(M, P)$ are true it results that $M = \emptyset$. This empty set constraint (ESC) - which empties all four subsets of M - is compatible with the universal premises of the VCAs of type 1 and 2 - but not with the ei on M. Nor is the $M = \emptyset$ ESC compatible with the type (3i) and (3ii) premises, $I(S^*, M)$ and $I(P^*, M)$. In fact, since the VCAs of the same type are all equivalent, it results that a complete discussion of the compatibility of various ESCs and VCAs may be reduced to examining just three representative cases (since Darai and Disamis are representatives of the same Darai pattern). Moreover, instead of firstly imposing an ESC, and then finding out the PCPs compatible with it, one can do it the other way around, by listing, for each VCA type, the ESCs with which that VCA type is compatible or incompatible. Darai's PCP, $A(M, P)I(S, M)$, means $MP' = \emptyset$, $SM \neq \emptyset$, and the LC is $SM = SMP + SMP' = SMP \neq \emptyset$. From the LC $SMP \neq \emptyset$, one may, with some loss of information, eliminate M, and re-express the LC as $I(S, P) = \text{"Some S is P"}$. Thus the PCP is incompatible with the $S = \emptyset$, $M = \emptyset$, and $P = \emptyset$ ESCs, but is compatible with the $S' = M' = P' = \emptyset$ ESCs, (which imply $S = M = P = U$; thus in this latter, extreme, case Darai's PCP and LC just assert that U is non-empty).

Darapti's PCP, $A(M, P)A(M, S)$, means $MP' = \emptyset$, $MS' = \emptyset$, and the LC is $M = MP + MP' = MP = MPS + MPS' = MPS$, which may be written as $A(M, SP)$. This time around one may eliminate M only via the ei hypothesis $M \neq \emptyset$, then re-express the LC as $I(S, P)$. Thus the ei hypothesis is incompatible with the $M = \emptyset$, $S = \emptyset$ and $P = \emptyset$ ESCs, but

is compatible with the $S'=M'=P'=\emptyset$ ESCs, (which imply $S=M=P=U$; therefore, in this latter, extreme case, Darapti's PCP plus the ei on M, assert that U is non-empty). Note that Darapti's PCP without the added ei condition is compatible even with $U=\emptyset$, in which case the PCP is just "chatter about empty sets".

Barbara's PCP, $A(M,P)A(S,M)$, means $MP'=\emptyset$, $SM'=\emptyset$, and the LCs are $S=SM+SM'=SM=SMP+ SMP'=SMP$, and $P'=P'M+ P'M'=P'M'= P'M'S+P'M'S'=P'M'S'$. The first LC may be written as $A(S,MP)$, or, with some loss of information, one may eliminate M, and write $A(S,P)=E(S,P')$, which now refers to two subsets of U instead of just one of the eight subsets of U. (A "precise" LC always pinpoints just one of the eight subsets of U.) The second LC may be written as $A(P', S'M')$, or, with some loss of information, one may eliminate M', and write $A(P',S') =E(S,P')$ – the same as the first LC. Since Barbara's PCP contains only universal premises, the PCP is compatible even with $U=\emptyset$ in which case all the deductions and the LCs – either "precise" or "classically expressed", are just "chatter about empty sets". One may then add an ei hypothesis, $S'\neq\emptyset$, to the 1st LC, and a different ei hypothesis, $P'\neq\emptyset$, to the 2nd LC, to obtain, after the M, resp. M', elimination the new ei LCs: $I(S,P)$, (Barbari), and resp., (the un-named), $I(S',P')$. The $S'\neq\emptyset$ ei hypothesis means, since $S=SPM$, that also $P\neq\emptyset$ and $M\neq\emptyset$, while the compatible ESCs are $S'=\emptyset$, or/and, $P'=\emptyset$, or/and, $M'=\emptyset$. The $S'=P'=M'=\emptyset$ constraint amounts to Barbari affirming $U\neq\emptyset$. The $P'\neq\emptyset$ ei condition implies that, also, $S'\neq\emptyset$ and $M'\neq\emptyset$. If both ei hypotheses are true then all the sets M, M', S, S', P, P' are non-empty, and there are no ESCs compatible with both ei hypotheses.

In conclusion any universal premise is compatible with any ESC. But any ei hypothesis or any LC of a VCA of type (3i) or (3ii), (containing one universal and one particular premise - both acting on either M or M'), specifies three sets that are non-empty, and thus pinpoints three ESCs with which the ei hypothesis or the "type (3i) or (3ii) LC" is incompatible. The above considerations are based on a sort of "temporal commutativity": instead of firstly applying the ESC to obtain a particular universe of discourse, and then searching for the LC in that universe, one writes down the LC in the usual 8-subset universe of discourse U, and one applies the ESC only afterwards, to see if it is compatible with the PCP and its LC.

9 How many sound VS or VCAs may one hope to construct out of three given terms, without imposing restrictions on the structure of the universal set U

When three specific terms are given, with one of them already designated as the middle term, one may consider all the 36 or 64 PCPs which can be constructed starting with these three specific terms, (out of which one is the designated middle term), and one can try to see what sound VS or VCAs one may construct out of the three terms. As one shows below, given three terms, with one of them already designated as the middle term, then, at most one sound VCA of type (1) or (2) may be built out of the three terms without restricting U to particular cases. Since that VCA can be formulated either as a Barbara or a Darapti if the terms are appropriately labeled, one

may say that given three terms, there exists at most one sound VCA of types (1) or (2) – either a Barbara or a Darapti – which can be constructed out of the given three terms, (again, if one of them was already designated as the middle term). If the three given terms generate, (modulo a relabeling), a sound Barbara, then a maximum of two other type (3i) and two other type (3ii) sound VCAs may perhaps be constructed with the same given three terms without restricting U to particular cases: these new sound VCAs have their universal premises “stolen” from Barbara and their possible particular premises place set elements on the four subsets adjacent to the four subsets emptied by Barbara’s two universal premises. One of these other possible four VCAs is a Darii/Datisi, and the other three have no names since they assert that subsets – other than the three subsets “preferred” as LCs by Classical Syllogistics (SPM, SPM, SPM') – are non-empty. If the three given terms generate, (modulo a relabeling), a sound Darapti, then only two other sound VCAs, one of type (3i) and one of type (3ii) may be constructed with the same given three terms without restricting U to particular cases: these two new VCAs, a Darii/Datisi and a Disamis/Dimaris will have their universal premises “stolen” from Darapti, and the same LC as the ei Darapti (after the middle term is eliminated): $SPM \neq \emptyset$. When the middle term is pre-determined, then two distinct PCPs of either type (1) Barbara, or type (2) Darapti, or one PCP of type (1) and one PCP of type (2), will necessarily contain either two distinct universal P-premises, or two distinct universal S-premises, which will impose a particular structure on the universal set U . For example, if $A(M,P)$ and $E(M,P)$, are both true, as P-premises in two different PCPs, that would imply M being empty, $M = \emptyset$. The relationships implied by the other five possible combinations of two universal P-premises being simultaneously true: $E \& E'$ imply $P = \emptyset$, $A' \& E'$ imply $M' = \emptyset$, $A \& A'$ imply $P' = \emptyset$, $A \& E'$ imply $P = M$, $A' \& E$ imply $P = M'$, and similar relationships hold for the “top face” of the S-cube, (if one places the four universal premises on the top faces of the P and S cubes, and the particular premises on the bottom faces of the two cubes). This shows, e.g., that Barbara, AE' , and Camestres, $E'E$, can both be sound, but uninteresting, since in that universal set, $P = M$ and $S = \emptyset$. Note that choosing another of the three terms as a middle term, leads to arguments and conclusions similar to the ones above: no two sound and distinct VCA of either type (1), or type (2) may be constructed with the same middle term, unless the universal set has a particular structure; no sound pair of a VCA of type (1) and one VCA of type (2) may be constructed with the same middle term, unless the universal set has a particular structure. Thus, the next task is to see if by using a term once as M , and a second time, say, as P , while the term firstly used as P , is afterward used as M , one can produce two distinct PCPs of type (1) without imposing, when all four premises are true, a particular structure on U . By adjoining the “standard” Barbara’s PCP, $A(M,P)A(S,M)=E(M,P')E(M',S)$, to each of the eight PCPs of type (1), having P as the middle term and S and M as end terms, $E(P^*,M^*)E(P'^*,S^*)$, one can show that, given three terms, at most one sound VCA of type (1) may be constructed with them, without imposing a particular structure on the universal set U , i.e., in short, one may say, that at most one of the terms, (out of three given terms), may be used as the middle term in a type (1) VCA. For example, if all the following four premises, $A(M,P)A(S,M)$ and $A(P,M)A(S,P)$, (where the second Barbara’s PCP is obtained by switching the roles which M and P played in the first

Barbara's PCP), are true, i.e., $MP'=SM'=PM'=SP'=0$, then, $M=MP'+MP=MP=MP+PM'=P$. For a complete proof, one may compare, two at a time, the eight PCPs of type (1) having P as the middle term, with the “standard” Barbara's PCP. From $E(M,P')E(M',S)$ and $E(P,M) E(P',S^*)$, it results $M=0$; from $E(M,P')E(M',S)$ and $E(P,M') E(P',S^*)$, it results $M=MP=P$; from $E(M,P')E(M',S)$ and $E(P',M) E(P',S^*)$, it results, if $S^*=S$, that $S=PS+P'S=P'S=P'SM+P'SM'=0+0=0$, and, if $S^*=S'$, that $M=MP=MPS=S$; from $E(M,P')E(M',S)$ and $E(P',M') E(P',S^*)$, it results $P'=0$. But one may easily see that the “standard” Barbara's PCP, $E(M,P')E(M',S)$, and this type (2) PCP having S as the middle term, $E(S,P')E(S,M')$, can be simultaneously sound without imposing a particular structure on U. Even three type (2) PCPs can be simultaneously sound without imposing a particular structure on U, under the condition that their middle terms are all different. For example, $E(M,P')E(M',S)$, $E(S,P')E(S,M)$, $E(P',M)E(P',S)$, use M, S, and resp. P' as middle terms, and out of their six premises only three are distinct. These three type (2) Darapti PCPs, empty a total of only four subsets of U – the same number as a single PCP of type (1) empties.

10 Conclusions

One saw that, based on set inclusions and set intersections, only three distinct patterns of valid syllogisms or valid categorical arguments (VCAs) do exist – the Barbara, Darapti and Darii patterns.

As it results from the last part of Section 4 and from Section 6, the Classical Syllogistics and the RofVS seem to work in a more contorted way than necessary – they become easier to grasp when one generalizes them to indefinite terms. The Classical Syllogistics extended to indefinite terms naturally provides the simple formulas (1)-(3ii) and (1')-(3ii') from Section 4, which completely display all the pairs of categorical premises (PCPs), (conforming to the Barbara, Darapti and Darii patterns), which entail logical consequences (LCs), and list their precise LCs, and their LCs out of which the middle term was eliminated. The Classical Syllogistics extended to indefinite terms also provides the formulas (4i) – (5ii) which completely list all the PCPs which do not entail any LCs – in accord with the intuitive criterion that if the PCP does not pinpoint a unique partitioning subset of the universe of discourse, U – then there no LC. The Rules of valid syllogism (RofVS) and the Rules of valid categorical syllogism (RofVCA) were developed into almost axiomatic “theories”. Postulating that some PCP subsets are inconclusive, i.e., do not produce LCs – any LCs at all, or LCs of the required type – one may find out, using the RofVCA, (resp. the RofVS), themselves, the entire set of the 32, (resp. the 21), inconclusive PCPs, out of 64 PCP matrix of the PCPs containing indefinite terms, (resp. out of the in the 36 PCP matrix of the PCPs containing only positive terms ca). The rest of the 32, (resp. the 15), possible PCPs from the 64 PCP matrix, (resp. the 36 PCP matrix), are all conclusive and the RofVCA, (resp. the RofVS), can “rapidly” predict their LCs out of which the middle term was eliminated. But the formulas (1)-(3ii) and (1')-(3ii') from Section 4, which list all possible valid categorical arguments (VCAs) and valid syllogisms (VS), including their precise “one partitioning subset of U” LCs, and their LCs out of which

the middle term was eliminated, can be probably used easier and faster, to decide for any PCP containing three concrete terms if the PCP is conclusive or not and to determine its LC if any, than if one tries to decide, in the RofVCA and RofVS fashion, firstly if the PCP contains only positive terms, secondly, if all the RofVCA or the RofVS, (as appropriate, considering the positivity or lack thereof of the terms appearing in the given PCP), are satisfied, and, finally, if it results that the proposed PCP is conclusive, to appropriately apply the RofVCA or the RofVS to predict the LC, (out of which the middle term was eliminated). (Probably the safest and fastest strategy is to compare the formulas (1)-(3ii), (and/or (1')-(3ii')), plus the formulas (4i)-(5ii) with a representation of the PCP on a Karnaugh map for $n=3$.)

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