On Fast Solving Spectral Method to Biharmonic Equation Of the Second Kind

Agah D. Garnadi

-

Department of Mathematics,
Faculty of Mathematics and Natural Sciences,
Bogor Agricultural University
Jl. Meranti, Kampus IPB Darmaga, Bogor, 16680 Indonesia

Abstract

This note is addressed to fast solving simply supported boundary value problem of biharmonic equation in the unit rectangle.

Key words: Biharmonic equation, simply supported boundary value problem, spectral method.

1 Introduction

This note is an extension of Fortunato and Townsend [6] work on fast solver for Poisson equation to homogeneous simply supported boundary value problem of biharmonic equation. See Henrici's work [2] on fast solver for Poisson's equation using finite difference method. The need to solve discretization of biharmonic equation is motivated by its occurrence from inpainting problem [citation needed], outside of classical applications such as elasticity and fluid flows [1, 3].

2 Biharmonic equations with simply supported boundary conditions.

Suppose that S is a unit square,

$$S = \{(\xi, \eta); 0 \leq \xi, \eta \leq 1\}.$$

We are going to solve biharmonic equation:

$$\Delta^2 u(x, y) = f(x, y),$$

in the rectangle S . Furthermore, we deal with homogeneous simply supported boundary value problem,

$$\left. \begin{array}{l} u = 0, \\ \Delta u = 0, \end{array} \right\} \text{ on } \partial\Omega .$$

Rewrite as coupled system:

$$\Delta u(x, y) = v; \quad \Delta v(x, y) = f(x, y), \quad (1)$$

with the boundary condition:

$$\left. \begin{array}{l} u = 0, \\ v = 0, \end{array} \right\} \text{ on } \partial\Omega .$$

To solve the problem using spectral methods on S , is easily executed since the availability of fast spectral method for Poisson Equation as given by [6].

3 Epilogue

We show an algorithmic development of solving biharmonic equations in unit rectangle. The implementation of spectral fast solver of Poisson equations in MATLAB in [6], made the fast spectral biharmonic problem with simply supported conditions is realizable.

References

- [1] M.M. Gupta, Numerical methods for viscous flow problems. In: A.S. Mujumdar(eds.), *Advances in Transport Processes*, Wiley Eastern, New Delhi, 1980.
- [2] P. Henrici, Fast Fourier methods in computational complex analysis, *SIAM Rev.* **21** (1979) 481–527.
- [3] I. Sneddon, *Elements in Partial Differential Equations*, McGraw-Hill, Tokyo, 1984.
- [4] G.D. Smith, *Numerical Solution of Partial Differential Equations*, Oxford Univ. Press, London, 1975.
- [5] Strikwerda, J.C.,2004, Finite difference schemes and partial differential equations, SIAM
- [6] Fortunato, Daniel and Townsend, Alex, 2017, Fast Poisson solvers for spectral methods, ,**arXiv preprint arXiv:1710.11259**