

Amortizing Cap Floor Model

An amortizing cap is a special type of cap whose notional principal amount declines during the life of the contract. An interest rate cap consists of a series of European call options (caplets) on interest rates.

An amortizing floor is a special type of floor whose notional principal amount declines during the life of the contract. An interest rate floor consists of a series of European put options (floorlets) on interest rates.

In other words, the notional amount corresponding to each caplet or floorlet is specified by an amortization schedule.

An amortizing cap is primarily used to hedge loans whose principal declines on a scheduled basis. Amortizing caps are frequently purchased by issuers of floating rate debt where the loan principal declines during the life.

We consider a cap option consisting of a series of caplets as follows. Here each caplet is specified by,

- settlement time, T ,
- set of Pibor fixing times, $\{\tau_i\}$, such that $0 < \tau_1 < \dots < \tau_m < T$,
- payoff at T of the form

$$N \times \Delta \times (L_i - X)^+$$

where

- L_i denotes the δ -period Pibor rate that sets at τ_i ,
- N is an FRF notional amount,
- Δ is an accrual period.

The present value of an amortizing or accreting cap is given by

$$PV(0) = \sum_{i=1}^n N_i \tau_i D_i (F_i \Phi(d_1) - K \Phi(d_2))$$

where

$D_i = D(0, T_i)$ – the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1 \right) / \tau_i$ – the forward rate for period (T_{i-1}, T_i) .

Φ – the accumulative normal distribution function

$$d_{1,2} = \frac{\ln \left(\frac{F_i}{K} \right) \pm 0.5 \sigma_i^2 T_i}{\sigma_i \sqrt{T_i}}$$

Consider a European style option of the form,

- maturity, T ,
- underlying, L ,
- strike, X ,
- payoff at maturity, $(X - L_T)^+$.

The option can be priced by an analytical formula,

$$d_T E^T \left[(X - L_T)^+ \right]$$

where

- d_T denotes the discount factor to time T ,
- $L_T = L_0 e^{-\frac{\sigma^2}{2} T + \sigma W_T}$, under the T -forward measure, with
 - σ a constant volatility parameter, and
 - W a standard Brownian motion.

If the underlying asset, L , is a Δ -period Libor rate, then L is a martingale under the $T + \Delta$ -**forward measure**. In this case, Black Scholes should be called with a zero-interest rate to maturity, so that d_T in (3.3.1) reduces to 1; the resulting option price can then be scaled by the discount factor to $T + \Delta$.

You can find more details at

<https://finpricing.com/lib/EqBarrier.html>