Open Journal of Mathematics and Physics | Volume 1, Article 20, 2019 | ISSN: 2674-5747
https://doi.org/10.31219/osf.io/rf6az | published: 10 Jun 2019 | https://ojmp.wordpress.com
AT

[microresearch]
Diamond Open Access

# $\frac{x+y}{2}$ coupled logistic map

Open Mathematics Collaboration*†

July 18, 2019

**Abstract**

I propose the discussion of one type of coupled logistic map and compare it with the usual logistic map in each iteration.

**keywords**: coupled logistic map, chaos, dynamical system, complex system

# Introduction

1. The motivation for this work is to apply coupled logistic map to population with bounds, like in the XY chromosomes evolution [1–3].

2. This discussion started in a section of [4], in the context of searching for chaos in the Y chromosome degeneration.

3. Consider the logistic map, $x_{n+1} = rx_n(1 - x_n)$, where $r$ is the dimensionless population growth factor and $x_n$ is the population of the $n$th generation [5,6].

4. From (3), we have $x_{n+1} = -rx_n^2 + rx_n$ [6].

*All authors with their affiliations appear at the end of this paper.
†Corresponding author: mplobo@uft.edu.br | Join the Open Mathematics Collaboration

5. Doing $y = x_{n+1}$ and $x = x_n$, the parabola of the logistic map is given by $y = -rx^2 + rx$ [6].

6. Depending on the value of $r$, there is chaos in the logistic map [6, 7].

## Is the logistic map in $y$ equals the logistic map in $x$ for each iteration?

7. Consider the following coupled logistic maps representing X and Y populations:

$$x_{n+1} = rx_n(1 - x_n); \qquad y_{n+1} = r\left(\frac{x_n + y_n}{2}\right)\left[1 - \left(\frac{x_n + y_n}{2}\right)\right].$$

8. The next following steps are calculations on $y_{n+1}$.

9. $y_{n+1} = \frac{r}{2}\left\{(x_n + y_n)\left[1 - \frac{1}{2}(x_n + y_n)\right]\right\}$.

10. $y_{n+1} = \frac{r}{2}\left\{x_n\left[1 - \frac{1}{2}(x_n + y_n)\right] + y_n\left[1 - \frac{1}{2}(x_n + y_n)\right]\right\}$.

11. $y_{n+1} = -\frac{1}{2}y_n^2 + (1 - x_n)y_n + \left(1 - \frac{x_n}{2}\right)x_n$.

12. In (11), we define: $a = -\frac{1}{2}$, $b = 1 - x_n$, $c = \left(1 - \frac{x_n}{2}\right)x_n$.

13. Note that in (12), we did not include the index $n$ in the definitions of $a$, $b$, and $c$, because we are considering that it holds of every iteration, i.e., for every value of $n$.

14. Then, $y_{n+1} = ay_n^2 + by_n + c$.

15. Consider $y_{n+1} = y'$, $y_n = x'$.

16. $y' = ax'^2 + bx' + c$.

17. Comparing (5) and (16), $y = -rx^2 + rx$ and $y' = ax'^2 + bx' + c$, we have $-r = a$, $r = b$, and $c = 0$.

18. From (12) and (17), $c = \left(1 - \frac{x_n}{2}\right) x_n = 0$, which means $x_n = 0$ or $x_n = 2$.

19. Considering $x_n = 0$, from (12), we have $b = 1$, and from (17) $r = 1$.

20. Considering $x_n = 0$, from (12), we have $a = -\frac{1}{2}$, and from (17), $-r = a$ leads to $r = \frac{1}{2}$.

21. From (19) and (20), we have a contradiction.

22. Considering $x_n = 2$, from (12), we have $b = -1$.

23. From (17) and (22), $r = -1$.

24. From (17) and (23), $r = \frac{1}{2}$.

25. (23) and (24) leads to a contradiction.

26. (21) and (25) shows that $x_n \neq 0$ and $x_n \neq 2$.

27. (26) means that the logistic map for $y$ NEVER behaves like the logistic map for $x$ in each iteration.

# Final Remarks

28. One question still persists: Is there chaos in the $y$ logistic map defined here?

# Open Invitation

Please *review* this article, *add* content, and *join* the **Open Mathematics Collaboration**. Contact mplobo@uft.edu.br.

# Ethical conduct of research

This original work was pre-registered under the OSF Preprints [8], please cite it accordingly [9]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

# References

[1] Lobo, M. P., and Roberto Nicolau Onody. "Degeneration of the Y chromosome in evolutionary aging models." *The European Physical Journal B-Condensed Matter and Complex Systems* 45.4 (2005): 533-537.

[2] Lobo, M. P. "Mathematical glimpse on the Y chromosome degeneration." *The European Physical Journal B-Condensed Matter and Complex Systems* 50.4 (2006): 613-615.

[3] Mackiewicz, Dorota, et al. "Role of recombination and faithfulness to partner in sex chromosome degeneration." *Scientific Reports* 8.1 (2018): 8978. https://www.nature.com/articles/s41598-018-27219-1.pdf

[4] Lobo, Matheus P., et al. "Chaos in the Y-chromosome evolution?." *arXiv preprint* arXiv:1612.00463 (2016). https://arxiv.org/pdf/1612.00463

[5] Wikipedia. *Logistic map*. https://en.wikipedia.org/wiki/Logistic_map

[6] Lobo, Matheus P. "Chaotic Logistic Map, Parabola, and Gravity." *OSF Preprints*, 9 June 2019. https://doi.org/10.31219/osf.io/g75we

[7] Groff, Jeffrey R. "Exploring dynamical systems and chaos using the logistic map model of population change." *American Journal of Physics* 81.10 (2013): 725-732.

[8] OSF. *Open Science Framework*. https://osf.io

[9] Lobo, Matheus P. "(x+y)/2 Coupled Logistic Map." *OSF Preprints*, 10 June 2019. https://doi.org/10.31219/osf.io/rf6az

## The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br)

Federal University of Tocantins (Brazil)