

Amortizing Bond Model

An amortizing bond is a bond with fixed rate coupon. The principal amount corresponding to each coupon date is specified by a variable schedule. If the principal decreases, it is an amortizing bond. If the principal increases, it is an accreting bond.

Each payment to the amortizing bond holder consists of a portion of interest and a portion of principal. An amortizing bond is used specifically for tax purposes as the amortized principal is treated as part of a company's interest expense.

The valuation of an amortizing bond needs to take the principal change into account. The present value of an amortizing bond is

- $$V(t) = \sum_{i=1}^n cP_i e^{-(r_i+s)T_i} + P_n e^{-(r^n+s)T_n}$$

where

t – valuation date

i – i^{th} cash flow from 1 to n

r_i – continuous compounded interest rate for period (t, T_i)

T_i – coupon payment date of the i^{th} cash flow

s – credit spread

P – principal amount or face value

c – coupon rate

The present value of a bond computed by model is the dirty price of the bond, which is the buying or selling price. The dirty price is the sum of the clean price and accrued interest.

Most of bonds are traded and quoted based on yield to maturity (YTM). The actual settlement clean price depends on the number of coupons available. For bonds with a single remaining coupon, the bond trades at a pure discount (i.e., like a money market instrument). For bonds with multiple remaining coupons, these are priced with a special formula.

Let

- N denote the number of remaining coupons,
- E denote the number of days in the first coupon period that includes the settlement date,
- DSC denote the number of days between settlement date and coupon payment, and
- Y denote the semiannual YTM of the bond.

The unadjusted dirty price of the bond (at settlement) is

$$P_d^* = \sum_{i=1}^N \frac{CF_i}{\left(1 + \frac{Y}{2}\right)^{\left(i-1 + \frac{DSC}{E}\right)}}.$$

The unadjusted dirty price is used, along with an unadjusted accrued interest, to compute the true clean price. The unadjusted accrued interest is computed

$$AA^* = \frac{E - DSC}{E} \times \frac{C}{2},$$

after which the true clean price is computed

$$P_c = P_d^* - AA^*.$$

Finally, the true dirty price may be computed

$$P_d = P_c + AccInt,$$

where $AccInt$ is calculated as described in Equation (A1). Assuming S days to settlement, the bond's dirty price at trade date is $({}_0df_S)P_d$.

You can find more details at

<https://finpricing.com/lib/EqConvertible.html>