Local Parameter Recovery Formula Around Point Optrodes in Diffuse Optics Tomography

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Abstract

We consider the inverse light diffusion problem in the case of piecewise constant diffusion and absorption with the isotropic light sources injected at point optrodes and the resulting light fields measured at the remaining optrodes.

Adapting the linear functional strategy from related problem in aquifer transmissivity, the diffusive and absorptive coefficient can be recovered on the zone of a domain containing the point optrodes. Such reconstruction method is needed as a preliminary step before full reconstruction of the diffusive and absorptive inside the domain from boundary measurements, as done in diffusive optical tomography.

1 Diffuse Optical Tomography as parameter identification problem

In the study of diffuse light propagation within a bounded object Ω , the governing equation is the following elliptic partial differential equation:

$$-\nabla(\kappa\nabla u) + \mu_a u = q \quad \text{in } \Omega \quad (1)$$

where $\Omega \in R^d, \{d = 2, 3\}$ is a bounded domain, $\kappa, \mu_a : \Omega \rightarrow R$ denotes the diffusive and absorptive coefficient respectively, $u : \Omega \rightarrow R$ is diffusive light fields, $q := I(\delta(r - s_i))$ is light impulse injection through point electrode at $s_i, 1 \leq i \leq N$, where N denoting the number of optrodes. When the current source term q and the coefficient (κ, μ_a) is given, with an appropriate prescribed boundary information for solution u so that the direct (forward) problem of u is uniquely solvable in Ω . Conversely, the recovery information about the coefficient of diffusive and absorptive pairs (κ, μ_a) given the pair (q, g) is given, where $g(r_k) := u(r_k)$ is measured light at point optrodes at r_k due to current injection pattern q , is an inverse problem with κ being sensitive due to perturbation in u and q . The recovery of κ from point optrodes measurements u and q , utilizing the equation (1), that is called *Diffuse Optics Tomography* with point optrode models.

2 The Chow-Anderssen Ansatz [3]

The essential ill-posedness in the recovery of information about the diffusive and absorptive pairs (κ, μ_a) from measurements of u and q relates to the fact that the observed light fields data u must be differentiated. This is clear from the structure of equation (1). The essence of the ansatz involves four steps:

- 32 a. the reformulation of the partial differential equations model in terms of a weak
33 form,
- 34 b. the application of integration by parts to move the differentiation of the observed
35 light fields onto the analytic test function,
- 36 c. the localization of the resulting linear functional representation for the diffusive
37 and absorptive to a subregion surrounding the point optrodes where the diffusive
38 and absorptive is assumed to be constant, and
- 39 d. the choice of the shape of the localization and the form of test function that
40 facilitates the computational performance of the resulting algorithm.

41 As explained in Chow and Anderssen [4] in terms of the notation of equation (1) and
42 the application of steps (a)-(d) of the ansatz, various linear functional formulas for the
43 estimation of κ on the subregion $\Lambda \subset \Omega$ can be derived which depends on the assumed
44 regularity of the solution u of equation (1) and the test function v .

45 The schemes to estimate local parameters provided in [3] is used here, i.e.

$$\hat{\kappa} \int_{\Lambda} \nabla u \cdot \nabla v \, dx + \hat{\mu}_a \int_{\Lambda} u \cdot v \, dx = - \int_{\Lambda} q v \, dx, \quad u \in L^2(\Lambda), \quad v \in H_0^1(\Lambda). \quad (2)$$

46 and

$$\hat{\kappa} \int_{\Lambda} u \cdot \Delta v \, dx + \hat{\mu}_a \int_{\Lambda} u \cdot v \, dx = - \int_{\Lambda} q v \, dx, \quad u \in L^2(\Lambda), \quad v \in H_0^2(\Lambda). \quad (3)$$

47 The special form that it take is a direct exploitation of the assumption that κ is a con-
48 stant in the subregion Λ . In this scheme, through an integration by parts in conjunction
49 with the regularity imposed on the test function v , the evaluation of $\hat{\kappa}$ does not involve
50 a differentiation of u since it has been transferred to v .

51 The Recovery Formula

52 In practice, for given light impulse injection $q := I(\delta(r - s_i))$ through point electrode
53 at s_i , let r_m is measurement point position such that $r_m \subset \Lambda$, $\text{supp}(q) \cap \Lambda \neq \emptyset$, and we
54 choose v is the solution of the following:

$$\Delta v = \delta(r - r_m), v|_{\partial\Lambda} = 0,$$

55 So, the scheme (3) resulted in:

$$\hat{\kappa} u(r_m) + \hat{\mu}_a \int_{\Lambda} u \cdot v \, dx = - \int_{\Lambda} q v \, dx.$$

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$$\hat{\kappa} u(r_m) + \hat{\mu}_a \int_{\Lambda} u \cdot v \, dx = -I v(s_i), \quad s_i, r_m \in \Lambda.$$

Observe that by taking two points measurement r_m^1, r_m^2 in two sub domain $\Lambda_{1,2}$, we arrive that:

$$\hat{\kappa}u(r_m^1) + \hat{\mu}_a \int_{\Lambda} u \cdot v \, dx = -I v(s_i), \quad s_i, r_m^1 \in \Lambda_1 .$$

$$\hat{\kappa}u(r_m^2) + \hat{\mu}_a \int_{\Lambda} u \cdot v \, dx = -I v(s_i), \quad s_i, r_m^2 \in \Lambda_2 .$$

Hence, we able to recover the diffusive and absorptive arounds point optrode at s_i .

3 Remarks

Similar to the problem of local recovery of conductivity of in Electrical Impedance Tomography [5], the remarks are also applicable to DOT problem addressed in this work.

- Note that the recovery formula also valid for general case of current patterns q under point electrode models in DOT, furthermore the assumption that the problem is under homogeneous background can be ruled out.
- In [4] was proposing adaptive methods, where adaptivity is worked out in shrinking zones manners. However, in the case of DOT with point optrode model, the techniques can be adapted in reverse by starting from a point expands into a larger constant zonation.

References

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