

Personal Income and Hierarchical Power

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Abstract

This paper examines the relation between personal income and hierarchical power. In the context of a firm hierarchy, I define hierarchical power as the number of subordinates under an individual's control. Using the available case-study evidence, I find that relative income within firms scales strongly with hierarchical power. I also find that hierarchical power affects income more strongly than any other factor for which data is available. I conclude that this is preliminary evidence for a hierarchical-power theory of personal income distribution.

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1 Introduction

What explains personal income? This paper tests the hypothesis that income is most strongly explained by hierarchical power. My approach represents a twist on the longstanding institutionalist view that income stems from power. The primary theoretical contribution of this paper is to restrict the study of power to that found in a hierarchy, and to offer a specific way of measuring this power. I define hierarchical power as the ability to influence subordinates within a hierarchical chain of command. I propose that this power can be measured in terms of the number of subordinates under an individual's control

Using this definition, I conduct the first investigation of how hierarchical power (within firms) affects individual income. To be clear, this type of investigation is in its infancy and relies on a relatively small sample of data. However, the evidence that does exist is unambiguous. I find that relative income within case-study firms scales strongly with hierarchical power. Furthermore, I find that grouping individuals by hierarchical rank (across society) affects income more strongly than any other factor tested here. I argue that this is preliminary evidence for a hierarchical power theory of personal income distribution.

The paper is organized into the following parts. In section 2, I outline the motivations behind my proposed hierarchical power-income hypothesis. In section 3, I look for a correlation between hierarchical power and relative income within case-study firms. In section 4, I investigate the strength of the hierarchical power-income effect (at the societal level) using a variation of the analysis of variance method. All methods and sources are documented in the Appendix. I conclude in sections 5 and 6 with thoughts on the significance of the hierarchical power-income relation, and I discuss avenues for future research.

2 Hierarchical Power and Income

I hypothesize that personal income can be explained most strongly by differentials in hierarchical power. Before diving into the specifics of this hypothesis, I want to provide a rationale based on the big picture of human history. I begin by asking a simple question: what aspects of human history suggest that hierarchical power might affect how humans distribute resources?

Let's begin with our deep history — the evolutionary backdrop of the human species. Humans are but one of a wide variety of social mammals, virtually all of which form dominance hierarchies, or 'pecking orders' (Barroso et al., 2000; Guhl et al., 1945; Kondo and Hurnik, 1990; Meese and Ewbank, 1973; Sapolsky, 2005;

Uhrich, 1938). A key characteristic of these dominance hierarchies is that high social rank is associated with preferential access to resources, particularly sexual mates (Bradley et al., 2005; Haley et al., 1994; Girman et al., 1997; Gerloff et al., 1999; Wroblewski et al., 2009).

Of course, human behavior is far more complex than even the most intelligent non-human primates. Just because we evolved from hierarchy-forming animals does not necessarily mean that hierarchical rank *still* plays a role in how we divide up the pie. However, there is good evidence that humans *do* have an instinctual behavior towards hierarchy formation. Several studies have shown that children and adolescents spontaneously form dominance hierarchies when placed into small groups (Frankel and Arbel, 1980; Savin-Williams, 1980; Strayer and Trudel, 1984). Other studies have shown that, like other social mammals, human reproductive success increases with social status (Hopcroft, 2006; Betzig, 2012). There is even evidence that social status at birth is epigenetically imprinted on human DNA (Borghol et al., 2012) — something that also occurs in Rhesus monkeys (Massart et al., 2016). Given our evolutionary heritage, it seems plausible that hierarchy plays a role in the way humans distribute resources.

Another reason to suspect that resource distribution has to do with hierarchy and power is the ubiquity of *inherited status* in human history. It is hard to justify the wealth of a hereditary aristocracy as stemming from anything but power and privilege. Interestingly, inherited status has surprisingly deep historical roots. There is tentative archaeological evidence for inherited status beginning in the neolithic era (Boric, 1996; Halstead, 1993; Van der Velde, 1990), and widespread evidence beginning in the bronze age around 5000 years ago (Aranda and Molina, 2006; Aranda-Jiménez et al., 2009; Graziadio, 1991; Kristiansen, 2000; Harding, 2000). It is around this time that the first Egyptian dynasty formed (Dee et al., 2013), followed later by dynasties in Mesopotamia (Reade, 2001) and China (Guo et al., 2000).

Since then, as Gaetano Mosca observes, the existence of a hereditary ruling class has been the norm:

There is practically no country of longstanding civilization that has not had a hereditary aristocracy at one period or another in its history. We find hereditary nobilities during certain periods in China and ancient Egypt, in India, in Greece before the wars with the Medes, in ancient Rome, among the Slavs, among the Latins and Germans of the Middle Ages, in Mexico at the time of the Discovery and in Japan down to a few years ago. (Mosca, 1939)

But while history may be sordid, there is always the possibility that modern societies have made a clean break with the past. Power may have played a central role

in the distribution of resources in past societies, but in modern societies *reciprocal exchange* is what matters most. This is the story that emerged in the writings of Adam Smith (1776) and was codified into neoclassical theory by Jevons (1879), Menger (1871), and Walras (1896). To paraphrase George Orwell (1972), this is now the prevailing orthodoxy that most right-thinking economists accept without question.

But what if there has *not* been a clean break with the past? What if power still plays an important role in shaping resource distribution? A wide variety of scholars have argued that this is the case. A non-exhaustive list would include Berle and Means (1932), Brown (1988), Commons (1924), Dugger (1989), Galbraith (1985), Huber et al. (2017), Lenski (1966), Mills (1956), Munkirs (1985), Nitzan and Bichler (2009), Peach (1987), Sidanius and Pratto (2001), Tool and Samuels (1989), Tool (2017), Veblen (1904, 1923), Weber (1978), and Wright (1979). These scholars argue that power plays a central role in shaping income distribution.

If there is to be a power-based theory of income distribution, what should it look like? According to Christopher Brown:

... [A] theory of distribution should be indistinguishable from a theory of power. A satisfactory theory of power would, beyond defining what power is, elucidate principles to explain how power is established, enlarged or diminished, protected and perpetuated, redistributed, exercised, and rendered legitimate or illegitimate. (Brown, 2005)

A full-fledged theory of power is a tall order. In this paper, I narrow the focus to look only at *hierarchical power* and its relation to personal income. My ideas stem from the work of Simon (1957) and Lydall (1959), who independently proposed income distribution models based on the hierarchical structure of firms.

The focus of Simon and Lydall's work is the *branching* nature of firm hierarchies, in which each superior controls multiple subordinates. This structure is unique to humans. All other animals form *linear* hierarchies — an ordinal ranking from top to bottom. The most important feature of a branching hierarchy is that it *concentrates* power in the hands of the few. I propose that differentials in hierarchical power can be used to explain differentials in income. The main theoretical contribution of this paper is to offer a quantifiable definition of hierarchical power that allows power differentials to be directly compared to income differentials.

2.1 Measuring Hierarchical Power

What is *hierarchical power*? I define it as the ability to control subordinates within a hierarchical chain of command. This definition builds on the common Weberian

definition of power, articulated by Reinhard Bendix (1998) as “the possibility of imposing one’s will upon the behavior of other persons”(cited in Wallimann et al. 1977). Or put another way, Raymond Aron (1964) defines the Weberian concept of power as “the chance of obtaining the obedience of others to a particular command” (cited in Wallimann et al. 1977).

The link between hierarchy and power is implicit in the etymology of the word ‘hierarchy’ itself, which derives from the Greek term *hierarkhēs*, meaning ‘sacred ruler’ (Verdier, 2006). In essence, a hierarchy is a nested set of power relations between a superior (a ruler) and subordinates (the ruled). The hierarchical chain of command confers the right of each superior to direct the activity of all those in subordinate positions. I propose that one’s power within a hierarchy is proportional to the *number of subordinates under one’s control*. I put this in formula form as:

$$\text{hierarchical power} = \text{number of subordinates} + 1 \quad (1)$$

The logic of this equation is that all individuals start at a baseline power of 1, indicating that they have control over themselves. Power then increases linearly with the number of subordinates.

If we had access to the exact chain of command structure of an institution, we could use this definition to measure the power of each individual within a hierarchy. Unfortunately, chain of command information is rarely available. Instead, existing case studies report *aggregate* hierarchical structure only — total employment by hierarchical level. While we cannot calculate the power of *specific* individuals, we can use this data to calculate the *average* power of all individuals in a specific hierarchical level:

$$\bar{P}_h = \bar{S}_h + 1 \quad (2)$$

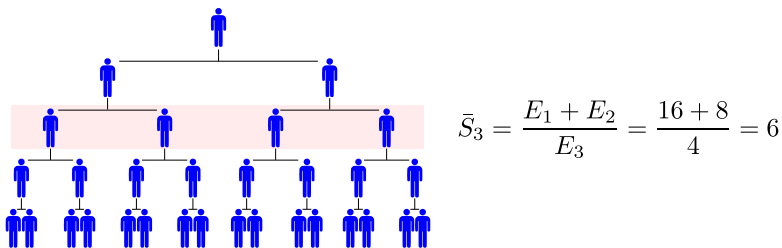


Figure 1: Calculating the Average Number of Subordinates

Here \bar{P}_h is the average power of individuals in hierarchical level h , and \bar{S}_h is the average number of subordinates below these individuals. The average number

of subordinates \bar{S}_h is equal to the sum of employment (E) in all subordinate levels, divided by employment in the level in question. Figure 1 shows a sample calculation of the average number of subordinates below individuals in the third hierarchical level. Each shaded individual has 2 direct subordinates, and 4 indirect subordinates, for a total of 6 subordinates. The average hierarchical power of individuals in level 3 is therefore 7.

Using summation notation, we can write the following general equation for the average number of subordinates in hierarchical level h (here $h = 1$ is the bottom hierarchical level):

$$\bar{S}_h = \sum_{i=1}^{h-1} \frac{E_i}{E_h} \quad (3)$$

Together, equations 2 and 3 allow us to define and measure the average power of individuals in a hierarchy.

2.2 Two Hypotheses

I propose that hierarchical power is a strong determinant of income. To refine this hypothesis, I break it down into two parts:

Hypothesis A: Relative income within a hierarchy is proportional to hierarchical power.

Hypothesis B: Hierarchical power affects income more strongly than any other factor for which data is available.

The reasoning behind this two-part hypothesis has mostly to do with the format of the available data. In principle, we could look at the proportionality between income and hierarchical power and test the strength of this effect all in one go. We would simply measure the correlation between individual income and hierarchical power, and compare the strength of this correlation to the correlation between other income-affecting factors. Unfortunately, few studies of firm hierarchy report individual level data. Those that do report this data do not report a wide range of other income-affecting factors against which to test the strength of the power-income effect.

The two-part hypothesis offers a way to deal with these data constraints by separating the measure of correlation between income and hierarchical power, and the measure of the strength of this effect. In section 3, I use the available firm

case-study evidence to investigate the correlation between relative income and hierarchical power. In section 4, I use a variant of the analysis of variance method to estimate the strength of the relation between hierarchical power and income.

3 Correlation Between Hierarchical Power and Relative Income Within Firms

Hypothesis A proposes that relative income within a hierarchy is proportional to hierarchical power. To test this hypothesis, I use the available firm case-study evidence to look for a correlation between hierarchical power and relative income within firms. I first look for a *static* correlation, followed by an analysis of the *dynamic* correlation between changes in income and changes in hierarchical power.

3.1 Static Power-Income Correlation

To look for a static correlation between income and hierarchical power, I use six case studies that cover firms in the United Kingdom, the United States, the Netherlands, and Portugal. (For a detailed discussion of these studies, see Appendix B). There are two important caveats to this analysis. First, the sample size is small. Having scoured the academic literature, these six studies are the only ones that I have found with the appropriate data. This paucity of data is partly due to the proprietary nature of firm payrolls. But more importantly, mainstream (neoclassical) economics has tended to ignore power, so there has been little academic incentive to study firm hierarchy. A second caveat to my analysis is that the case-study firms are all relatively large. This is not a methodological choice; instead, all of the available case studies have focused on large firms. The public sector is also excluded from analysis and left as a topic for future research.

Figure 2 shows the correlation between relative income and hierarchical power within the six case-study firms. Each point represents a single firm-year observation, with the different case-study firms indicated by shape/color. Note that this figure plots *average* income (by hierarchical level) against *average* hierarchical power (by hierarchical level). In order to make comparisons across firms (and across time), I normalize incomes so that the mean income in the bottom hierarchical level of each firm is equal to one. Although the firm sample is small, the evidence is unambiguous: there is a strong correlation between relative income and hierarchical power in these case-study firms.

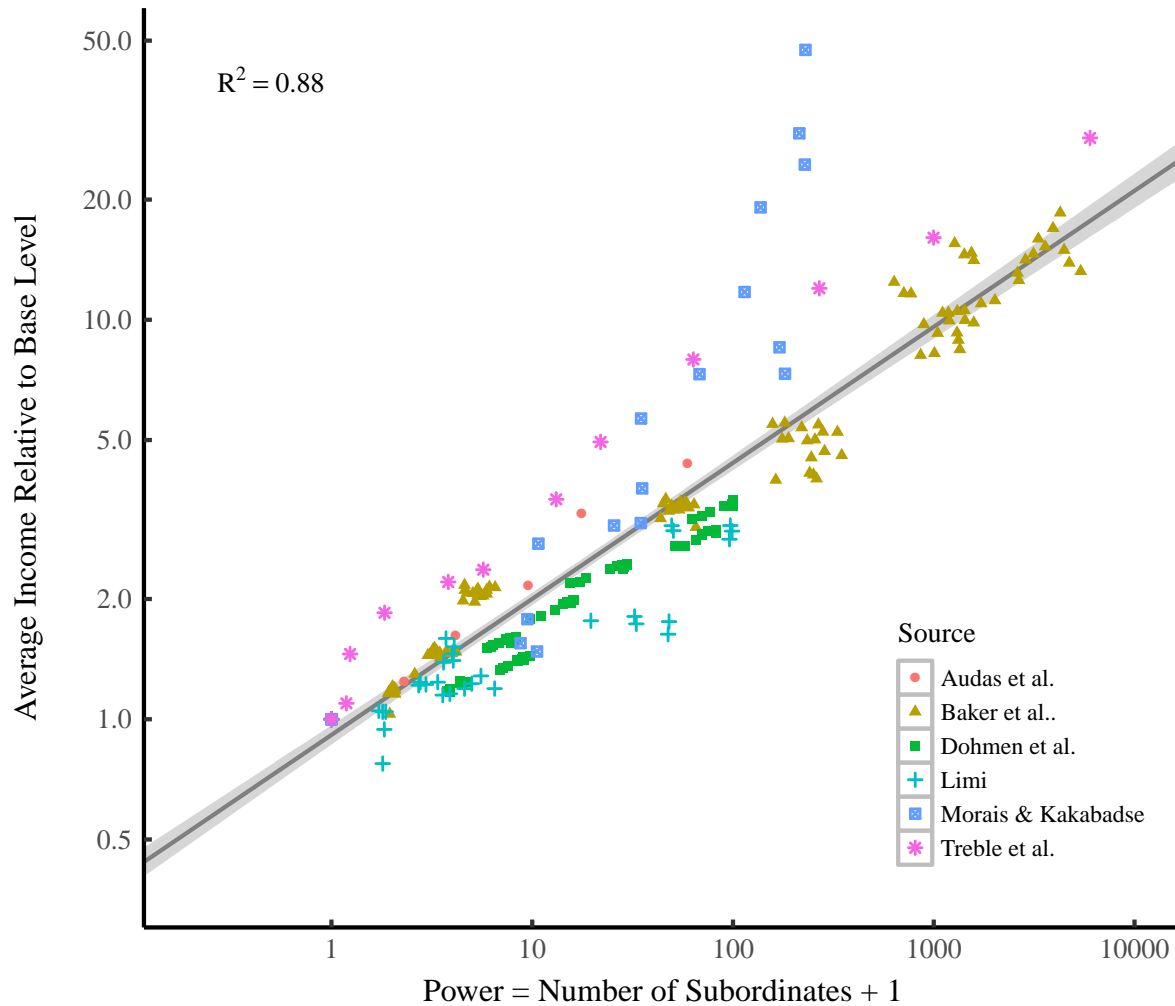


Figure 2: Average Income vs. Average Hierarchical Power in Case-Study Firms

This figure plots average income against average hierarchical power for six case-study firms (Audas et al., 2004; Baker et al., 1993; Dohmen et al., 2004; Lima, 2000; Morais and Kakabadse, 2014; Treble et al., 2001). Average income is normalized to equal one in the base hierarchical level. Average hierarchical power is calculated using Eq. 2 and 3. Each point represents a single firm-year observation, and shape/color indicates the particular case study. The line indicates a log-log regression, while the grey region indicate the 95% confidence interval.

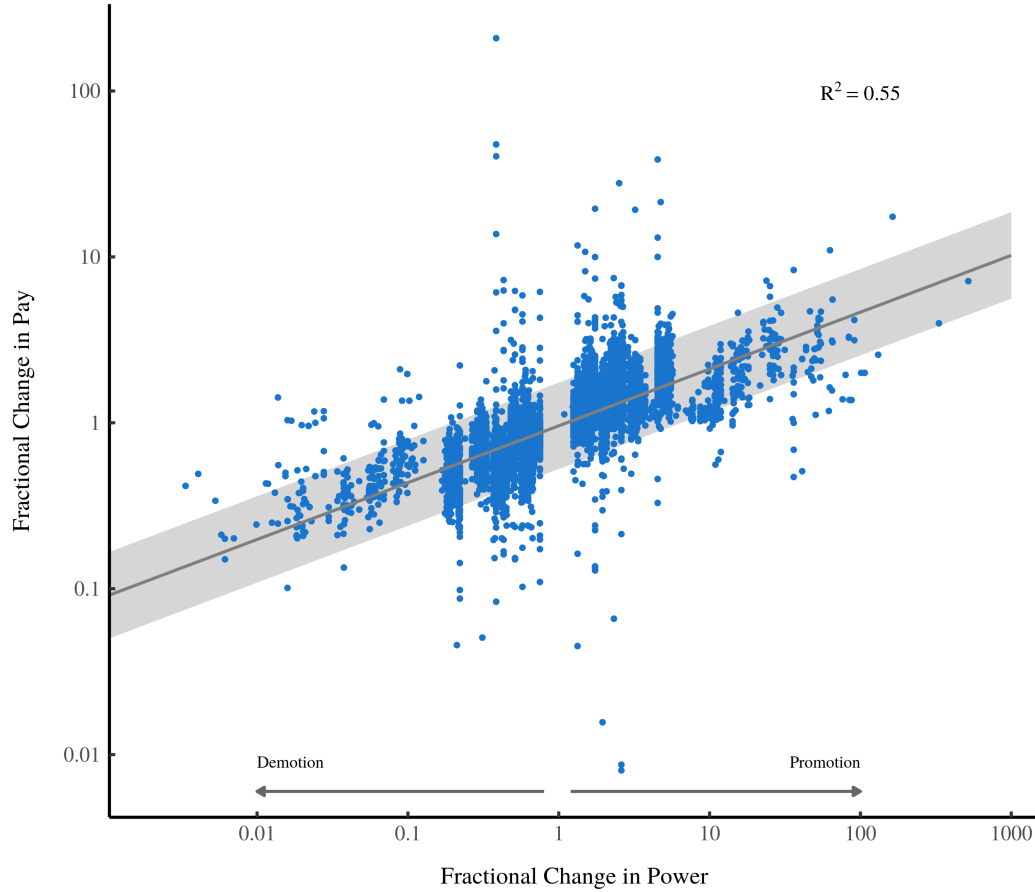


Figure 3: Changes in Hierarchical Power and Pay During Intra-Firm Promotions

This figure plots the fractional change in pay (Eq. 5) versus the fractional change in hierarchical power (Eq. 4) for individual promotions/demotions in the Baker, Gibbs, and Holmstrom (BGH) dataset (Baker et al., 1993). Each point represents the resulting change in pay and power of a *single* individual. Over 16,000 promotion/demotion events are plotted here. The grey region indicates the 95% prediction interval of a log-log regression. The BGH data comes from an anonymous US firm over the period 1969-1985. The dataset is available at <http://faculty.chicagobooth.edu/michael.gibbs/research/index.html>

Dynamic Correlation

I test for a dynamic correlation between changes in relative income and changes in hierarchical power using data published by Baker, Gibbs, and Homstrom ([Baker et al., 1993](#)) — the ‘BGH dataset’. This dataset contains raw personnel data for a large US firm over the years 1969-1985. Importantly, the BGH dataset tracks the income and hierarchical level of individuals over time. This allows the analysis of changes in income and hierarchical power when individuals are promoted or demoted. I define a promotion/demotion as a change in hierarchical level.

Since the chain of command is unknown, I begin by assigning all individuals the *average* hierarchical power (\bar{P}) of their respective hierarchical level. For each promotion/demotion event, I then calculate the fractional change in an individual’s hierarchical power ($\Delta\bar{P}$) as the following ratio:

$$\Delta\bar{P} = \frac{\bar{P}_{\text{after}}}{\bar{P}_{\text{before}}} \quad (4)$$

Here \bar{P}_{after} is hierarchical power after the promotion and \bar{P}_{before} is hierarchical power before the promotion. For each promotion/demotion event, I also calculate the fractional change in income (ΔI):

$$\Delta I = \frac{I_{\text{after}} / \bar{I}_{\text{after}}}{I_{\text{before}} / \bar{I}_{\text{before}}} \quad (5)$$

Here I_{after} is individual income after the promotion, and I_{before} is individual income before the promotion. In order to isolate the effect of the promotion from the effects of inflation and/or general wage increases, I measure after-before incomes *relative* to the firm mean income in the appropriate year. Here \bar{I}_{after} is firm mean income after the individual’s promotion, while \bar{I}_{before} is firm mean income before the individual’s promotion.

Figure 3 show the results of this dynamic analysis. Each point represents the fractional change in pay and hierarchical power for the promotion/demotion of a *single individual*. Over 16,000 promotions/demotion events are shown. Within the BGH data, a highly significant correlation exists between changes in hierarchical power and changes in individual income. Interestingly, the correlation holds both for promotions and for *demotions*, the latter occurring when an individual drops hierarchical levels. The relative pay reductions accompanying these demotions are difficult to understand from neoclassical marginal productivity perspective. Do these individuals suddenly experience a drastic reduction in ability/productivity? The evidence in Figure 3 suggests a better explanation: within the BGH firm, pay

is largely a function of the power of a specific hierarchical *position*, irrespective of the person holding this position.

To summarize, the case-study evidence indicates that relative income within firms is both statically and dynamically correlated with hierarchal power. This evidence is consistent with the hypothesis that relative income within a hierarchy is proportional to hierarchical power.

4 How Strongly Does Hierarchical Power Affect Income?

Having found a correlation between hierarchical power and income, the next step of the analysis is to measure the strength of this effect at the societal level. Hypothesis B proposes that hierarchical power affects income more strongly than any other factor for which data is available. To test Hypothesis B, we need to relate hierarchical power's effect on income to the effect-size of a variety of other factors. To do this, I use a signal-to-noise ratio similar to Cohen's f^2 . As with the test of Hypothesis A, a caveat to this test of Hypothesis B is that the available evidence on firm hierarchy is limited. Therefore, this analysis should be considered preliminary.

4.1 Measuring Effect Size With A Signal-to-Noise Ratio

I measure income effect size using a group-based signal-to-noise ratio. This method has two parts. First, we organize individuals into an income-affecting grouping (for instance, by individuals' sex). To measure the effect that this grouping has on income, we then calculate the signal-to-noise ratio by comparing dispersion in average income *between* groups to the average income dispersion *within* groups (Eq. 6). The larger the signal-to-noise ratio, the larger the grouping's effect on income.

$$\text{signal-to-noise ratio} = \frac{\text{between-group income dispersion}}{\text{average within-group income dispersion}} \quad (6)$$

Figure 4 shows an example of how a two-group factor like individuals' sex might affect income. Dispersion between groups is evident as the difference between mean incomes of each sex (dotted lines). Within-group dispersion is visible in terms of the average spread (width) of each sex's respective income distribution. Figure 4A shows a *small* effect on income — evident as a small difference between group means and large within-group dispersion. Conversely, Figure 4B shows a *large* effect on income — evident as a large difference between group means and small within-group dispersion. While the signal-to-noise ratio is most easily illustrated for a two-group factor, it can be generalized to an income-affecting factor with any number of groups.

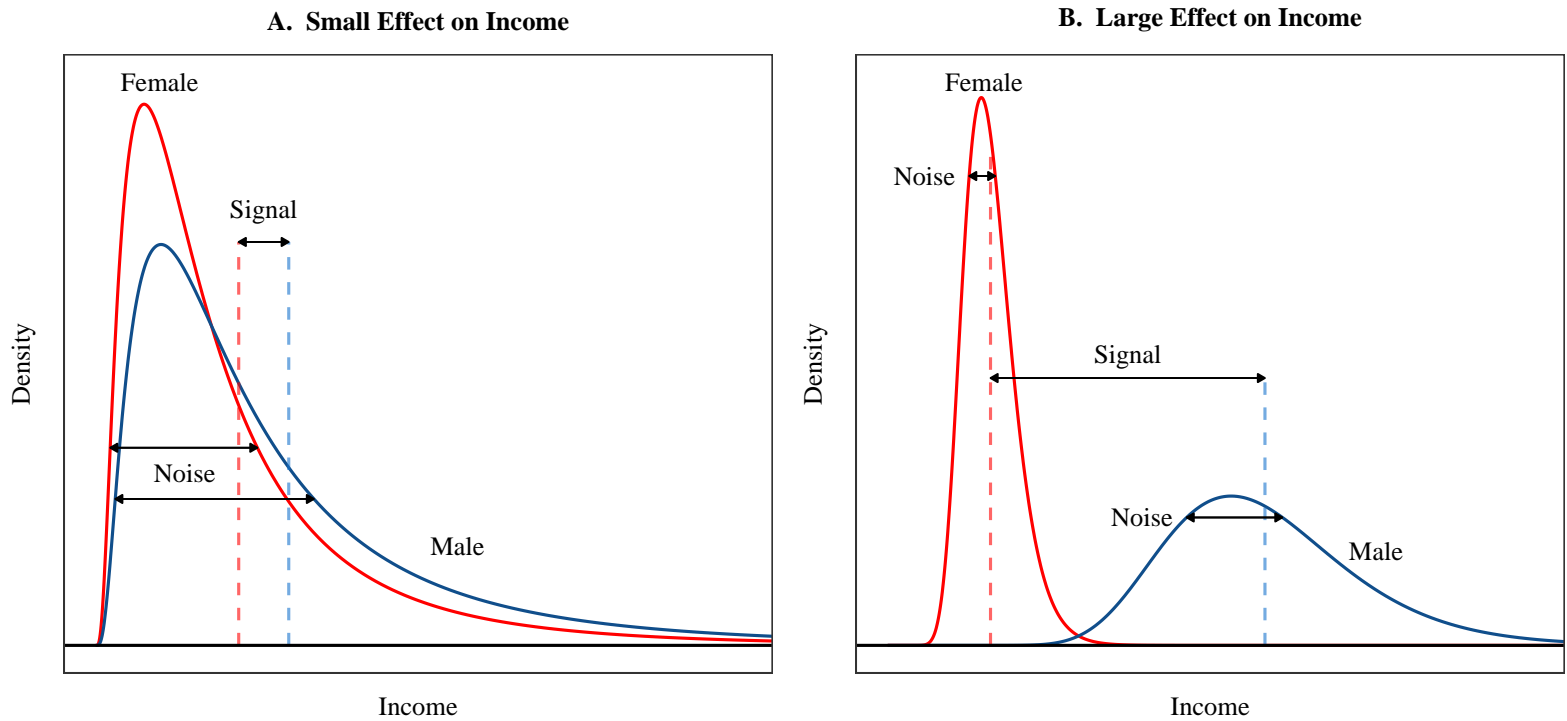


Figure 4: Visualizing Effect Size

This figure shows an example of how a two-group factor like a person's sex might affect income. Each panel shows a hypothetical income distribution of both males and females. We can judge the income effect size of being male vs. female by comparing the 'signal' to the 'noise'. The signal is the income difference *between* group mean incomes (dotted lines), while the noise is the income dispersion *within* groups (visualized here as the standard deviation). The larger the signal is relative to the noise, the larger the effect on income. Individual's sex has a small effect on income in Panel A and a large effect in Panel B.

Although there are many ways of measuring income effect size, an advantage of this group-based approach is that it is easily applicable to the qualitative variables that are well-known to effect income (such as 'sex', 'race', 'occupation', 'education', etc.). To compare the income effect of two factors such as 'sex' and 'race', we compare the signal-to-noise ratio of grouping individuals by their sex to the signal-to-noise ratio of grouping individuals by their race. Another advantage of this group-based approach is that we do not need to have a single sample of individuals with an exhaustive list of their characteristics and income (like we would need for a multivariate regression analysis). Instead, we can use different datasets for each respective income-affecting factor, provided that each dataset is representative of the general population.¹

In standard analysis of variance, the signal-to-noise ratio is called Cohen's f^2 ,

and is calculated using *variance* as the measure of dispersion (Fleishman, 1980; Steiger, 2004). However, I do not use Cohen's f^2 to measure income effect size because variance is not commonly reported in income distribution statistics. Instead, statistical agencies typically report the Gini index of income dispersion within groups. Because of its ubiquity, I use the Gini index to calculate the signal-to-noise ratio:²

$$\text{signal-to-noise ratio (Gini)} = \frac{\text{Gini index of group means}}{\text{average within-group Gini index}} \quad (7)$$

For a detailed discussion of this metric (and its relation to Cohen's f^2), see Appendix H.

Grouping Individuals By Hierarchical Level

In order to measure the income effect of hierarchical power using the signal-to-noise ratio, we must group individuals into different classes of hierarchical power. My method is to group individuals by *hierarchical level* across all firms, as illustrated in Figure 5. This method is theoretically attractive because hierarchical level is the principle determinant of hierarchical power. If a firm has a constant 'span of control' (the number of subordinates controlled by each superior) then hierarchical power will increase *exponentially* with hierarchical level). This grouping method is also empirically convenient because the available data on firm hierarchies is limited, and the most commonly reported statistic is the distribution of income by hierarchical level.

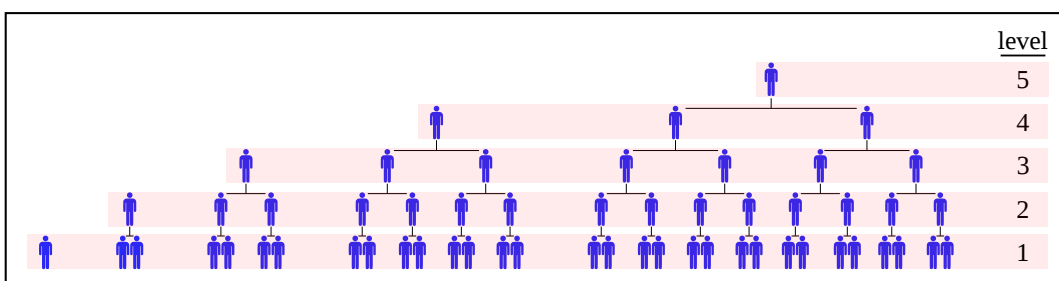


Figure 5: Grouping Individuals By Hierarchical Level

This figure shows how I group individuals by hierarchical power. My groups consist of all individuals (regardless of firm) that share the same hierarchical level. In this figure, each hierarchy represents a different firm. Groups are indicated by shaded regions.

4.2 Methods

Hypothesis B proposes that hierarchical power affects income more strongly than any other factor for which data is available. To test this hypothesis, I use the signal-to-noise ratio (Eq. 7) to quantify the income effect of each factor shown in Table 1. All data (except those discussed below) come from the United States. If Hypothesis B is correct, grouping individuals by hierarchical level (across society) should produce the largest signal-to-noise ratio.

My method is as follows. For each income-affecting factor, I divide a sample of the US population into the appropriate groups. For example, the factor ‘sex’ groups the US population into males and females. Similarly, the factor ‘occupation’ groups the US population into 55 different occupations, the factor ‘education’ groups the US population into 9 levels of education, and the factor ‘hierarchical level’ groups the US population into 12-14 hierarchical levels. (For more details about sources and methods, see Appendix A). Once we have the grouping for a specific factor, we calculate income dispersion between groups and compare it to average income dispersion within groups (using the Gini index). The ratio of these two quantities — the signal-to noise ratio — indicates the income effect size of the factor in question. We can then use these signal-to-noise ratios to rank the income effect sizes of the factors in Table 1.

The point of this analysis is to determine if grouping individuals by hierarchical level has the greatest effect on income. The problem, as I have stated previously, is that the data on firm hierarchy is quite limited. I am not aware of any dataset on hierarchical rank and income for a large sample of US citizens. Because of this data shortage, I am forced to use model-dependent data. I build a model that extrapolates the available case-study data on firm hierarchy to estimate the hierarchical pay structure of 713 US firms in the Compustat database (covering the years 1992-2015). This ‘Compustat Model’ is discussed in detail in the Appendix. The model generates synthetic data from which we can estimate the signal-to-noise ratio of grouping individuals by hierarchical level. I also use the Compustat model to estimate the signal-to-noise ratio of grouping individuals by firm.

I supplement this model-dependent data with two non-US studies that are purely empirical. The first source is a seminal study by Mueller, Ouimet, and Simintzi (2016) that reports income distribution by hierarchical level for 880 United Kingdom firms over the period 2004-2013. The second source is a study by Fredrik Heyman (2005) that analyzes the pay distribution of the top 4 levels of management in 560 Swedish firms in the year 1995. Heyman’s data comes with the caveat that it does not represent all hierarchical levels — just the top four. (For this reason, I mark

Table 1: Income-Affecting Factors Used to Test Hypothesis B

Geographic	Physical Attribute	Socioeconomic
Census Block Group	Age	Education
Census Tract	Cognitive Score*	Employee vs. Self-Employed
County	Race	Firm*
Urban vs. Rural	Sex	Full vs. Part Time
		Hierarchical Level*
		Home Owner vs. Renter
		Occupation
		Parents' Income Percentile
		Public vs. Private Sector
		Religion
		Type of Income (Labor/Property)

* Indicates variables that use model-dependent data (at least in part)

For sources and methods, see Appendix A.

Heyman's results with an asterisk). I use this non-US data as an empirical check on the Compustat model's results.

4.3 Results

The results of my test of hypothesis B are shown in Figure 6, which plots signal-to-noise ratios for each of the 19 different income-affecting factors. To reiterate, the signal-to-noise ratio is the ratio of between-group dispersion to average within-group dispersion. A larger signal-to-noise ratio indicates that grouping individuals by the factor in question has a larger effect on income. For all factors except religion and cognitive score, the boxplots indicate the variation of the signal-to-noise ratio over time (typically the last 20 years). For religion, the boxplot range indicates uncertainty in the signal-to-noise ratio. For cognitive scores, the boxplot range represents variation between different studies.

This test of hypothesis B yields unambiguous results. Of the 19 different income-affecting factors tested, the evidence suggests that grouping individuals by hierarchical level has the strongest effect on income. Importantly, the Compustat model produces a signal-to-noise ratio that is consistent with the purely empirical (non-US) studies of firm hierarchy. This supports the model's results.

In addition to the support for hypothesis B, Figure 6 reveals a few other notable findings. Firstly, physical attributes (age, cognitive score, race, and sex) have a

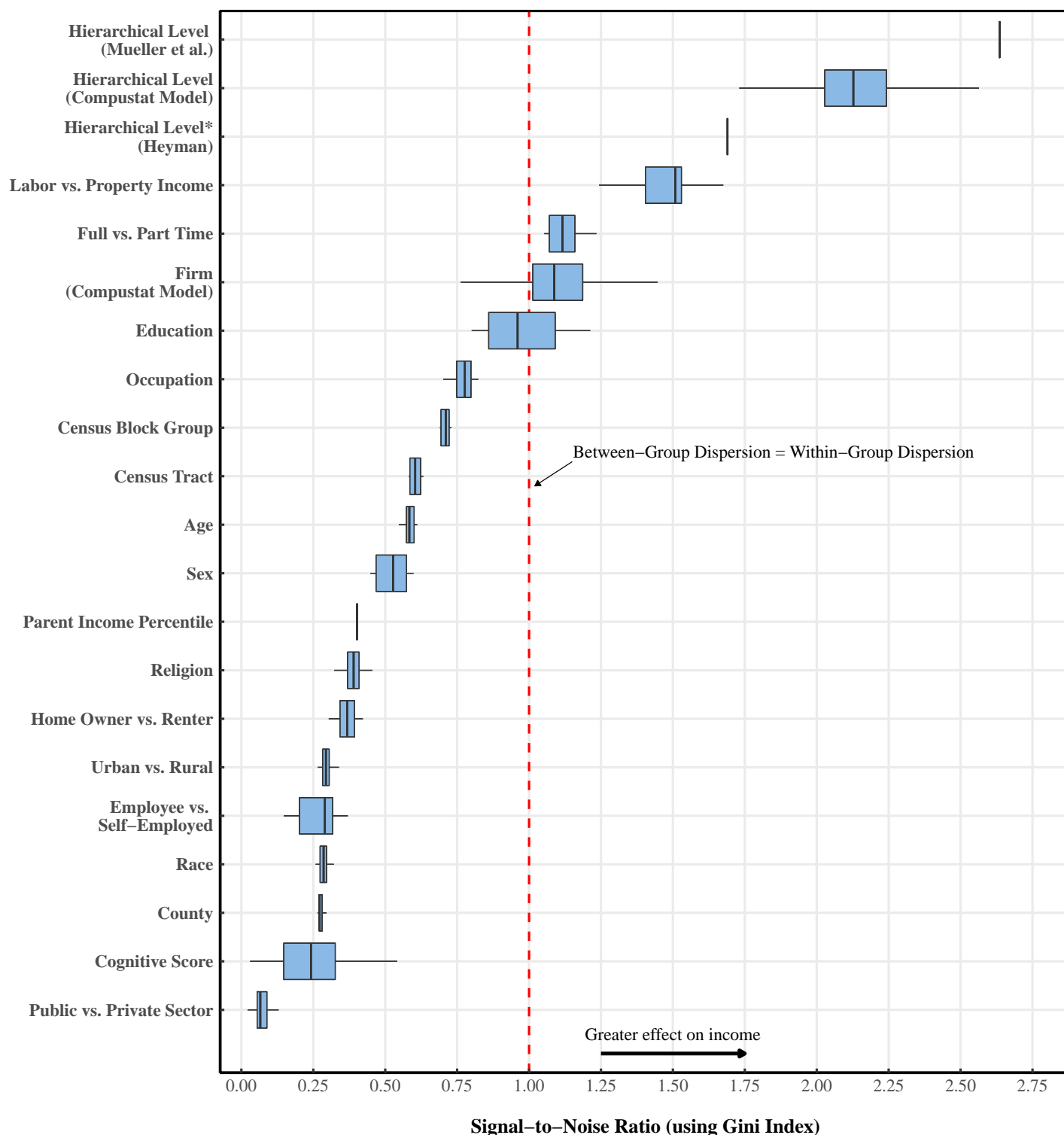


Figure 6: The Signal-to-Noise Ratio for Different Income-Affecting Factors

This figure shows signal-to-noise ratios for each of the 19 income-affecting factors used to test Hypothesis B (which proposes that hierarchical power affects income more strongly than any other factor for which data is available). The signal-to-noise ratio indicates the income effect of grouping individuals according to the factor in question. The boxplots indicate the total range (horizontal line), 25th to 75th percentile range (the box), and the median (vertical line) of the signal-to-noise ratio. With the exception of hierarchical level data from Mueller et al. (2016) and Heyman (2005), all data is from the United States. For sources and methods, see Appendix A.

* Includes only top 4 hierarchical levels

relatively insignificant effect on income. Geographic effects are also quite small, although they become larger as the geographic area *decreases*. (Geographic factors ranked from largest to smallest area are: county, tract, block group).

Of symbolic interest is the one-to-one value for the signal-to-noise ratio — the point where the signal is of equal size to the noise. Besides hierarchical level, only two other factors have an income effect size that is significantly larger than this one-to-one threshold: labor vs. property income and full vs. part time. The latter is easily understandable. Part-time individuals work significantly fewer hours than full-time individuals, so we would expect significant income differentials between the two groups. Added to this effect is the fact that part-time jobs are often in sectors (such as retail) that have lower wages than in sectors where full-time employment is the norm (like mining).

But what should we make of the significant effect of *functional* income type (property vs. labor)? At first glance, this may seem to support many political economists' deeply held convictions about functional income distribution: capitalists tend to be much wealthier than workers. While this may be true, the results shown here indicate something different. They indicate that property income is on average *much less* than labor income. This result is best thought of as an artifact of the US Census accounting method. In the Census data, 'property income' includes *anyone* with some form of dividend, interest, or rental income. This means that average property income is trivially small — about 8% of the average income from wages/salaries. This is because many people earn small amounts of property income in the form of interest on savings or dividends from small investments. Since these people likely earn income from other sources, this comparison of Census data for labor and property income has little meaning. However, I include it here for the sake of completeness.³

To summarize, the available evidence (which is preliminary) suggests that grouping individuals by hierarchical level affects income more strongly than any of the other 18 factors tested here.

5 Discussion

The evidence presented here raises many questions for future research. For instance, how general is the correlation between hierarchical power and relative income within firms? Does this correlation generalize to the public sector? Does it vary between countries? And related to this — does the income-effect of grouping individuals by hierarchical level vary by country (or over time)? When better data

on firm/government hierarchy becomes available, we can begin to answer these questions.

Another important question to ask is – what are the mechanisms that cause income to be correlated with hierarchical power? My hunch is that there are no simple answers to this question, because there are many ‘pathways to power’ ([Price and Feinman, 2010](#)). A hierarchical chain of command can be sustained in many different ways, ranging from the pure use of ideology to the pure use of force. In the purely ideological case, the hierarchy functions because subordinates simply agree that their superior has legitimate authority. In this case, subordinates likely believe that their superior deserves to earn more than them. On the other end of the spectrum, the history of slavery indicates that a hierarchy can also function through brutal repression. In this case, the income of superiors is an outcome of the judicious use of force. In both cases, the superior has power over subordinates, but the appearance (and justification) of this power is very different. An intriguing possibility is that greater inequality within a hierarchy might be associated with a greater use of intimidation and fear (rather than the use of a legitimizing ideology).

It is also plausible that belief in the legitimacy of a hierarchy increases with hierarchical rank ([Schmidt, 2001](#)). If this is true, it should manifest in opinions about income. Interestingly, a recent survey reveals that a majority of Americans question the legitimacy of CEO income ([Larcker et al., 2016](#)). Only 16% of the general public agree that CEOs are “paid the correct amount relative to the average worker”. Yet a majority of Fortune 500 CEOs (64%) thought that CEO pay was ‘correct’. It would be fascinating to expand this type of survey to see if there is a gradient of opinion by hierarchical rank.

Another complexity is that firms are not islands unto themselves — there are power relations *between* institutions as well as within them ([Bichler and Nitzan, 2017](#)). Government regulation, for instance, can have a significant impact on CEO pay. CEOs in the highly regulated US utility sector have significantly lower pay than CEOs in other sectors (see Appendix F, as well as [Joskow et al. \(1993\)](#)). There is also evidence that CEO pay has a class-like cohesiveness. The average compensation of top US CEOs moves coherently with the capitalization of large firms ([Mishel and Davis, 2014](#)). This raises interesting implications for integrating the concept of hierarchical power with Nitzan and Bichler’s (2009) ‘capital as power’ hypothesis, in which capital is conceived as a symbolic representation of power.

Lastly, the evidence presented here raises questions about recent increases in income inequality that have occurred in the United States and other countries (see [Piketty and Saez 2001, 2006; Piketty 2014; Atkinson and Piketty 2010; Alvaredo et al. 2013](#)). To what extent has this increase been due to an increase in hierarchical

pay inequality? Because the empirical study of the relation between income and hierarchical power is in its infancy, the avenues for future research are expansive.

6 Conclusions

The hypothesis that income is related to power has been proposed numerous times over the last century. However, in its general form a power-income hypothesis is difficult to test. Power is simply too broad a concept to pin down empirically. In this paper, I have attempted to overcome this difficulty by narrowing the focus to hierarchical power only, which I define as the number of subordinates under an individual's control.

Although the data on firm hierarchy is limited (and thus results should be considered preliminary), the available evidence is clear. There is a strong correlation between relative income and hierarchical power within case-study firms. Moreover, the available US evidence suggests that grouping individuals (across society) by hierarchical level affects income more strongly than any other factor tested. I suggest that this is preliminary evidence for a hierarchical power theory of personal income distribution.

I conclude by offering some thoughts on the ideological implications of such a theory. Regardless of their scientific merit, all theories of income distribution evoke some form of ethics that either justifies income *redistribution*, or justifies the *status quo*. Conventional (neoclassical) theories of income distribution illicit an ethics of fairness — “*To each according to what he and the instruments he owns produces*”, as Milton Friedman famously put it [Friedman \(1962\)](#). The effect of this theory is to justify as fair any conceivable distribution of income. The result is an innate bias towards the status quo, whatever it may be.

A hierarchical power theory of income distribution is very different. If we parallel Friedman's language, we might state that a hierarchical power theory elicits the following ethos: “*To each according to his/her power to take*”. Few would argue that this is fair — it is the basic recipe for *despotism*. But if individual income is most strongly determined by hierarchal power, then acts of income redistribution can be considered largely as *checks* on power — no different than the checks and balances that form the governmental basis of most liberal democracies.

Notes

¹Readers trained in econometrics will observe that my method for measuring effect-size does not *isolate* the income-effects of a given factor. It does not show that, when *all* other factors are held constant, a change in factor *A* by amount *x* affects income by amount *y*. I make no attempt to do this because I think it is the wrong approach. As Keynes (1939) long ago argued, the only conceivable way that an econometric model can *isolate* an effect is if the model includes a *complete* list of causal factors. But since we can never be sure that our causal list is complete, we can never know if our econometric model is wrong (Nitzan, 1992). My thinking is more pragmatic. Given the complexities of human behavior, we can likely never isolate a factor to find its ‘true’ effect on income. But we can *rank* effect-size with the full understanding that when we measure one factor’s effect on income, enumerable other factors are included in this measurement. In the face of enumerable confounding variables, Occam’s razor would suggest that we simply chose the factor with the largest effect on income and use it to build a theory income distribution.

² A well-known shortcoming of the Gini index is that it has a *downward bias* for small sample sizes. If the sample size is *n*, the maximum possible Gini index is:

$$G_n^{max} = \frac{n-1}{n} \quad (8)$$

Thus a sample size of $n = 2$ has a maximum Gini index of $G_2^{max} = 0.5$. To correct for this bias, I use the method proposed by George Deltas (2003). The bias-adjusted Gini index (G^{adj}) is defined by dividing the unadjusted Gini (G) by the maximum possible Gini (G_n^{max}):

$$G^{adj} = \frac{G}{G_n^{max}} \quad (9)$$

I use the adjusted Gini index for all between-group Gini calculations in this paper.

³To compare the income-effect of functional income type, what we really need to do is group individuals by the *proportion* of income coming from property sources. Based on the work of Piketty (2014), it is reasonable to expect that this would strongly affect income. Piketty shows how the proportion of capitalist income increases with income fractile in the United States. But this grouping is the reverse of what would be required to apply my analysis of variance method. Piketty groups individuals by income size, while the method used here would require grouping individuals by the proportion of capitalist income. At present, I am not aware of the data sources that would allow such a grouping.

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Appendices

for

Personal Income and Hierarchical Power

Supplementary materials for this paper are available at the Open Science Framework repository:

<https://osf.io/en4rz/>

The supplementary materials include:

1. Data for all figures appearing in the paper;
2. Raw source data;
3. R code for all analysis;
4. Compustat model code.

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A Data Sources

Age

Age mean income and within-group Gini index data is from US Census Tables PINC-02 over the years 1994-2015. Age is grouped into the following 4 categories: 18-24, 25-44, 45-64, 65 and older.

Census Blocks

Census blocks data comes from the US Census American Community Survey (ACS) over the years 2010-2014. This data is tabulated at the *household* (rather than individual) level. Neither mean household income nor household Gini index data is directly available from the ACS at the census block level. I calculate mean household income by dividing aggregate household income by the number of households.

Within-group Gini indexes are estimated from binned income data using the R ‘[binequality](#)’ package. I construct two different estimates: one using a parametric method and the other using the midpoint method. For the parametric method, I fit either a lognormal or gamma distribution (whichever is best) to the binned data. Gini indexes are then calculated from this fitted distribution. The midpoint method uses midpoints of the bins to estimate the Gini index. The midpoint of the upper bin (which has an open upper bound) is estimated from a best-fit power law (again, implemented in the R [binequality](#) package).

Census Tracts

Census tract data comes from the US Census American Community Survey (ACS) over the years 2010-2015. Mean income data comes from series S1902, while within-group Gini indexes come from series B19083.

Cognitive Score

The signal-to-noise ratio for cognitive score is estimated using data from Figure 6 in Bowles et al. (2001). Bowles’ figure presents 65 different estimates (from 24 studies between 1963 and 1992) of the relation between individual income and cognitive score. The strength of this relation is quantified using the beta coefficient (β) of a log-linear regression. This coefficient represents the slope of the regression equation shown in Eq. 1, where the logarithm of income – $\log(I)$ – and cognitive

score (S) have first been normalized to have a mean of 0 and standard deviation of one.

$$\log(I) = \alpha + \beta S \quad (1)$$

I use Engauge Digitizer to extract data from Bowles' graph. I then use a model to estimate the signal-to-noise ratio from Bowles' reported beta coefficients. The model creates a stochastic log-linear scaling relation between income and cognitive score. By adjusting the strength of this relation, we can create modeled data that has an equivalent beta coefficient to any of the points in Bowles' figure. I then use the model to calculate the signal-to-noise ratio for this beta coefficient.

The model assumes that cognitive score (S) is a normally distributed random variate with a mean of 100 and standard deviation of 15:

$$S \sim \mathcal{N}(100, 15) \quad (2)$$

We assume that the natural log of mean income ($\ln \bar{I}$) scales exponentially with cognitive score (Eq. 3). Since there is no evidence that extreme IQs lead to extreme incomes (at either the bottom or top end), I do not include them in the model. I model only those individuals with scores that are within two standard deviations of the mean ($70 < S < 130$). The parameter a determines how strongly cognitive score affects average income.

$$\ln(\bar{I}) = a(S - 70) \quad \text{for} \quad 70 < S < 130 \quad (3)$$

We assume that individual income (I) is a stochastic variable that is distributed according to a lognormal distribution defined by the location parameter μ and scale parameter σ :

$$I \sim \ln \mathcal{N}(\mu, \sigma) \quad (4)$$

Equation 5 shows how mean income \bar{I} is related to μ and σ .

$$\bar{I} = e^{\mu + \frac{1}{2}\sigma^2} \quad (5)$$

By taking the logarithm and solving for μ , Eq. 5 can be transformed into the following:

$$\mu = \ln(\bar{I}) - \frac{1}{2}\sigma^2 \quad (6)$$

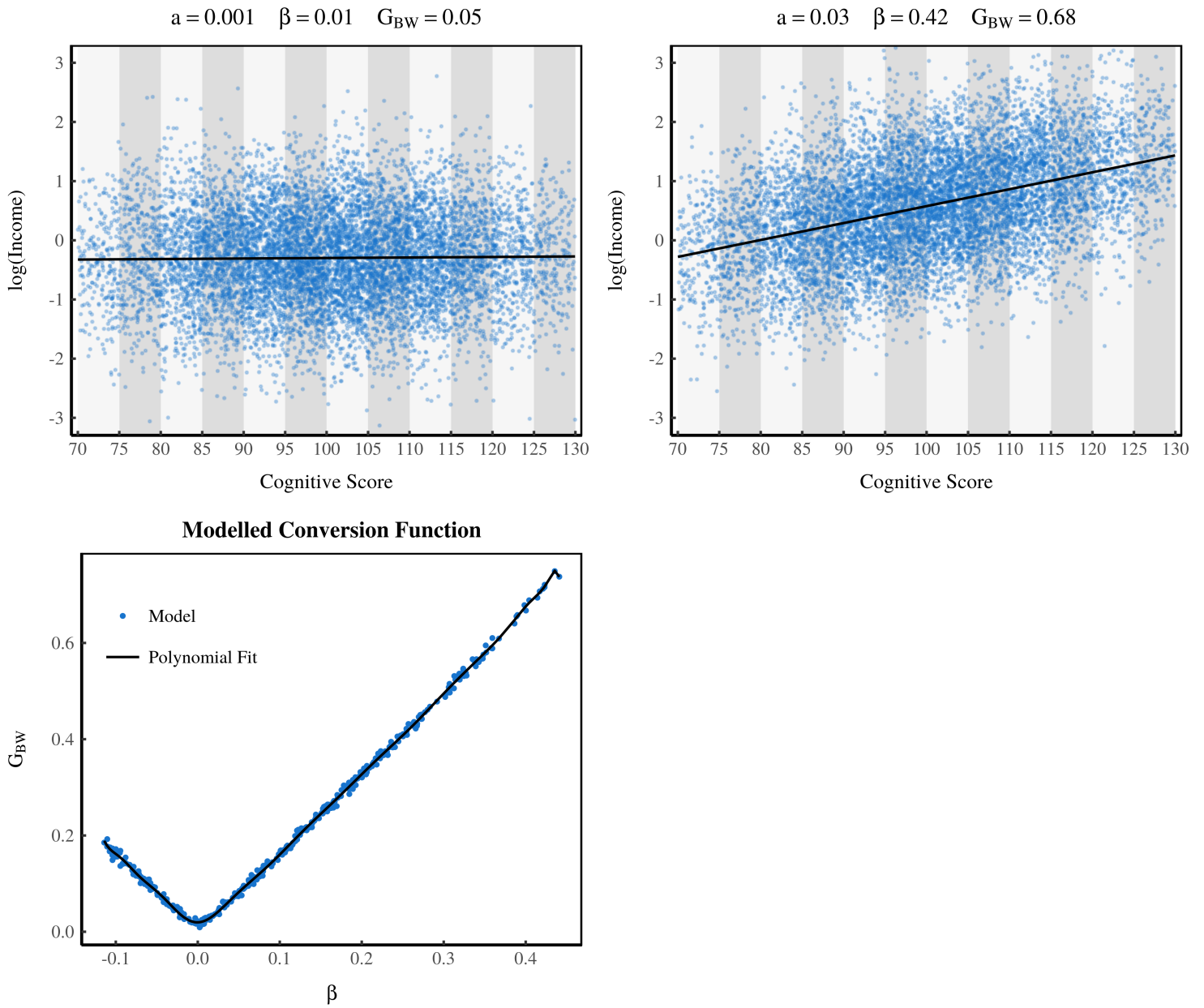


Figure 1: Cognitive Score Method — Estimating the Signal-to-Noise Ratio from Normalized Regression Coefficients

This figure shows an example of the model for converting cognitive score regression data from Bowles et al. (2001) to a signal-to-noise ratio. The signal-to-noise ratio (G_{BW}) is the ratio of the between-group Gini index to the average within-group Gini index. Using equations 2-7, I create a stochastic scaling relation between the logarithm of individual income and cognitive score. The strength of this scaling relation is determined by the parameter a , and is quantified by the normalized regression coefficient β . The top left panel shows a weak scaling relation, while the top right shows a strong scaling relation. I then group individuals into cognitive score intervals of 5 (vertical grey bars) and calculate signal-to-noise ratio (G_{BW}). The bottom left panel shows the resulting relation between G_{BW} and β that is used to convert Bowles' data.

We then substitute Eq. 3 into Eq. 6 to define μ in terms of cognitive score:

$$\mu = a(S - 70) - \frac{1}{2}\sigma^2 \quad (7)$$

The algorithm for the model is as follows. We first generate a random cognitive score S , drawn from the normal distribution defined by Eq. 2. We then take this score and use Eq. 7 to define the parameter μ . Finally, we generate a random income for this cognitive score, drawn from the lognormal distribution defined by Eq. 4. This process is then repeated as many times to generate a stochastic dataset relating income to cognitive score.

The model has 2 free parameters: a and σ . Parameter a affects the rate at which income scales with cognitive score, while σ determines the amount of dispersion around the mean income \bar{I} . The parameter σ strongly affects the level of ‘global’ inequality in the model, while a has only a slight effect. For this reason, it is important to choose σ such that the model has a realistic level of inequality. I chose $\sigma = 0.8$. Over the chosen range of $-0.007 < a < 0.03$, this produces global Gini indexes that range between 0.43 and 0.47, which is roughly consistent with US data for the second half of the 20th century.

For any given value of a , the model generates a stochastic relation between cognitive score and income I . Two examples are shown in Figure 1. In Figure 1A, the small value of a produces a very weak relation between income and cognitive score. In Figure 1B, the larger value of a produces a stronger relation between income and cognitive score.

The strength of the relation is indicated by the beta coefficient β . The purpose of this model is to convert the values of β reported by Bowles et al. into the signal-to-noise ratio that is used in this paper. To make this conversion, we must group individuals by their cognitive score. The bin-size of this grouping is arbitrary; I construct groupings of 5 point cognitive score intervals (indicated by the grey vertical bands in Fig. 1A-B). For each group, we calculate the mean income and within-group Gini index. The signal-to-noise ratio (G_{BW}) is then calculated by the method outlined in the main paper.

I repeat this process for many different values of a , which produces the modeled relation between G_{BW} and β shown in Figure 1C. I then fit this relation with a high order polynomial that serves as the function for converting Bowles’ β values into the signal-to-noise values used in this paper.

Counties

US County data comes from the American Community survey for the years 2006-2015. County Gini indexes are from series B19083, while mean income is from series S1902.

Education

Mean income and within-group Gini indexes by educational level come from US Census tables PINC-03 over the years 1994-2014. Educational level is categorized into the following groups:

- Less Than 9th Grade
- 9th to 12th Nongrad
- High school Graduate (Incl GED)
- Some College
- Associate Degree
- Bachelor's Degree
- Master's Degree
- Professional Degree
- Doctorate Degree

Employees vs. Self-Employment

To calculate mean income and within-group Gini indexes for employees and self-employed workers, I use US Census table PINC-07 between 1994 and 2015. This table contains three categories: *Government Wage And Salary Workers*, *Private Wage And Salary Workers*, and *Self-Employed Workers*. Table 1 shows how I have mapped these categories onto the 'employees' and 'self-employed' sectors.

Table 1: Grouping Categories of Census Table PINC-07

Employees	Self-Employed
<ul style="list-style-type: none"> • Government Wage And Salary Workers • Private Wage And Salary Workers 	<ul style="list-style-type: none"> • Self-Employed Workers

Self-employed mean income and within-group Gini index come directly from PINC-07. To calculate the mean income of employees, I use the average of the means of government workers and private workers, weighted by the size of each group.

Since Gini indexes are not additive, I estimate the inequality among employees from binned data. I first add the binned income counts of both government and private wage/salary workers to get a binned income distribution for all 'employees'.

From this binned data, I then use the the R ‘[binequality](#)’ package to estimate private sector Gini indexes.

I construct two different estimates: one using a parametric method and the other using the midpoint method. For the parametric method, I fit various theoretical distributions to the binned data. Gini indexes are then calculated from the best-fitting distribution. The midpoint method uses midpoints of the bins to estimate the Gini index. The midpoint of the upper bin (which has an open upper bound) is estimated from a best-fit power law (again, implemented in the R [binequality](#) package).

Firms

Firm signal-to-noise ratio calculations use the Compustat database, and are a combination of empirical and modeled data. Firm mean income is calculated directly from Compustat data by dividing Total Staff Expenses (series XLR) by the number of employees (series EMP). Firm internal inequality is estimated using the Compustat Model. See Appendix [B-G](#) for a detailed discussion.

Full and Part Time Workers

Full and part time worker mean income and within-group inequality data comes from US Census tables PINC-05 from 1994-2015.

Parent Income Percentile

‘Parent income percentile’ refers to grouping individuals by the income percentile of their parents. My calculations are done using Table 1 and 2 from the [online data tables](#) of Chetty et al. (2014) — a seminal study of US intergenerational mobility. For every parent income percentile x , Table 1 gives the probability $p(x, y)$ that the corresponding child will have an income in percentile y . Table 2 gives the mean income (\bar{I}_y) of each child percentile y .

My method for estimating group mean incomes and within-group inequality is shown in equations [8](#) and [9](#). The first step is to convert the probability $p(x, y)$ into an integer $w(x, y)$ that can be used to weight incomes. Since the probabilities in Table 1 contain 7 decimal places, I multiply $p(x, y)$ by 10^7 (Eq. [8](#)).

$$w(x, y) = p(x, y) \times 10^7 \quad (8)$$

For each each income percentile x , we then create a vector of child incomes (\mathbf{I}_x)

by repeating each child percentile mean income \bar{I}_y by the weighting factor $w(x, y)$. Here the notation $\times^{w(x,y)}$ indicates that the value \bar{I}_y is repeated $w(x, y)$ times.

$$\mathbf{I}_x = (\bar{I}_1^{\times^{w(x,1)}}, \bar{I}_2^{\times^{w(x,2)}}, \dots, \bar{I}_{100}^{\times^{w(x,100)}}) \quad (9)$$

We can think of \mathbf{I}_x as an estimated income distribution for children of parents in income percentile x . Mean income and within-group inequality of parent group x are then estimated by calculating the mean and Gini index (respectively) of \mathbf{I}_x .

Note that this method neglects the income dispersion *within* each child income percentile (Chetty et al. do not provide this data). Thus, our estimated Gini index will have a slight *downward* bias.

Hierarchical Level — Heyman

This data comes from Fredrik Heyman's (2005) study of 560 Swedish firms in the year 1995. His dataset includes only the top 4 levels of management. I include Heyman's results in the paper with the caveat that his data does not represent all hierarchical levels.

Heyman (Table A.1) provides the mean and standard deviation of the *logarithm* of incomes in each level. I estimate mean income (\bar{I}) and Gini index (G) by hierarchical level by assuming that intra-hierarchical level income is lognormally distributed. Under this assumption, the *mean* of log income is equal to the lognormal location parameter μ , while the *standard deviation* of log income is equal to the scale parameter σ . Equations 10 and 11 then define the mean income and Gini index (respectively) of each hierarchical level.

$$\bar{I} = e^{\mu + \frac{1}{2}\sigma^2} \quad (10)$$

$$G = \text{erf}\left(\frac{\sigma}{2}\right) \quad (11)$$

Figure 2 shows how the implied aggregate inequality within the Heyman's sample compares to Swedish empirical data. Heyman's sample implies a bit less inequality than the empirical data. This is not surprising, however, as Heyman's data includes only the top 4 levels of management.

Hierarchical Level — Mueller et al.

This data comes from Mueller et al. (2016), who study the hierarchical pay structure of 880 United Kingdom firms over the period 2004-2013. For each hierarchical

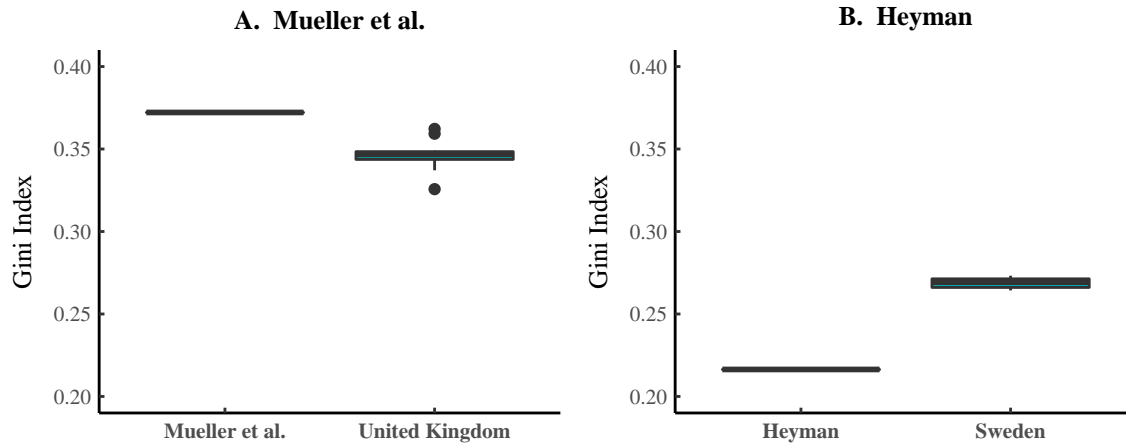


Figure 2: Aggregate Inequality Implied by Hierarchy Data

This figure compares levels of inequality implied by the Mueller et al. and Heyman firm samples against the inequality in their respective countries. UK inequality data is over the period 2004-2013, the same as covered by Mueller’s data. Heyman’s study covers the year 1995, while Swedish data is from 2004-2013. UK and Sweden Gini data is from the World Bank, series SI.POV.GINI.

level, Mueller et al. provide the mean income as well as the 25th, 50th, and 75th income percentiles. To estimate intra-level inequality, I adapt R code written by [Andrie de Vries](#) to find the best-fit theoretical distribution for each hierarchical level. Intra-hierarchical level inequality is then calculated from the best-fit distribution.

Figure 2 shows how aggregate inequality within the Mueller et al. sample compares to UK data over the same period. Although the Mueller et al. data is slightly more unequal than the UK as a whole, it is a reasonably representative sample.

Hierarchical Level — Compustat Model

The Compustat model is discussed extensively in Appendix [B-G](#).

Labor and Property Income

‘Labor’ income is defined as wages and salaries, while ‘property’ income is defined as the sum of interest, dividends, rents, royalties, and estates or trust income. Mean income and within-group inequality data comes from US Census tables PINC-08 from 2003-2015.

Occupation

Data for mean income and within-group inequality by occupation comes from US Census tables PINC-06 (income by occupation of longest job) between 2007 and 2015. This table classifies occupations by major type, minor type, and detailed type. I use *detailed* categories only, which amounts to between 53 to 55 different occupation groups (depending on the year).

The US Bureau of Labor Statistics also publishes occupational wage estimates (available at <https://www.bls.gov/oes/tables.htm>). For the sake of completeness, I analyze this data here, but do not use it for the results published in the paper. The BLS data differs from Census data in the ways shown in Table 2.

Because the BLS does not report within-occupation Gini indexes directly, I estimate them via the reported values for 10th, 25th, 50th, 75th, and 90th income percentiles. Using an adaption of R code written by [Andrie de Vries](#), I fit a variety of theoretical distributions to this percentile data. Within-occupation Gini indexes are calculated from the best-fit theoretical distribution.

The resulting signal-to-noise ratio is shown in Figure 3A, alongside the results from Census occupation data. The two calculations differ starkly. Census data indicates that between-occupation inequality is *less* than within-occupation inequality; however, the BLS data indicate the *reverse*.

Which result is correct? The answer to this question depends on the type of income inequality we are interested in explaining. The BLS data covers only full-time, non-self-employed workers earning labor income. Census data, on the other hand, includes *all* individuals. For the purposes of this paper, the Census data is a better choice.

Table 2: Contrasting the US Census and BLS Occupational Income Data

Census Data	BLS Data
Includes self-employed workers	Does <i>not</i> include self-employed workers
Income for all full and part-time work	Income is for full-time equivalent workers only (hourly wage \times 2080 hours).
Includes non-labor income	Does <i>not</i> include non-labor income
53-55 detailed occupation types	700-800 detailed occupational types
Reports Gini index directly	Reports 10th, 25th, 50th, 75th and 90th income percentiles

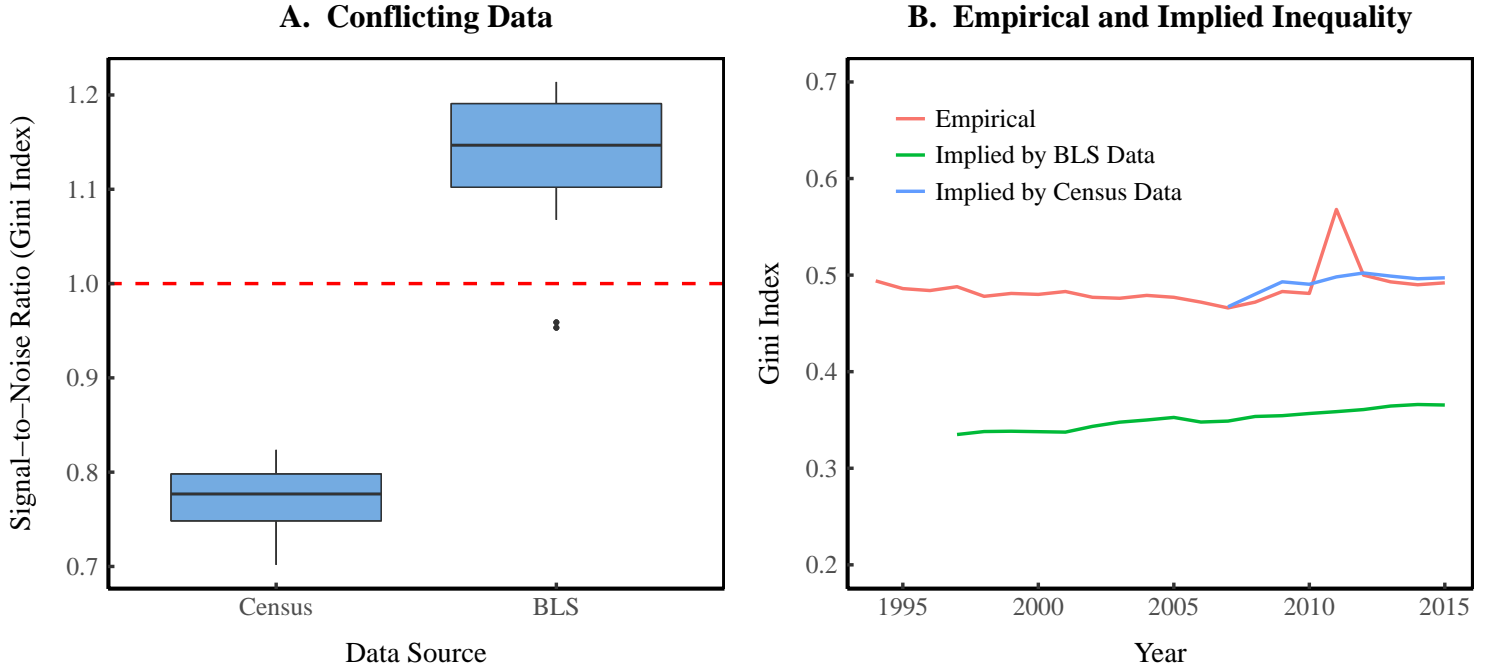


Figure 3: Inequality by Occupation — Data Discrepancies

This figure shows differences in the occupation income data published by the US Census versus that published by the US Bureau of Labor Statistics (BLS). Panel A shows calculations of the Gini index signal-to-noise ratio for both BLS and Census data. The BLS data gives a much higher signal-to-noise ratio, meaning between-occupation income dispersion is far greater (relative to within-occupation income dispersion) in BLS data than it is in the Census data. Why? The two datasets imply very different levels of aggregate (society-wide) inequality, as shown in panel B. This is because the BLS data includes only full-time wage/salary earners, while the Census data includes all individuals. The level of aggregate inequality implied by the Census data closely matches actual levels. I use Census data only in this paper.

To demonstrate the differences between BLS and Census data, we can calculate the aggregate inequality that is implied by the data. To do this, I make the simplifying assumption that all occupations have lognormal income distributions. Given the mean income (\bar{I}) and within-group Gini index (G) of a particular occupation, we can define the lognormal location (μ) and scale (σ) parameters:

$$\sigma = 2 \cdot \text{erf}^{-1}(G) \quad (12)$$

$$\mu = \ln(\bar{I}) - \frac{1}{2}\sigma^2 \quad (13)$$

If the number of individuals engaged in this occupation is n , we can create a simulated occupational income distribution by generating n values from the lognormal distribution defined by μ and σ . We repeat this process for every occupation, and then aggregate all of the simulated occupational income distributions. The Gini index of this aggregated distribution is the level of inequality that is *implied* by the data.

The results of this analysis are shown in Figure 3B. As expected, the inequality that is implied by Census data closely matches actual levels of inequality between all individuals. However, the inequality implied by BLS data is *much* lower — a clear result of the restrictions underlying the BLS methods. For the purposes of this paper, the Census data is the correct choice.

Owner vs. Renter

Mean income and within-group Gini indexes by home-ownership status come from US Census table PINC-01 between 1994 and 2015. I use the following two categories: (1) Owner Occupied; and (2) Renter Occupied.

Public vs. Private Sector

To calculate mean income and within-group Gini indexes for public and private sector workers, I use US Census table PINC-07 between 1994 and 2015. This table contains three categories: *Government Wage And Salary Workers*, *Private Wage And Salary Workers*, and *Self-Employed Workers*. Table 3 shows how I have mapped these categories onto the ‘public’ and ‘private’ sectors.

Table 3: Grouping Categories of Census Table PINC-07

Public Sector	Private Sector
• Government Wage And Salary Workers	• Private Wage And Salary Workers
	• Self-Employed Workers

The mean income and Gini index of the public sector is equivalent to the values for government wage/salary workers. Private sector mean income is calculated as the average of the means of private wage/salary worker income and self-employed worker income, weighted by the size of each group.

Since Gini indexes are not additive, I estimate the inequality of private sector income from binned data. I first add the binned income counts of both private

wage/salary workers and self-employed workers to get a binned income distribution for the private sector. From this binned data, I then use the R ‘[binequality](#)’ package to estimate private sector Gini indexes.

I construct two different estimates: one using a parametric method and the other using the midpoint method. For the parametric method, I fit various theoretical distributions to the binned data. Gini indexes are then calculated from the best-fitting distribution. The midpoint method uses midpoints of the bins to estimate the Gini index. The midpoint of the upper bin (which has an open upper bound) is estimated from a best-fit power law (again, implemented in the R [binequality](#) package).

Race

Data for mean income and within-group inequality by race comes from US Census tables PINC-01 between 1994 and 2015. Data for 2002-2015 contain the following four categories: Asian, Black, Hispanic, and White. Data for 1994-2001 contains only three categories: Black, Hispanic, and White.

Religion

Religion income data comes from the Pew Research Center 2007 U.S. [Religious Landscape Survey](#) (RLS). I use the following groups:

- Agnostic
- Atheist
- Baptist
- Buddhist
- Church of Christ, or Disciples of Christ
- Congregational or United Church of Christ
- Episcopalian or Anglican
- Hindu
- Holiness (Nazarenes, Wesleyan Church, Salvation Army)
- Jewish
- Lutheran
- Methodist
- Mormon
- Muslim
- Nondenominational or Independent Church
- Nothing in particular
- Orthodox
- Pentecostal
- Presbyterian
- Reformed (include Reformed Church in America; Christian Reformed; Calvinist)
- Roman Catholic

The RLS reports the binned income of each respondent. I use the R ‘[binequality](#)’ package to estimate group mean income and Gini indexes (using the midpoint

method). Because some religions have a very small sample size, I use the bootstrap method ([Efron and Tibshirani, 1994](#)) to estimate a plausible range of values for group mean incomes and within-group income inequality.

Sex

Data for mean income and within-group inequality by sex (male/female only) comes from US Census tables PINC-01 between 1994 and 2015.

Urban vs. Rural

Data for urban/rural mean income and within-group Gini index comes from US Census tables PINC-01 between 1994 and 2015. I define ‘urban’ as individuals inside metropolitan statistical areas, and ‘rural’ as individuals outside these areas.

B Hierarchical Structure and Pay Within Case-Study Firms

In this section I review the case-study evidence of firm hierarchy used in this paper. Table 4 summarizes the data sources, while Figure 4 shows the hierarchical employment and pay structure of these firms. The firms remain anonymous, and are named after the authors of the case studies. Although the exact shapes vary, all the firms in this sample have a roughly pyramidal employment structure and inverse pyramid pay structure.

Figure 5 dissects these trends to allow further analysis. Figure 5A shows how the span of control (the employment ratio between adjacent hierarchical levels) changes as a function of hierarchical level. In these firms, the span of control is not constant, but instead tends to *increase* with hierarchical level. Similarly, Figure 5B shows the ratio of mean pay between adjacent levels. Like the span of control, the pay ratio tends to increase with hierarchical level. Lastly, Figure 5C shows income dispersion within hierarchical ranks of each firm (measured with the Gini index). Note that income dispersion within levels is quite low and there is no evidence of a trend.

In addition to case-study data of single firms, several studies have reported the aggregate hierarchical structure of a sample of firms (see Table 5 and Figure 6). The data from these firms reveals the same general trends as the case studies. However, the aggregate data is less useful because these studies capture only the top few hierarchical ranks within firms.

From the case-study evidence, I propose the following ‘stylized’ facts about firm employment and pay structure:

1. The span of control tends to *increase* with hierarchical level.
2. The inter-level pay ratio tends to *increase* with hierarchical level.
3. Intra-level income inequality is approximately *constant* across all hierarchical levels.

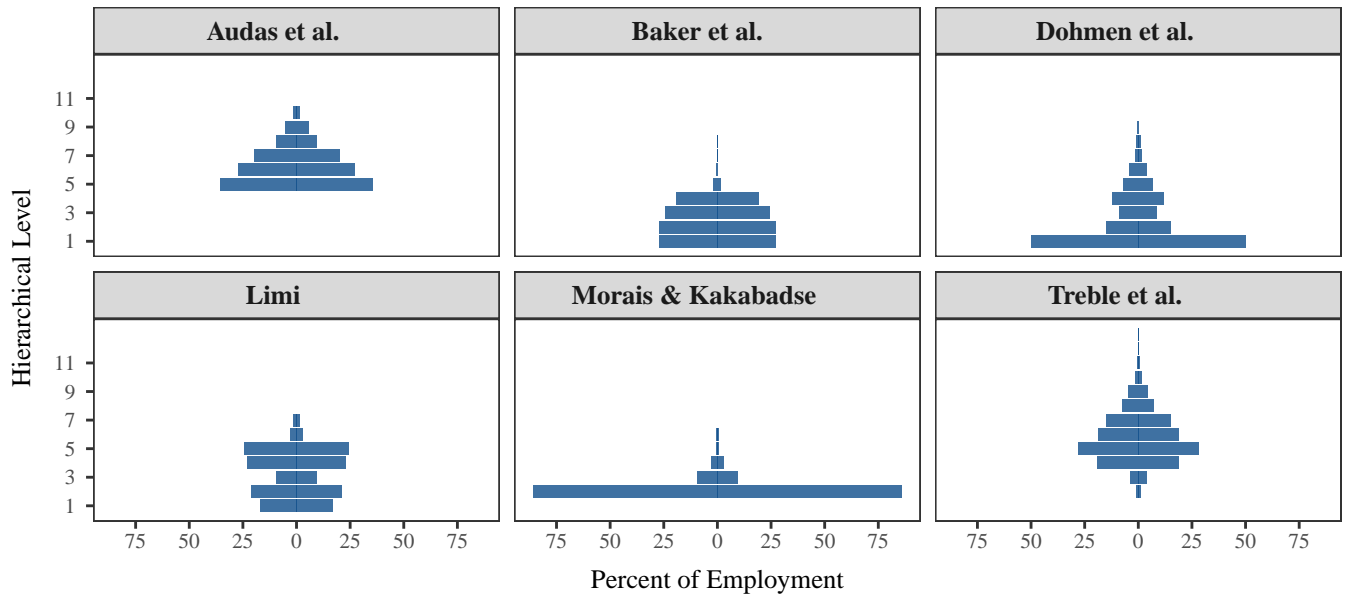
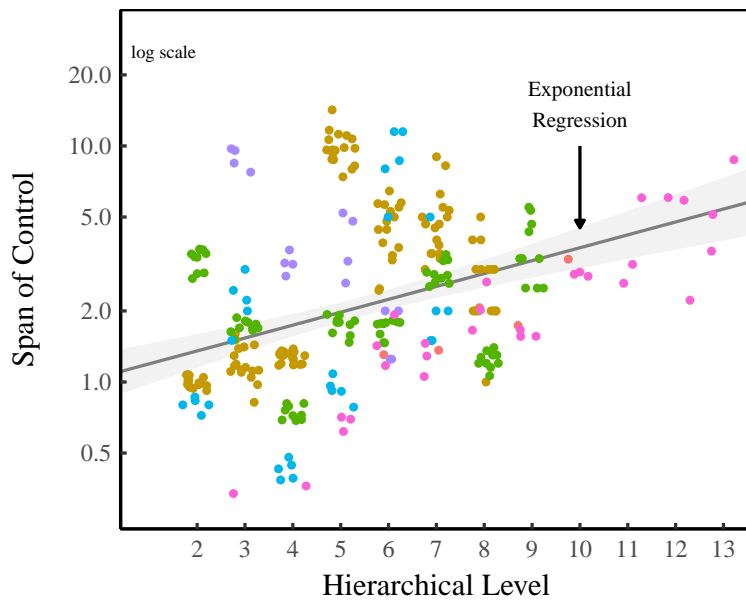
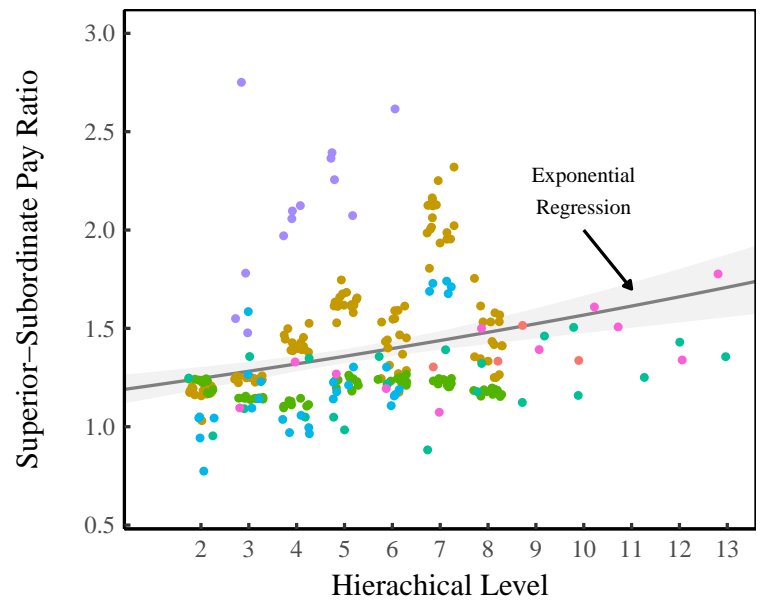
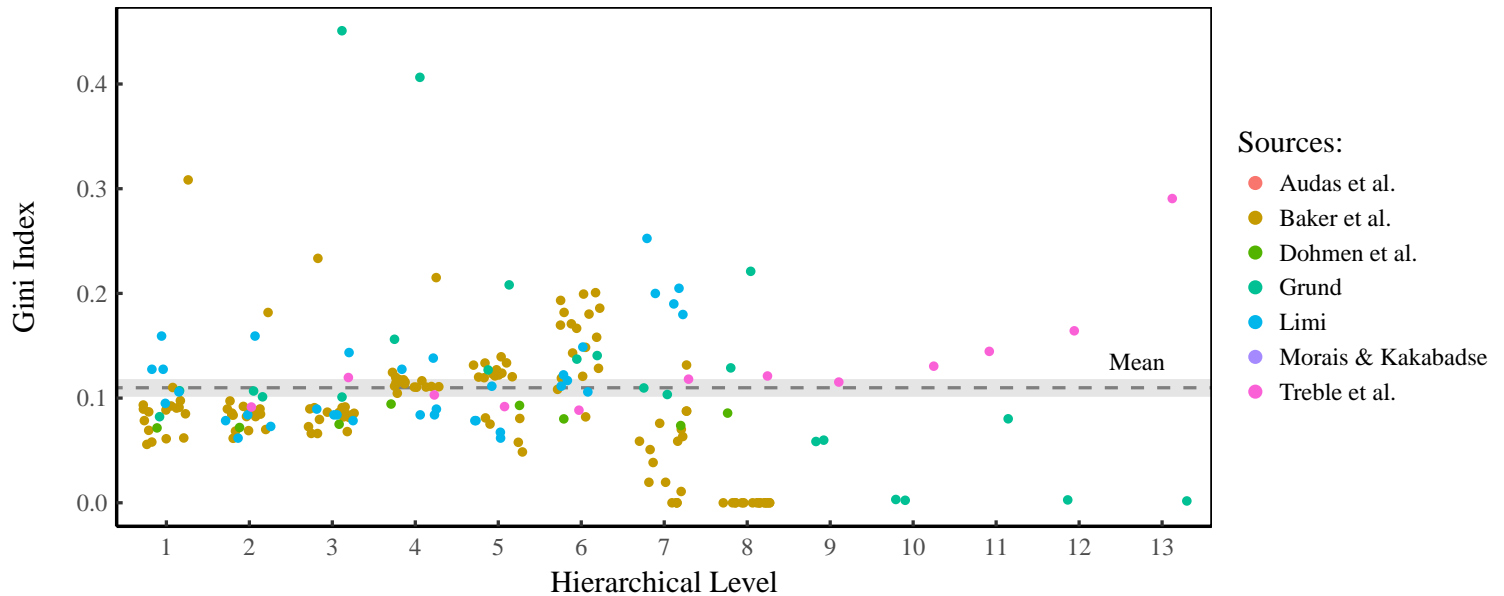
A. Firm Hierarchical Employment Structure**B. Firm Hierarchical Pay Structure**

Figure 4: The Hierarchical Employment and Pay Structure of Six Different Firms

This figure shows the hierarchical employment and pay structure of six different case-study firms. Panel A shows the hierarchical structure of employment, while Panel B shows the hierarchical pay structure.

A. Span of Control By Hierarchical Level**B. Pay Ratio By Hierarchical Level****C. Income Dispersion Within Each Level****Figure 5: Case Studies of Firm Hierarchical Structure**

This figure shows data from 7 case-study firms. Panel A shows how the span of control (the subordinate-to-superior employment ratio between adjacent levels) varies with hierarchical level. Note the log scale on the y-axis. Panel B shows how the superior-to-subordinate pay ratio varies with hierarchical level. In Panels A and B, the *x*-axis corresponds to the *upper* hierarchical level in each corresponding ratio. Panel C shows the Gini index of income inequality within each hierarchical level. Different case-study firms are indicated by color, with names indicating the study author. Note that horizontal ‘jitter’ has been introduced in all three plots in order to better visualize the data (hierarchical level is a discrete variable). The lines in Panels A and B indicate exponential regressions, while the line in Panel C shows the average Gini index. Grey regions correspond to the 95% confidence intervals.

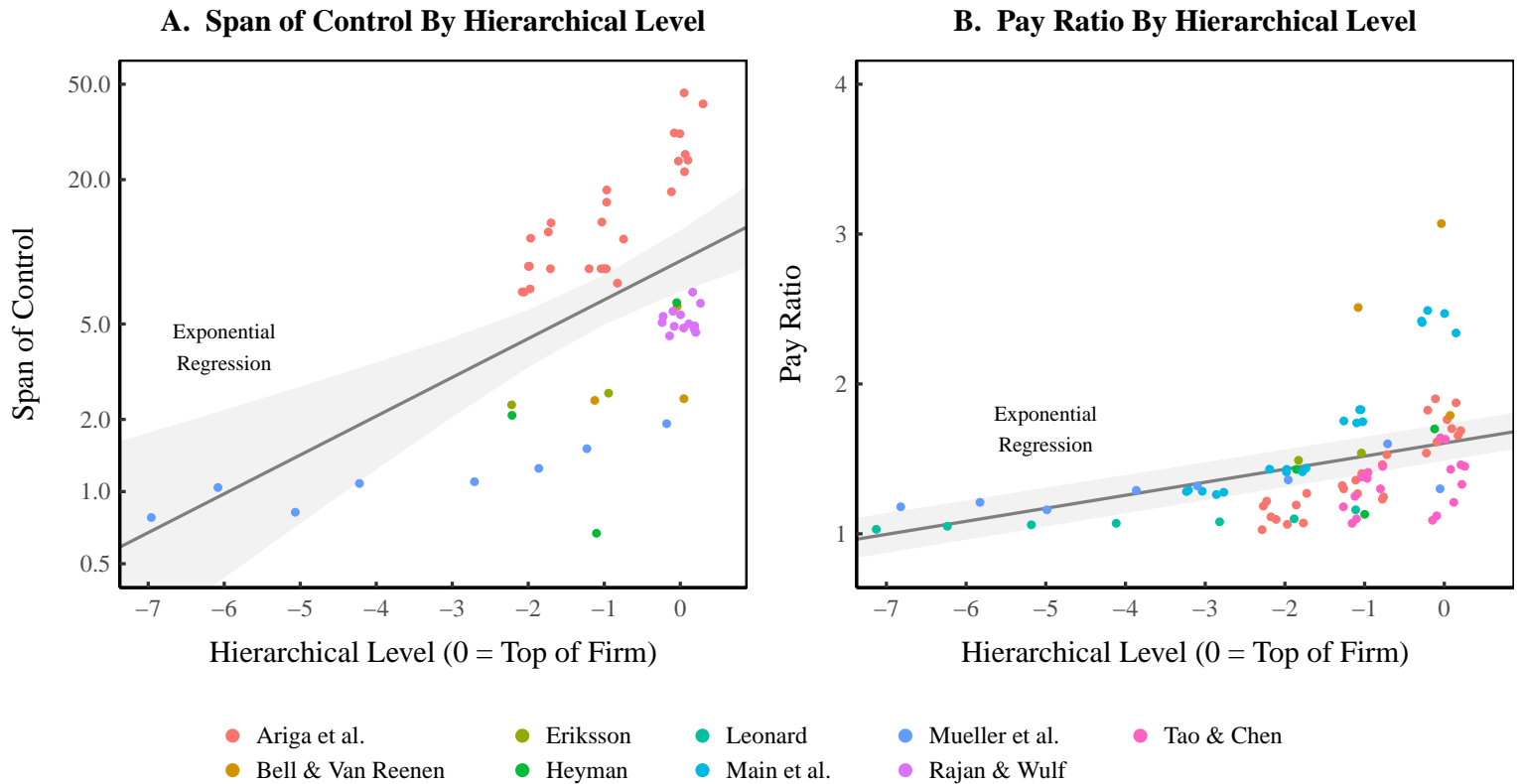


Figure 6: Aggregate Studies of Firm Hierarchical Structure

This figure shows data from 9 different aggregate firm studies. Most of these studies only survey the top several hierarchical levels in each firm. Because of this, I order hierarchical levels from the top down, where the CEO is level 0, the level below is -1, etc. Panel A shows how the span of control (the employment ratio between adjacent levels) relates to hierarchical level. Panel B shows how the pay ratio between adjacent levels varies with hierarchical level. In both plots, horizontal ‘jitter’ has been introduced in order to better visualize the data (hierarchical level is a discrete variable). Grey regions correspond to the 95% confidence interval for regressions.

Table 4: Firm Case Studies

Source	Years	Country	Firm Levels	Span of Control	Level Income	Level Income Dispersion
Audas et al. (2004)	1992	Britain	All	✓	✓	
Baker et al. (1993)	1969-1985	United States	Management	✓	✓	✓
Dohmen et al. (2004)	1987-1996	Netherlands	All	✓	✓	✓
Grund (2005)	1995 & 1998	US and Germany	All		✓	✓
Lima (2000)	1991-1995	Portugal	All	✓	✓	✓
Morais and Kakabadse (2014)*	2007-2010	Undisclosed	All	✓	✓	
Treble et al. (2001)	1989-1994	Britain	All	✓	✓	✓

Notes: This table shows metadata for the firm case studies displayed in Fig. 5. ‘Firm Levels’ refers to the portion of the firm that is included in the study. ‘Management’ indicates that only management levels were studied.

*For the analysis conducted in this paper I discard (as an outlier) the bottom hierarchical level in Morais and Kakabadse’s data.

Table 5: Firm Aggregate Studies

Source	Years	Number of Firms	Country	Firm Levels	Span of Control	Level Income
Ariga et al. (1992)	1981-1989	unknown	Japan	All	✓	✓
Bell and Van Reenen (2012)	2001-2010	552	United Kingdom	Top 3	✓	✓
Eriksson (1999)	1992-1995	210	Denmark	Management	✓	✓
Heyman (2005)	1991,1995	560	Sweden	Management	✓	✓
Leonard (1990)	1981-1985	439	United States	Top 9		✓
Main et al. (1993)	1980-1984	200	United States	Top 4		✓
Mueller et al. (2016)	2004-2013	880	United Kingdom	All	✓	✓
Rajan and Wulf (2006)	1986-1998	261	United States	Top 2	✓	
Tao and Chen (2009)	1986-1998	8101	Taiwan	Top 2		✓

Notes: This table shows metadata for the aggregate studies displayed in Fig. 6. ‘Firm Level’s refers to the portion of the firm that is included in the study. ‘Top 2’, ‘Top 3’, etc. indicates that only the top n levels were included in the study (where the top level is the CEO).

Table 6: Income Inequality Within Case Study Firms

Source	Years	Mean Gini Index
Baker et al. (1993)	1969-1985	0.32
Dohmen et al. (2004)	1991	0.18
Lima (2000)	1991-1995	0.15
Morais and Kakabadse (2014)	2007-2010	0.23
Treble et al. (2001)	1989-1997	0.26

B.1 Inequality Within Case Study Firms

I report here my estimates for inequality within the case study firms. Of the seven case studies summarized in Table 4, only one (Morais and Kakabadse) directly reports a firm Gini index. However, four other studies — Baker et al., Dohmen et al., Lima, and Treble et al. — provide enough data to allow estimates of firm internal inequality. I outline my calculation methods below. The resulting Gini estimates are shown in Table 6.

Baker et al.

Baker et al. (1993) have made their raw personnel data publicly available at the site below. I use this raw data to calculate the firm internal Gini index.

<http://faculty.chicagobooth.edu/michael.gibbs/research/index.html>

Dohmen et al.

Dohmen et al. (2004) report the following data that I use to estimate the firm Gini index:

1. Fraction of employment by hierarchical level (Tbl. 1);
2. Density plots of income distribution by hierarchical level (Fig. 5).

I use the *Engauge Digitizer* program to digitize and pull data from the density plots. I then use the resulting numerical density functions to estimate the firm Gini index.

We define $f_h(x)$ as the income density function for hierarchical level h . The income density function of the entire firm $f_T(x)$ is then defined by Eq. 14 – the sum

of the density functions for each hierarchical level, weighted by the fraction of total employment (E_h/E_T).

$$f_T(x) = \sum_{h=1}^n \frac{E_h}{E_T} \cdot f_h(x) \quad (14)$$

The firm Gini index is then defined by Eq. 15-17. Equation 15 defines the mean income of the firm (\bar{I}), while equation 16 defines the cumulative income distribution function $F(x)$. Equation 17 then defines the Gini index (G). I use numerical integration implemented in R to evaluate these integrals.

$$\bar{I} = \int_0^{\infty} x \cdot f_T(x) dx \quad (15)$$

$$F(x) = \int_0^{\infty} f_T dx \quad (16)$$

$$G = \frac{1}{\bar{I}} \int_0^{\infty} F(x)(1 - F(x))dx \quad (17)$$

Grund

Grund (2005) does not provide enough information to calculate firm-wide inequality. However, I am able to calculate intra-level income dispersion, (which appears in Fig. 5C). I use data from Grund's Fig. 1, which shows mean income by level, as well as what I assume to be 5th and 95th percentiles. After digitizing this data, I use the best-fit theoretical distribution to estimate the Gini index.

Lima

Lima (2000) provides the following summary statistics, which I use to estimate a firm Gini index:

1. Employment within each hierarchical level (Tbl. 1);
2. Mean pay within each hierarchical level (Fig. 2);
3. Wage coefficient of variation by hierarchical level (Tbl. 6).

I use the *Engauge Digitizer* program to digitize and pull data from Fig. 2. To calculate the firm Gini index, I assume income within each hierarchical level is lognormally distributed. For each hierarchical level h , I then use equation 18

to define the lognormal scale parameter σ that produces a distribution with an equivalent coefficient of variation, c_v :

$$\sigma_h = \sqrt{\ln(c_v^2 + 1)} \quad (18)$$

Once we have σ_h , we use equation 19 to calculate the lognormal location parameter μ for each hierarchical level . Here \bar{I}_h is the mean pay in hierarchical level h (which Lima reports directly).

$$\mu_h = \ln(\bar{I}_h) - \frac{1}{2}\sigma_h^2 \quad (19)$$

Once we have the appropriate lognormal parameters for each hierarchical level, we use these distributions to create a simulated payroll. To do this, we draw E_h numbers (employment in level h) from each lognormal distribution $\ln \mathcal{N}(\mu_h, \sigma_h)$. I then calculate the Gini index from this simulated payroll.

Treble et al.

Treble et al. (2001) report the following summary statistics, which I use to estimate a firm Gini index:

1. Employment within each hierarchical level (Fig. 2);
2. Mean pay within each hierarchical level (Fig. 3);
3. 5th and 95th wage percentile by hierarchical level (Fig. 4).

Again, I use *Engauge Digitizer* to pull data from all graphs. To estimate the intra-level Gini index, I adapt code writtent by [Andrie de Vries](#) to fit a parameterized distribution to the mean and 5th/95th percentiles.

C The Compustat Data

To model the hierarchical structure of US firms, I use the Compustat data series shown in Table 7. Selected statistics from this dataset are shown in Figure 7. For each Compustat firm (in each year), I calculate the following statistics:

$$\text{Employee Mean Income} = \frac{\text{Total Staff Expenses}}{\text{Employees}} \quad (20)$$

$$\text{CEO Pay Ratio} = \frac{\text{Top Exec Pay}}{\text{Employee Mean Income}} \quad (21)$$

Note that ‘CEO pay’ is defined as the income of the top-paid executive in a given firm.

Table 7: Compustat Data Series

Database	Series ID	Description
ExecuComp	TDC1	Executive Total Compensation
Fundamentals Annual	XLR	Total Staff Expenses
Fundamentals Annual	EMP	Employees

Notes: Executive compensation series TDC1 = Salary + Bonus + Other Annual + Restricted StockGrants + LTIPPayouts + All Other + Value of Option Grants

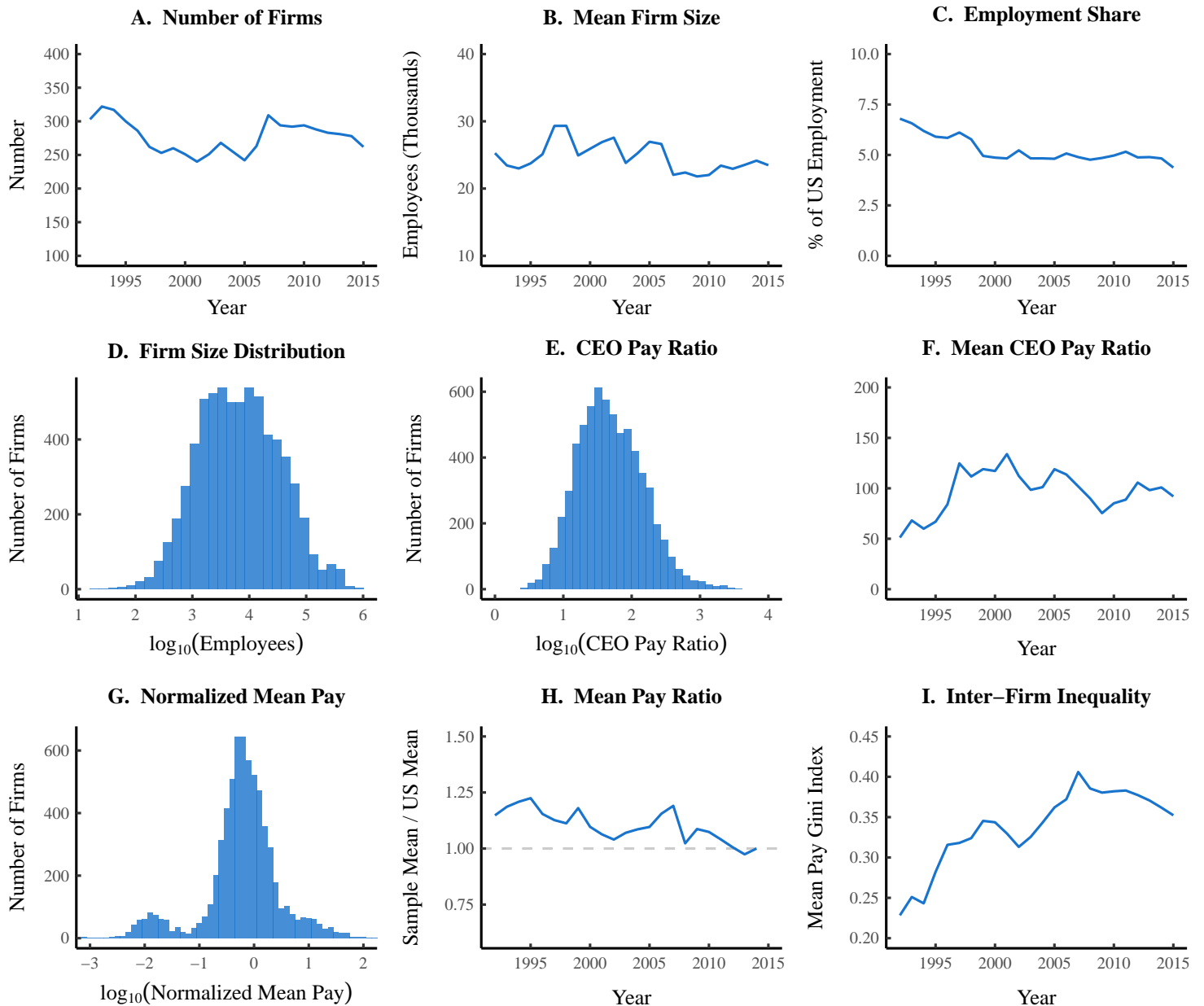


Figure 7: Selected Statistics from the Compustat Firm Sample

This figure shows statistics for the Compustat firm sample, which consists of US firms for which the data series in Table 7 are available. In panel H, US mean income per worker is calculated from national accounts (BEA Table 1.12, National Income by Type of Income) by dividing the sum of employee and proprietor income by the number of workers (BEA Table 6.8C-D, persons engaged in production).

D A Hierarchy Algorithm

In this section, I outline the mathematics underlying my hierarchical model of the firm. The model assumptions, outlined below, are based on the stylized facts gleaned from the real-world firm data in section B.

1. Firms are hierarchically structured, with a span of control that increases *exponentially* with hierarchical level.
2. The ratio of mean pay between adjacent hierarchical levels increases *exponentially* with hierarchical level.
3. Intra-hierarchical-level income is lognormally distributed and constant across all levels.

Using these assumptions, I first develop an algorithm that describes the hierarchical employment within a model firm, followed by an algorithm that describes the hierarchical pay structure.

Table 8: Notation

Symbol	Definition
a	span of control parameter 1
b	span of control parameter 2
C	CEO to average employee pay ratio
E	employment
F	cumulative distribution function
G	Gini index of inequality
h	hierarchical level
\bar{I}	average income
μ	lognormal location parameter
n	number of hierarchical levels in a firm
p	pay ratio between adjacent hierarchical levels
r	pay-scaling parameter
s	span of control
σ	lognormal scale parameter
T	total for firm
\downarrow	round down to nearest integer
\prod	product of a sequence of numbers
\sum	sum of a sequence of numbers

D.1 Generating the Employment Hierarchy

To generate the hierarchical structure of a firm, we begin by defining the span of control (s) as the ratio of employment (E) between two consecutive hierarchical levels (h), where $h = 1$ is the *bottom* hierarchical level. It simplifies later calculations if we define the span of control in level 1 as $s = 1$. This leads to the following piecewise function:

$$s_h \equiv \begin{cases} 1 & \text{if } h = 1 \\ \frac{E_h}{E_{h-1}} & \text{if } h \geq 2 \end{cases} \quad (22)$$

Based on our empirical findings in Section B, we assume that the span of control is *not* constant; rather it increases *exponentially* with hierarchical level. I model the span of control as a function of hierarchical level (s_h) with a simple exponential function, where a and b are free parameters:

$$s_h = \begin{cases} 1 & \text{if } h = 1 \\ a \cdot e^{bh} & \text{if } h \geq 2 \end{cases} \quad (23)$$

As one moves up the hierarchy, employment in each consecutive level (E_h) *decreases* by $1/s_h$. This yields Eq. 24, a recursive method for calculating E_h . Since we want employment to be *whole* numbers, we round down to the nearest integer (notated by \downarrow). By repeatedly substituting Eq. 24 into itself, we can obtain a non-recursive formula (Eq. 25). In product notation, Eq. 25 can be written as Eq. 26.

$$E_h = \downarrow \frac{E_{h-1}}{s_h} \quad \text{for } h > 1 \quad (24)$$

$$E_h = \downarrow E_1 \cdot \frac{1}{s_2} \cdot \frac{1}{s_3} \cdot \dots \cdot \frac{1}{s_h} \quad (25)$$

$$E_h = \downarrow E_1 \prod_{i=1}^h \frac{1}{s_i} \quad (26)$$

Total employment in the whole firm (E_T) is the sum of employment in all hierarchical levels. Defining n as the total number of hierarchical levels, we get Eq. 27, which in summation notation, becomes Eq. 28.

$$E_T = E_1 + E_2 + \dots + E_n \quad (27)$$

$$E_T = \sum_{h=1}^n E_h \quad (28)$$

In practice, n is not known beforehand, so we define it using Eq. 26. We progressively increase h until we reach a level of zero employment. The highest level n will be the hierarchical level directly *below* the first hierarchical level with zero employment:

$$n = \{h \mid E_h \geq 1 \text{ and } E_{h+1} = 0\} \quad (29)$$

To summarize, the hierarchical employment structure of our model firm is determined by 3 free parameters: the span of control parameters a and b , and base-level employment E_1 .

D.2 Generating Hierarchical Pay

To model the hierarchical pay structure of a firm, we begin by defining the inter-hierarchical pay-ratio (p_h) as the ratio of mean income (\bar{I}) between adjacent hierarchical levels. Again, it is helpful to use a piecewise function so that we can define a pay-ratio for hierarchical level 1:

$$p_h \equiv \begin{cases} 1 & \text{if } h = 1 \\ \frac{\bar{I}_h}{\bar{I}_{h-1}} & \text{if } h \geq 2 \end{cases} \quad (30)$$

Based on our empirical findings in Section B, we assume that the pay ratio increases *exponentially* with hierarchical level. I model this relation with the following function, where r is a free parameter:

$$p_h = \begin{cases} 1 & \text{if } h = 1 \\ r^h & \text{if } h \geq 2 \end{cases} \quad (31)$$

Using the same logic as with employment (shown above), the mean income I_h in any hierarchical level is defined recursively by Eq. 32 and non-recursively by Eq. 33.

$$\bar{I}_h = \frac{\bar{I}_{h-1}}{p_h} \quad (32)$$

$$\bar{I}_h = \bar{I}_1 \prod_{i=1}^h p_i \quad (33)$$

To summarize, the hierarchical pay structure of our model firm is determined by 2 free parameters: the pay-scaling parameter r , and mean pay in the base level (\bar{I}_1).

D.2.1 Useful Statistics

Two statistics are used repeatedly within the model: mean firm pay, and the CEO-to-average-employee pay ratio. Mean income for all employees (\bar{I}_T) is equal to the average of hierarchical level mean incomes (\bar{I}_h) weighted by the respective hierarchical level employment (E_h):

$$\bar{I}_T = \sum_{h=1}^n \bar{I}_h \cdot \frac{E_h}{E_T} \quad (34)$$

To calculate the CEO pay ratio, we define the CEO as the person in the top hierarchical level. Therefore, CEO pay is simply \bar{I}_n , average income in the top hierarchical level. The CEO pay ratio (C) is then equal to CEO pay divided by average pay:

$$C = \frac{\bar{I}_n}{\bar{I}_T} \quad (35)$$

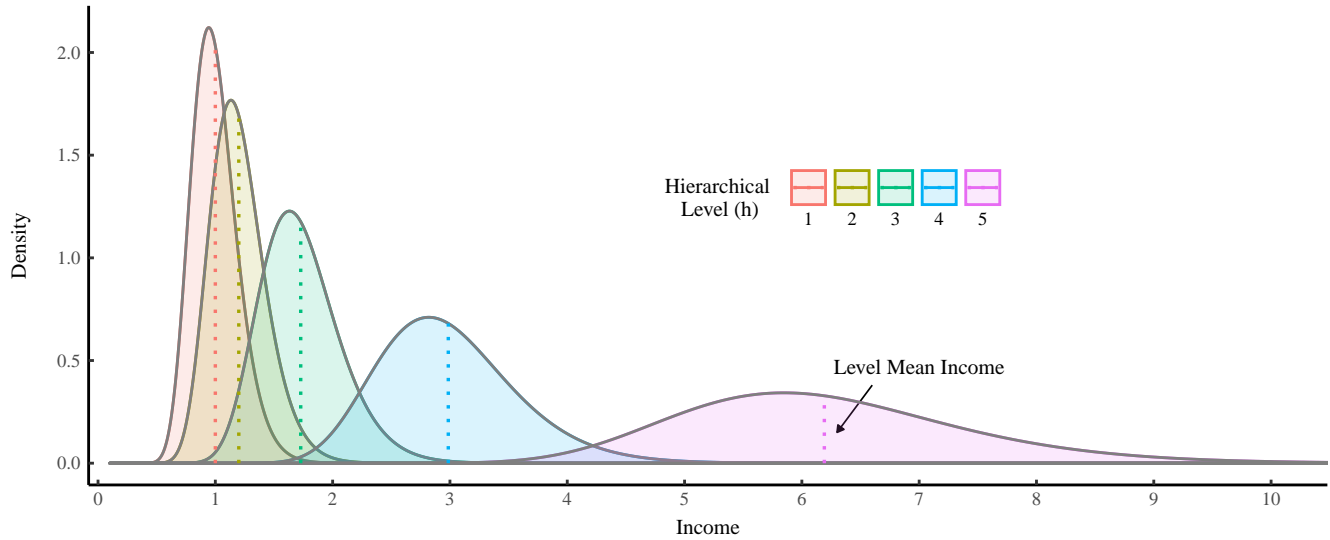
D.3 Adding Intra-Level Pay Dispersion

Up to this point, we have modeled only the *mean* income within each hierarchical level of a firm. The last step in the modeling process is to add pay *dispersion* within each hierarchical level.

I assume that pay dispersion within hierarchical levels is *lognormally* distributed. The lognormal distribution is defined by location parameter μ and scale parameter σ . Our empirical investigation of firm case studies indicated that pay dispersion with hierarchical levels is relatively constant (see Fig. 5C). Given this finding, I assume *identical inequality* within all hierarchical levels. This means that the lognormal scale parameter σ is the same for all hierarchical levels.

In order to add dispersion within each hierarchical level, I multiply mean pay \bar{I}_h by a lognormal random variate with an expected mean of one. Formally, this is represented by Eq. 36. Since the mean of a lognormal distribution is equal to

A. Pay Dispersion Within Each Hierarchical Level of a Firm



B. Relative Contribution to Within-Firm Income Distribution

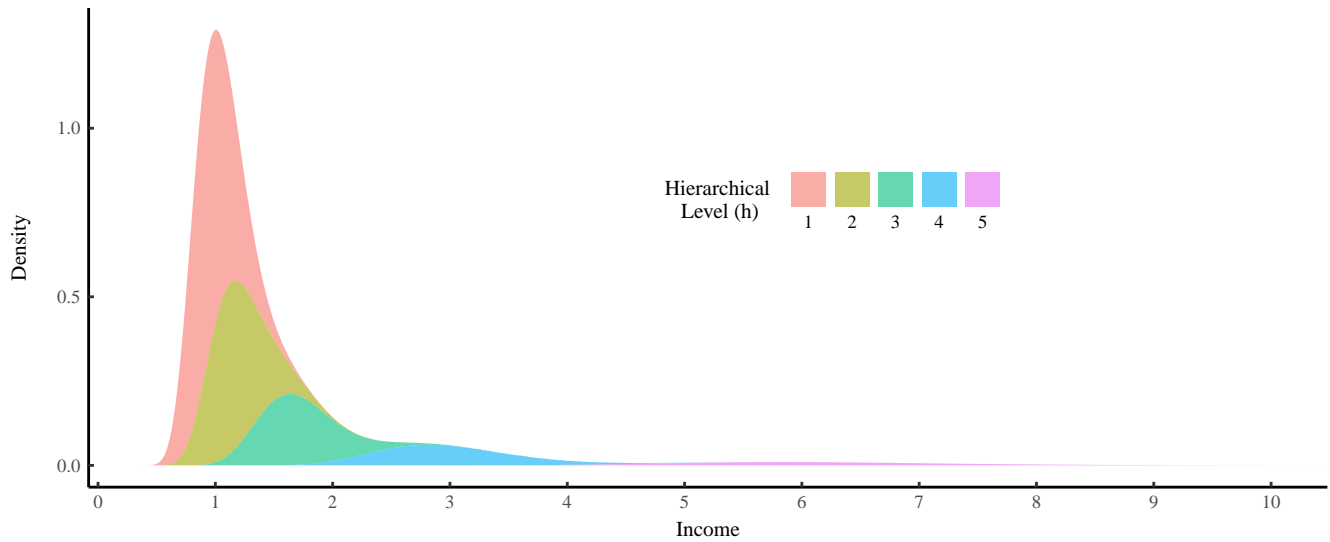


Figure 8: Adding Intra-Level Pay Dispersion to a Model Firm

This illustrates a model firm with lognormal pay dispersion in each hierarchical level. Panel A shows the separate distributions for each level, with mean income indicated by a dashed vertical line. Panel B shows contribution of each hierarchical level to the resulting income distribution for the whole firm (income density functions are summed while weighting for their respective employment).

$e^{\mu + \frac{1}{2}\sigma^2}$, I leave it to the reader to show that a mean of one requires that μ be defined by Eq. 37.

$$I_h = \bar{I}_h \cdot \ln \mathcal{N}(\mu, \sigma) \quad (36)$$

$$\mu = -\frac{1}{2}\sigma^2 \quad (37)$$

Given a value for σ (which is a free parameter), we can define the pay distribution within any hierarchical level of a firm. This process is shown graphically in Figure 8. Figure 8A shows the lognormal income distributions for each hierarchical level of a 5-level firm. Figure 8B shows the size-adjusted contribution of each hierarchical level to the overall intra-firm income distribution. Lower levels have more members, and thus dominate the overall distribution.

E The Compustat Model

The Compustat model takes the available firm case-study data and uses it to estimate the hierarchical structure of US firms in the Compustat database. The model assumes that each Compustat firm has an employment hierarchy shaped like Figure 9. This is the shape that is implied by fitting a trend to the span of control data for case-study firms (Fig. 5A). This restricting assumption allows us to use the available data on firm employment, mean pay, and CEO pay to estimate the hierarchical pay structure of each Compustat firm. The idea is that when the hierarchical employment structure of a firm is pre-specified (using case-study data), the CEO pay ratio allows us to calculate how income increases by hierarchical level. Of course, for any specific firm, this estimation method is likely not that accurate. However, the hope is that on average, this model provides insights into the hierarchical structure of Compustat firms.

The Compustat model uses the algorithms discussed in Appendix D with parameters summarized in Table 9. In the following sections, I outline how I restrict each model parameter.

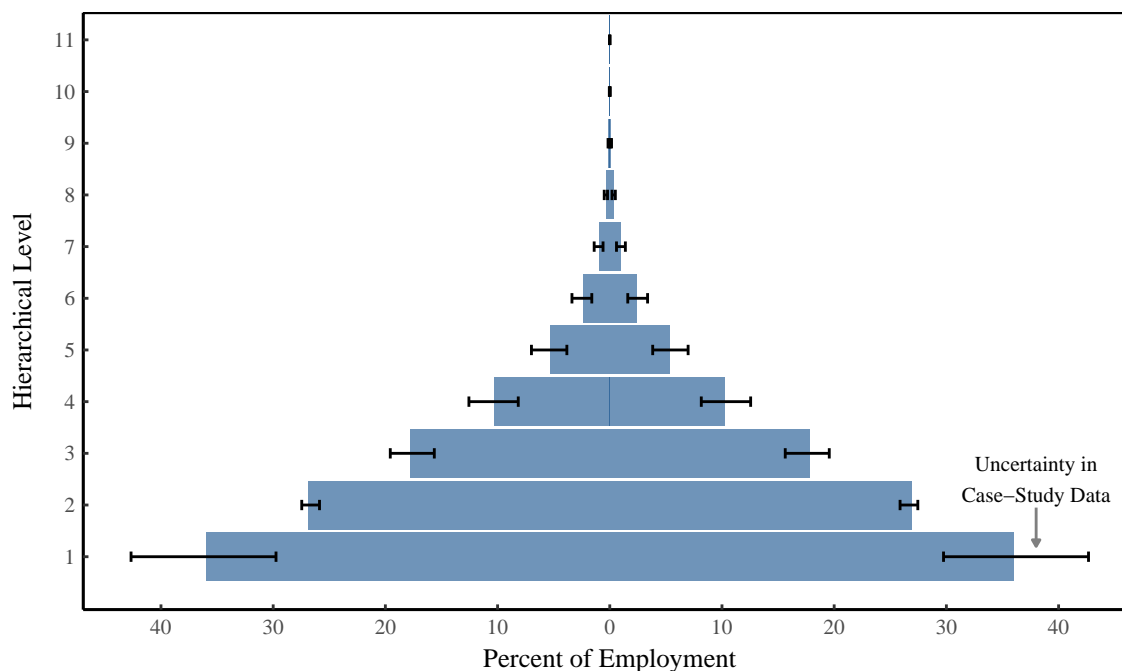


Figure 9: Idealized Firm Employment Hierarchy Implied by Case Studies

This figure shows the idealized firm hierarchy that is implied by fitting trends to case-study data (Fig. 5A). Error bars show the uncertainty in the hierarchical shape, calculated using a bootstrap resample of case-study data.

Table 9: Model Parameters

Parameter	Definition	Action	Scope
a, b	Span of control parameters	Determines the shape of the firm hierarchy.	Identical for all firms.
E_1	Employment in base hierarchical level	Used to build the employment hierarchy from the bottom up. Determines total employment.	Specific to each firm.
r	Pay-scaling parameter	Determines the rate at which mean income (within a firm) increases by hierarchical level.	Specific to each firm.
\bar{I}_h	Mean pay in base hierarchical level	Sets the base level income of the firm, which determines firm average pay.	Specific to each firm.
σ	Intra-hierarchical level pay dispersion parameter	Determines the level of inequality within hierarchical levels of a firm.	Identical for all firms.

E.1 Span of Control Parameters

The parameters a and b together determine the shape of firm employment hierarchy. These parameters are estimated from an exponential regression on case-study data (Fig. 5A). The model assumes that parameters a and b are *constant* for all Compustat firms.

Because the case-study sample size is small, there is considerable uncertainty in these values. I incorporate this uncertainty into the model using the *bootstrap* method (Efron and Tibshirani, 1994), which involves repeatedly resampling the case-study data (with replacement) and then estimating the parameters a and b from this resample. Figure 10 shows the probability density distribution resulting from this bootstrap analysis. I run the Compustat model many times, each time with parameters a and b determined by a bootstrap resample of case-study data.

E.2 Base-Level Employment

Given span of control parameters a and b , each Compustat firm hierarchy is constructed from the bottom hierarchical level up. Therefore, we must know base-level employment in each firm. However, the Compustat database provides data for *total* firm employment only. To estimate base-level employment, I use the model to re-

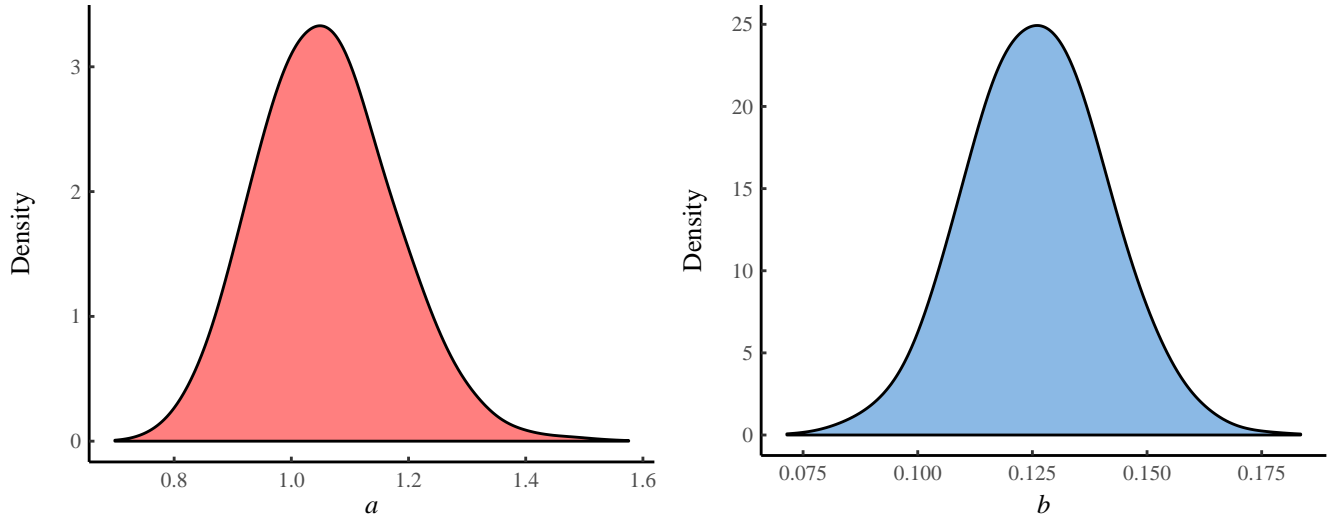


Figure 10: Density Estimates for Span of Control Parameters

This figure shows density estimates for the parameters a and b , which together determine the ‘shape’ of the firm hierarchy. These parameters are determined from regressions on firm case-study data (Fig. 5). The density functions are estimated using a bootstrap analysis, which involves resampling (with replacement) the case-study data many times, and calculating the parameters a and b for each resample.

verse engineer the problem. I input a range of different base employment values into equations 23, 26, and 28 and calculate total employment for each value. The result is a discrete mapping relating base-level employment to total employment. I then fit a high order polynomial to this relation. This function then allows us to predict base-level employment E_1 , given total employment E_T , and span parameters a and b .

E.3 Pay-Scaling Parameter

The pay-scaling parameter r determines the rate at which mean pay increases by hierarchical level. Unlike the span of control parameters, I allow the pay-scaling parameter to vary between firms. I fit the pay-scaling parameter r to each Compustat firm using the CEO-to-average-employee pay ratio (C). The first step of this process is to build the employment hierarchy for each Compustat firm using parameters a , b , and E_1 (the latter is determined from total employment). Given this hierarchical employment structure, the CEO pay ratio in the modeled firm is uniquely determined by the parameter r . Thus, we simply choose r such that the model produces a CEO pay ratio that is equivalent to the empirical ratio.

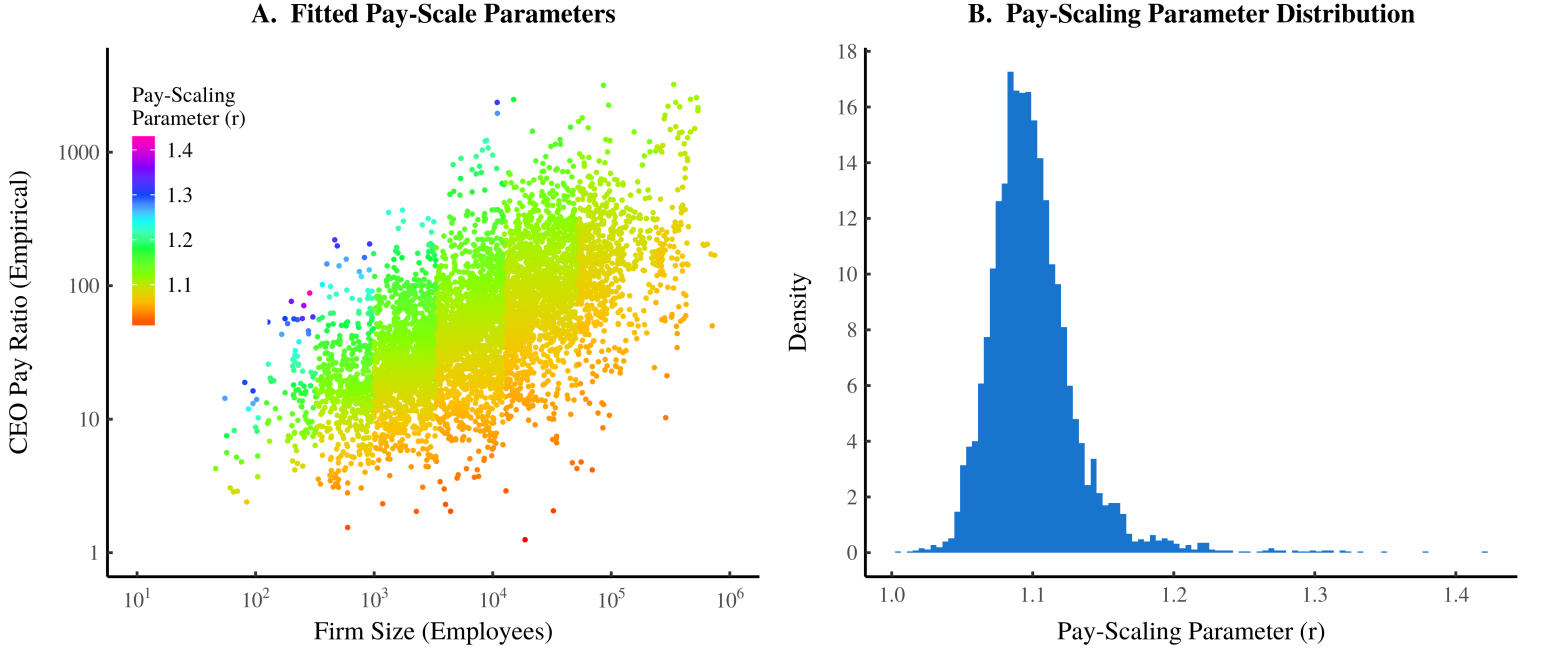


Figure 11: Fitting Compustat Firms with a Pay-Scaling Parameter

This figure shows the fitted pay-scaling parameters (r) for all Compustat firms. Panel A shows the relation between the CEO pay ratio and firm size, with the fitted pay-scaling parameter indicated by color. The discrete changes in color (evident as vertical lines) correspond to changes in the number of hierarchical levels within firms. The pay-scaling parameter distribution for all firms (and years) is shown in Panel B.

To solve for this r value, I use numerical optimization to minimize the error function shown in Eq. 38. Here $C_{\text{Compustat}}$ and C_{model} are Compustat and modeled CEO pay ratios, respectively.

$$\epsilon(r) = |C_{\text{model}} - C_{\text{Compustat}}| \quad (38)$$

For each firm, the fitted value of r minimizes this error function. To ensure that there are no large errors, I discard Compustat firms for which the best-fit r parameter produces an error that is larger than $\epsilon = 0.01$). Example fitted results for r are shown in Figure 11.

E.4 Base-Level Mean Pay

Having already fitted a hierarchical pay structure to each Compustat firm (in the process of finding r), we can use this data to estimate base pay for each firm. To do

this, we set up a ratio between base-level pay (\bar{I}_1) and firm mean pay (\bar{I}_T) for both the model and Compustat data:

$$\frac{\bar{I}_1^{\text{Compustat}}}{\bar{I}_T^{\text{Compustat}}} = \frac{\bar{I}_1^{\text{model}}}{\bar{I}_T^{\text{model}}} \quad (39)$$

The modeled ratio between base pay and firm mean pay ($\bar{I}_1^{\text{model}}/\bar{I}_T^{\text{model}}$) is *independent* of the choice of base pay. This is because the modeled firm mean pay is actually a *function* of base pay (see Eq. 33 and 34). If we run the model with $\bar{I}_1^{\text{model}} = 1$, then Eq. 39 reduces to:

$$\frac{\bar{I}_1^{\text{Compustat}}}{\bar{I}_T^{\text{Compustat}}} = \frac{1}{\bar{I}_T^{\text{model}}} \quad (40)$$

We can then rearrange Eq. 40 to solve for an estimated base pay for each Compustat firm ($\bar{I}_1^{\text{Compustat}}$):

$$\bar{I}_1^{\text{Compustat}} = \frac{\bar{I}_T^{\text{Compustat}}}{\bar{I}_T^{\text{model}}} \quad (41)$$

E.5 Intra-Hierarchical Level Income Dispersion

Intra-hierarchical level income dispersion within Compustat firms is modeled with a lognormal distribution, with the amount of inequality determined by the scale parameter σ . The model assumes that σ is constant for all hierarchical levels within all firms.

I estimate σ from the case-study data shown in Figure 5C. This data uses the Gini index as the metric for dispersion. To estimate σ , we first calculate the mean Gini index of all data (\bar{G}). We then use Eq. 42 to calculate the value σ . This equation corresponds to the lognormal scale parameter that would produce a lognormal distribution with an equivalent Gini index. This equation is derived from the definition of the Gini index of a lognormal distribution: $G = \text{erf}(\sigma/2)$.

$$\sigma = 2 \cdot \text{erf}^{-1}(\bar{G}) \quad (42)$$

Because the case-study sample size is small, there is considerable uncertainty in these values. I quantify this uncertainty using the *bootstrap* method (Efron and Tibshirani, 1994), which involves repeatedly resampling the case-study data (with replacement) and then estimating the parameter σ from this resampled data.

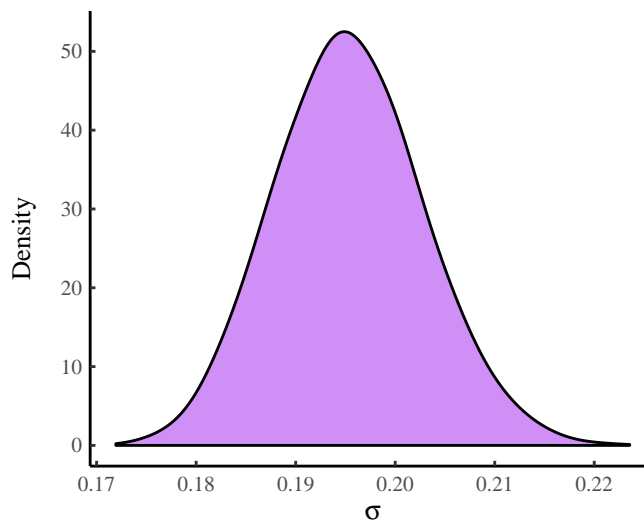


Figure 12: Density estimates for Intra-Hierarchical Level Pay Dispersion Parameter σ

This figure shows the distribution of the lognormal scale parameter σ , which determines pay dispersion within all hierarchical levels of all firms. The distribution is calculated using the bootstrap method.

Figure 12 shows the probability density distribution resulting from this bootstrap analysis. In order to incorporate this uncertainty, I run the model many times, with each run using a different bootstrapped value for σ .

E.6 Visualizing the Compustat Model

In order to aid the intuitive understanding of the Compustat model, Figure 13 visualizes the model in landscape form. Here I show selected firms from the year 2010. Each pyramid represents a separate firm with volume proportional to total employment. The vertical axis corresponds to hierarchical level. Income is indicated by color.

Year: 2010

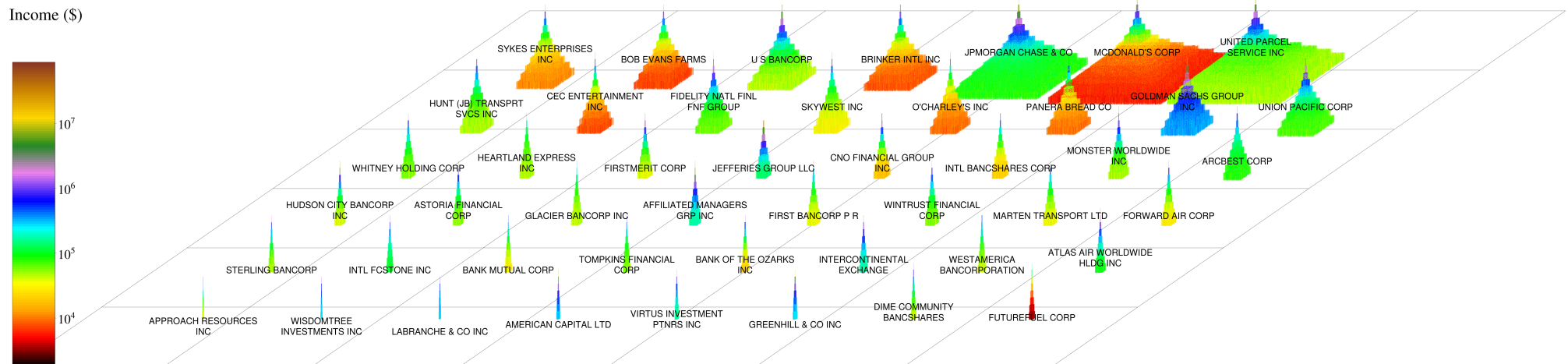


Figure 13: Visualizing the Compustat Model

This figure visualizes the results of the Compustat Model for selected US firms in the year 2010. Each pyramid represents a separate firm with volume proportional to total employment. The vertical axis corresponds to hierarchical level. Income is indicated by color.

F Compustat Model Results

I review here the results of the Compustat model that are not discussed in the main paper. All results are generated using 5000 bootstrap model runs over different values for the parameters a , b , and σ . I review here the following: (1) estimates for income inequality within Compustat firms; (2) estimates for income by hierarchical level; and (3) aggregate inequality of all firms in the model.

F.1 Inequality Within Compustat Firms

Figure 14 shows estimates of income inequality within Compustat firms. In Figure 14A, I illustrate how firm Gini indexes are related to both the CEO pay ratio and firm size. Note that the CEO pay ratio is a reliable indicator of firm inequality only for firms of the *same size*. A general feature of a hierarchical firm model is that when internal inequality is held constant, the CEO pay ratio nonetheless tends to increase with firm size (a feature first demonstrated by Herbert Simon (1957)). In Figure 14A, this feature is evident as color contours of constant within-firm Gini indexes that scale with both firm size and the CEO pay ratio.

Figure 14B shows the overall distribution of all firm Gini indexes. According to our model, 90% of Compustat firms have internal Gini indexes between 0.2 and 0.5. Note that the distribution is right-skewed — a small minority of firms have extremely unequal pay.

In Figure 14C I compare firm inequality in the Compustat model to inequality within the case-study firms discussed in Appendix B. The results indicate that Compustat firms are slightly more unequal than the case-study firms. However, because the case-study sample size is small, this difference is not statistically significant. A Kolmogorov-Smirnov test gives a p-value of 0.20, indicating that there is a reasonable (20%) probability that the two firm samples (model and case study) come from the *same* distribution. Thus, under the conventional 5% significance level, we cannot reject the null-hypothesis that these samples come from the same distribution.

Figure 14D shows the time evolution of average inequality within Compustat firms. During the late 1990s inequality rapidly increased, followed by relative stability from 2000 onward. While the *trend* is clear, there is significant uncertainty in the *absolute* level of inequality (as indicated by the shaded region). This uncertainty is due to the small case-study sample size on which key model parameters are based (see Appendix E).

Finally, Figure 15 shows Gini index estimates for the 50 most equal and 50

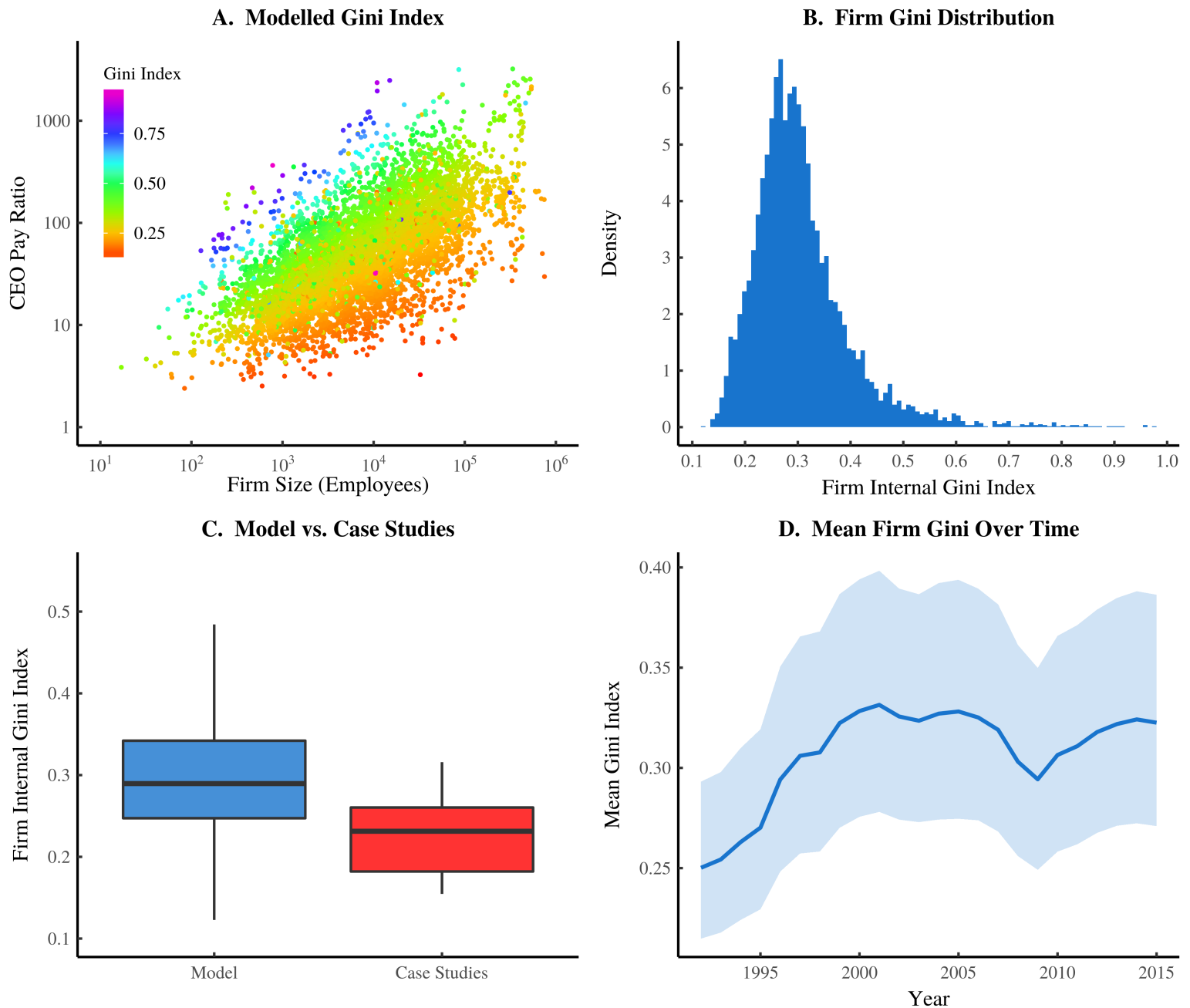
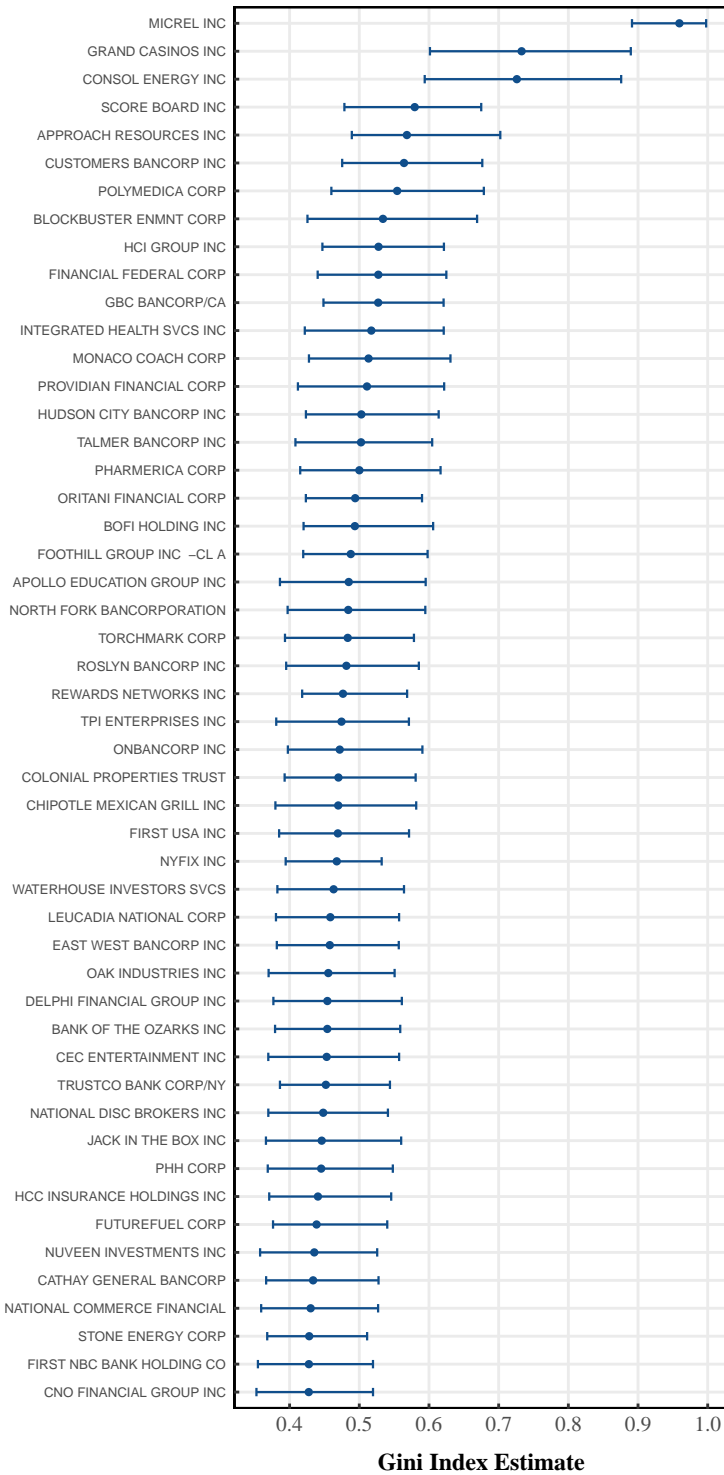


Figure 14: Compustat Model Results for Intra-Firm Inequality

This figure shows the firm internal Gini index results of the Compustat model. Panel A shows how firm internal inequality (indicated by color) is related to the CEO pay ratio and firm size. Panel B shows the distribution of modeled Gini indexes for all firms. Panel C compares model results to the Gini index of case-study firms (see section B.1 for case-study methods). Panel D shows time evolution of the average Gini index of all modeled firms. The shaded region indicates the 95% confidence interval. All results are computed from 5000 model runs, each with different bootstrapped parameters a , b , and σ .

A. 50 Most Unequal Firms



B. 50 Most Equal Firms

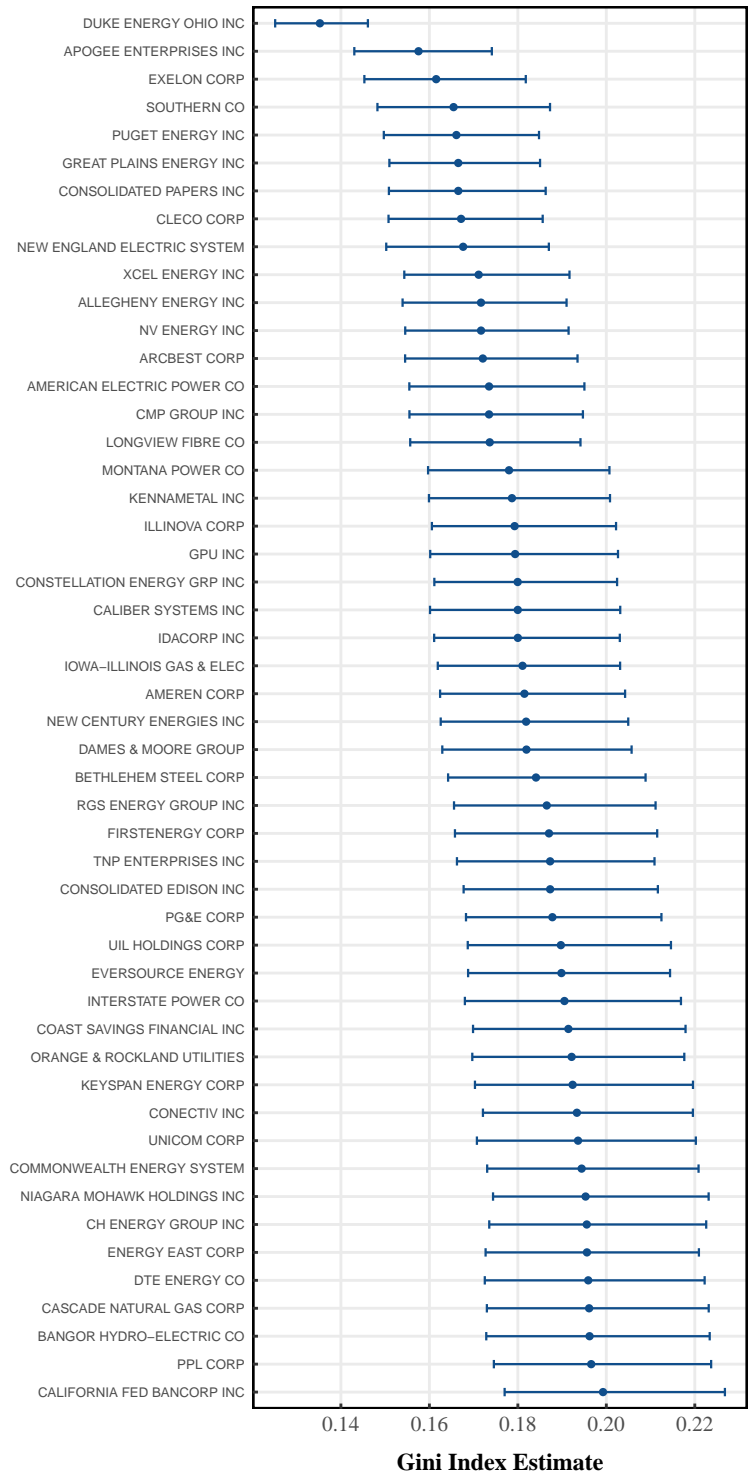


Figure 15: The Most Equal and Unequal Compustat Firms

This figure shows the 50 most unequal (Panel A) and 50 most equal firms (Panel B). Points indicate the mean Gini index for each firm, while the error bars show the 95% confidence interval calculated from 5000 bootstrap model runs.

most unequal firms. What is most interesting about these results is the *sectoral composition* of the 50 most equal firms. The vast majority (80%) are energy/utility companies. In the United States, firms in the utility sector are highly regulated, which leads to far more scrutiny over executive pay. Previous studies have found similar results — executives in regulated firms earn far less than those in unregulated firms (Joskow et al., 1993). This finding has important implications for a hierarchical power theory of income distribution. It suggests that government regulation serves as a *check* on power, limiting the degree to which elites are able to use their status to amass wealth.

F.2 Income By Hierarchical Level

The main purpose of the Compustat model is to estimate the income effect of grouping individuals by hierarchical level. Here I review summary statistics of the model's hierarchical pay structure. Figure 14A shows how mean income (across all firms in the model) increases by hierarchical level. The Compustat model's results are compared to the UK data documented by Mueller et al. (2016). In both the Compustat model and Mueller's data, mean income increases super-exponentially with hierarchical level — it increases *faster* than an exponential function. Figure 14B shows how intra-level income inequality changes by hierarchical level. For hierarchical levels 1-10, both the Compustat model and Mueller's data show similar trends.

The similarities between the model's results and Mueller's data lend credence to the model. However, what explains the differences? One key factor is that the Compustat data comes from the US, which has much more inequality than the United Kingdom, where Mueller's sample is taken. In this light, the differences in Figure 14A make sense. In the more unequal United States, income scales more rapidly with hierarchical level than in the United Kingdom. The results in Figure 14B can be similarly explained. In the more unequal United States, intra-hierarchical level income dispersion is greater than in the UK.

Another interesting result in Figure 14 is the conspicuous change in model trends for hierarchical levels above 11. Above this level, mean income no longer increases with hierarchical level. This may simply be an artifact of the particular Compustat firm sample. Going back to Figure 14A, note that the four largest firms have particularly low CEO pay ratios. Given the model's assumptions, only the very largest firms will have more than 11 hierarchical levels. Since the 4 largest firms have particularly low CEO pay ratios, resulting mean income in hierarchical levels 12-14 will be relatively low.

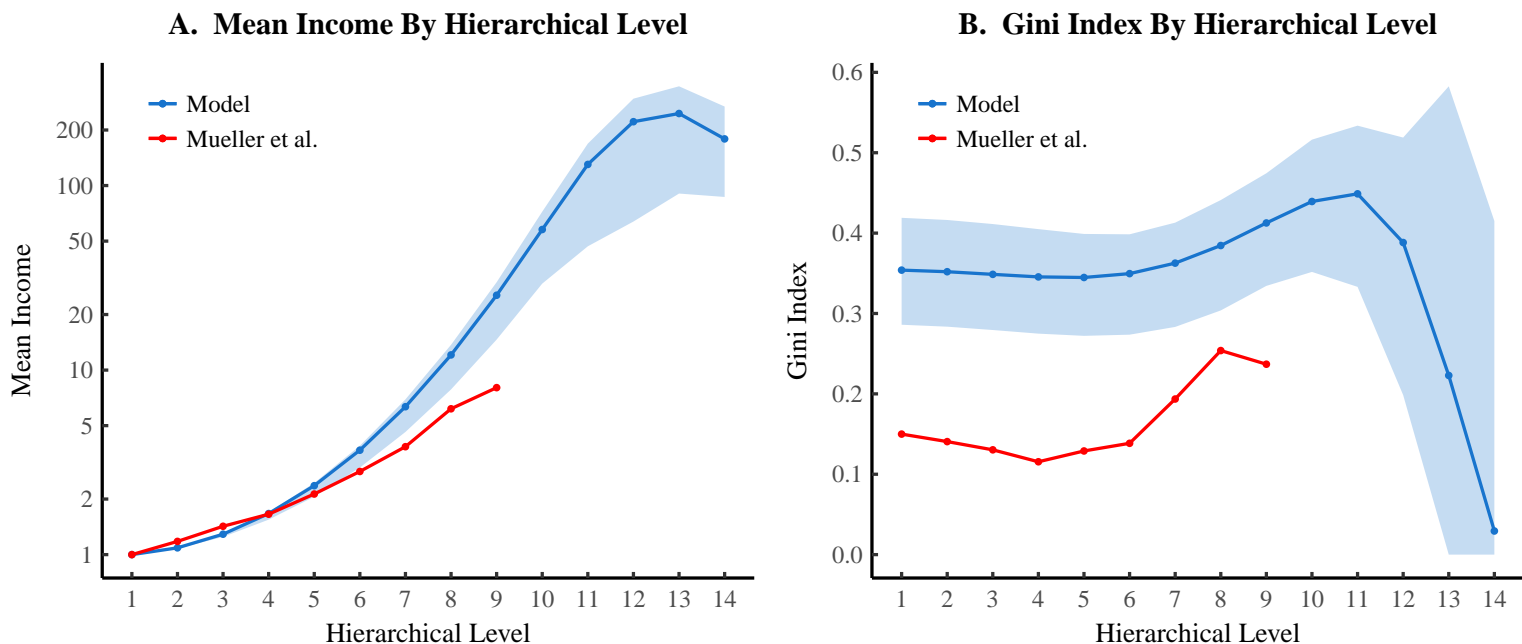


Figure 16: Compustat Model Results for Income by Hierarchical Level

This figure compares the results of the Compustat model to the UK data from Mueller et al. (2016). Panel A shows average income by hierarchical level (across all firms) indexed to pay in level 1. Panel B shows how intra-level inequality changes by hierarchical level. Shaded regions indicate the 95% confidence region of the model, estimated from 5000 bootstrap runs (see Appendix E).

What about the drop in intra-level inequality for hierarchical levels 12-14? This is a mathematical artifact. Individuals in upper hierarchical levels become exponentially rare. In some iterations of the Compustat model, there are only a few individuals in level 13, and only one in level 14. The low Gini index for these upper hierarchical levels results from the fact that a sample size of one has zero inequality, by definition.

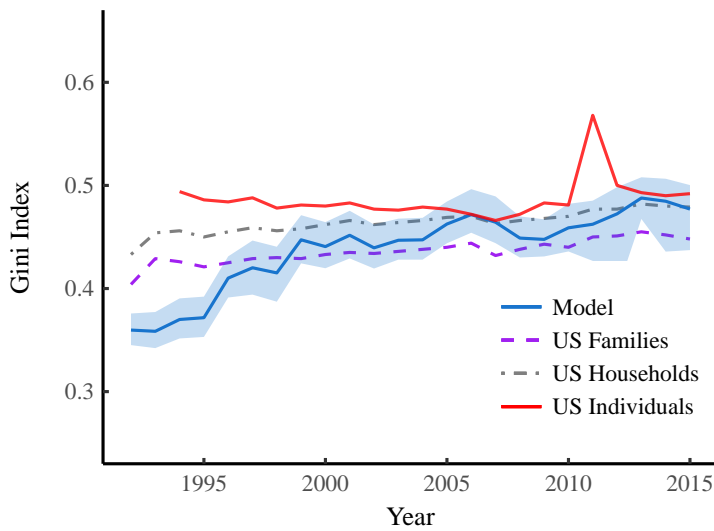
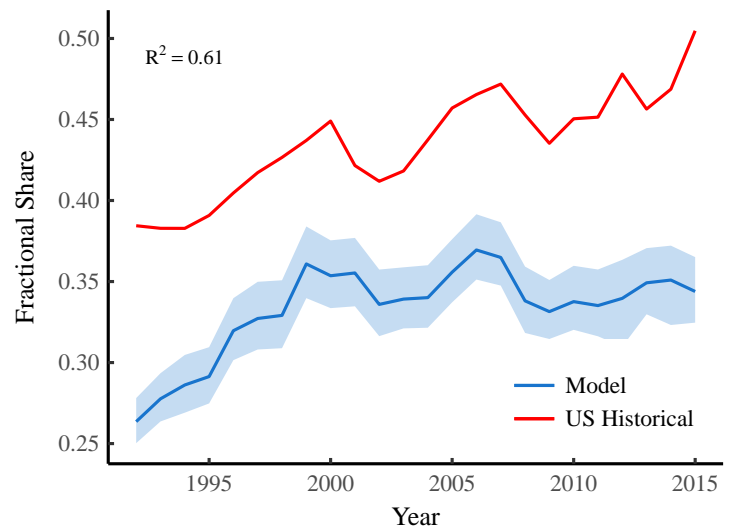
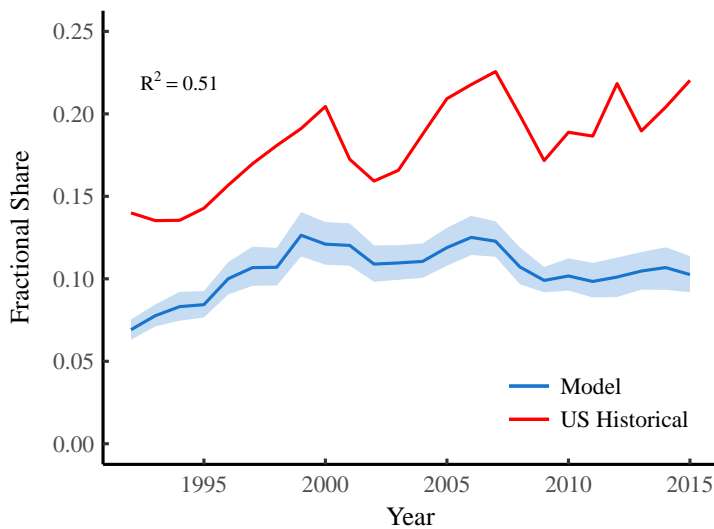
F.3 Aggregate Inequality

An important test of the Compustat model is to see if it produces aggregate levels of inequality that are comparable to US empirical data. Figure 17 shows the results of such a test. Here I plot the time-series trends in both US historical inequality and aggregate inequality in the Compustat model. This latter metric is calculated by aggregating (by year) all individuals in the model into a single sample, and then calculating the inequality of the resulting income distribution.

Figure 17A compares the model's aggregate Gini index against three different types of data published by the US Census: Gini by *individual*, *family*, and *household*. Two findings are evident. Firstly, the model is roughly consistent with the US empirical data over the period 2000-2015. However, the model produces too little inequality during the 1990s. Secondly, the US empirical data shows *contradictory* trends — roughly *constant* inequality among individuals, but secularly *increasing* inequality among families and households. The model reproduces the secular trend.

The problem with US Census data is that it is based on survey data, and individuals have a systemic tendency to under-report their incomes (especially if it is large). As an alternative to survey-based data, Thomas Piketty (2014) has focused on measuring inequality in the tail of the income distribution using income tax data. Figures 17B and 17C show Piketty's series for the top 10% and 1% income share in the United States. Both series show secularly increasing inequality over the period in question. The model reproduces these *trends* quite accurately, but at a lower absolute level of inequality.

Although the empirical data is itself contradictory, the important finding here is that the Compustat model produces a level of inequality that is consistent with official US data. This means that it is fair to compare the model's results to other results derived from official data.

A. Gini Index**B. Top 10% Income Share****C. Top 1% Income Share****Figure 17: Compustat Model Aggregate Inequality vs. US Historical Data**

This figure compares estimates of aggregate income inequality in the Compustat model to US historical data. Panel A compares the model Gini index to three different US measures (the Gini of individuals, families and households). Panel B shows the income share of the top 10%, while Panel C shows the top 1%. The shaded regions indicate the 95% confidence interval of the model, estimated over 5000 bootstrap runs. US Gini index data is for individuals, and comes from US Census table PINC-05. The 2011 outlier in US data is likely a statistical error. Families and Household Gini indexes are from the Federal Reserve Bank, series GINIALLRF and GINIALLRH, respectively. US top 10% and top 1% share data is from the World Wealth and Income Database, series *sptinc992j*.

G A Sensitivity Analysis of the Compustat Model

The Compustat model relies on three parameters (a , b , and σ) that are determined from regressions on firm case-study data (see Appendix E). Parameters a and b define the span of control, and ultimately determine the ‘shape’ of each firm’s hierarchy. The parameter σ determines the level of income dispersion within each hierarchical level of a firm.

Because the case-study analysis contains only seven firms, there is a great deal of uncertainty in these parameters. Given this uncertainty, it is important to understand the ‘sensitivity’ of our model results to changes in the parameters a , b , and σ . I measure this sensitivity using a bootstrap analysis of the model’s properties. I run the model many times, each time with a different resample of the case-study data (leading to different values of a , b , and σ). I use this bootstrapped data to analyze how each parameter affects the following metrics:

1. The signal-to-noise ratio of grouping individuals by hierarchical level;
2. The signal-to-noise ratio of grouping individuals by firms;
3. Aggregate levels of inequality within the entire model.

Figure 18 shows the results of this analysis. We can immediately conclude that the model is not sensitive to the value of σ , which has virtually no effect on any of the above metrics. However, the model appears to be highly sensitive to parameters a and b . This sensitivity is least pronounced for the signal-to-noise ratio for grouping individuals by hierarchical level. However, for grouping by firms, changes in a and b have a strong effect on the signal-to-noise ratio.

This is an important finding. It suggests that our hierarchical level results (used to test the power-income hypothesis) are relatively robust. A different firm case-study sample would likely not lead to significant changes in our findings. Our firm results, however, are less robust. A different firm case-study sample could lead to very different results.

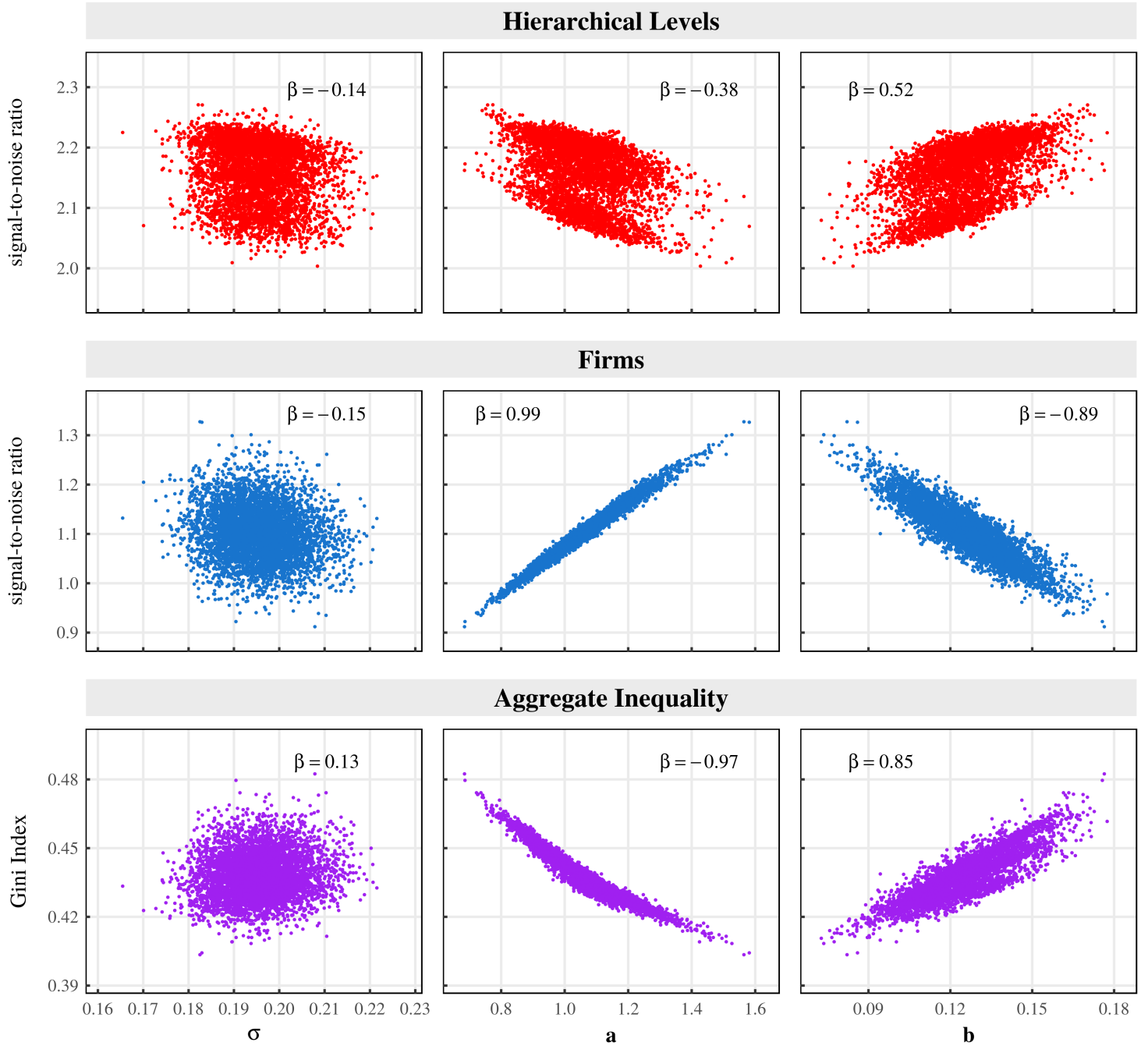


Figure 18: A Sensitivity Analysis of Compustat Model Parameters

This figure shows the results of a sensitivity analysis of Compustat model parameters. The top row shows the effect that each parameter has on the signal-to-noise ratio (using the Gini index) for grouping individuals by hierarchical level. The middle row shows the same for firms, and the bottom row for aggregate inequality in the model. Each plotted point indicates a different parameter combination. The normalized regression coefficient β (which can range from -1 to 1) quantifies the model sensitivity.

H Measuring Effect Size

In this section, I discuss what is meant by ‘effect size’ (in the context of this paper) and how the signal-to-noise ratio (using the Gini index) relates to more standard measures of effect size.

What is Meant By ‘Effect Size’

In the context of income distribution, there are two possible ways that we might define effect size:

Definition A: How much a factor affects *total inequality*.

Definition B: How much a factor affects *individual income*.

Effect size definition A refers to what we might call ‘inequality accounting’. For instance, we might ask: how much do differences in pay between two groups contribute to total inequality? The point is that this definition attempts to measure how a given factor affects *total inequality*. In general, inequality accounting depends crucially on the *size* of the various groups.

Figure 19 shows this phenomenon. In both panels, groups A and B have equal differences in mean income and equal within-group income dispersion. However the total inequality obtained by merging the two groups varies dramatically depending on the relative size of A to B. In Fig. 19A, the two groups are of equal size, while in Fig. 19B, group B is 50 times smaller than group A. The resulting merger of A and B produces much more inequality when the two groups are of equal size than when they are not.

Effect size definition B is concerned only with the effect on *individual income*, not on accounting for total inequality. The key difference is that for definition A, we care about group size, while for definition B we do not. In more technical terms, effect size definition B should be calculated by drawing *equal sized samples* from each group. Perhaps the simplest and most intuitive metric of effect size definition B is *Cohen’s d* (Eq. 43). This is defined as the difference in means (\bar{x}) between two group samples (A and B), divided by the within-group standard deviation (s_W).

$$d = \frac{\bar{x}_B - \bar{x}_A}{s_W} \quad (43)$$

Cohen’s d can be interpreted as a *signal-to-noise ratio*. The ‘signal’ is the difference in means (the effect we want to measure), while the ‘noise’ is the dispersion within groups, as measured by the standard deviation. In the case of income, the

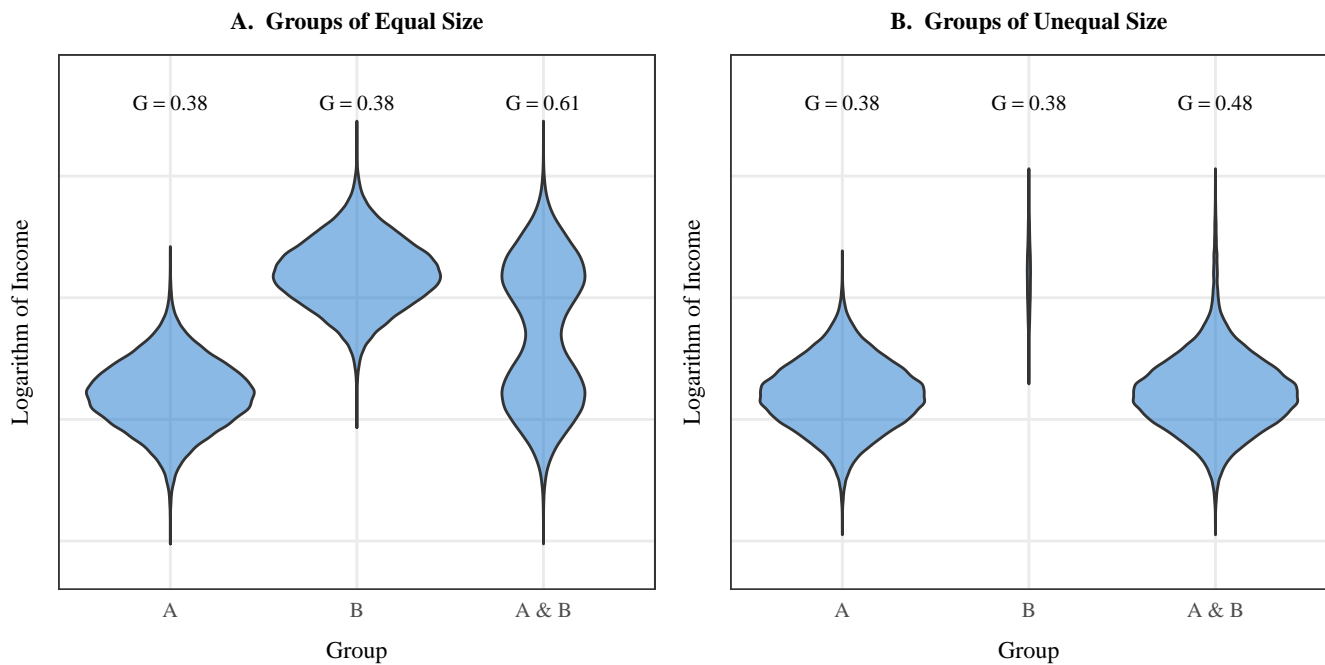


Figure 19: How Group Size Affects Inequality Accounting

This figure shows how differences in group size affect total inequality. In both panels, the income distribution of two different groups (A and B) are shown. The distributions are displayed as ‘violin’ plots, where the thickness of the violin indicates the number of individuals with that income. In both panels, groups A and B have identical differences in mean income, and identical within-group income dispersion (Gini indexes are shown above each violin). In the left panel, both groups have the same size. In the right panel, group B is 50 times smaller than group A. The rightmost violin plot in each panel shows the income distribution produced by merging groups A and B. Far more inequality is produced when the two groups are of equal size than when there are large differences in size.

size of the signal-to-noise ratio indicates how accurately we can predict someone's income based only on knowledge of their group membership (either A or B). The larger the signal-to-noise ratio, the more accurate the prediction.

In the example shown in Figure 19, group A and B have the same difference in means and the same within-group standard deviation in both the left and right panel. Therefore Cohen's d would measure an identical effect size. To be clear, this is the effect on *individual income* (definition B), not the effect on inequality (definition A).

The Signal-to-Noise Ratio

In this paper, I am concerned only with effect size definition B — the effect on individual income. I measure this effect size with a signal-to-noise ratio that uses the Gini index as a measure of dispersion:

$$G_{BW} = \frac{G_B}{\bar{G}_W} \quad (44)$$

Here G_B is the Gini index of group means and \bar{G}_W is the mean of all within-group Gini indexes.

How does this metric relate to more standard measures of effect size? It amounts to a signal-to-noise ratio that is similar to Cohen's f^2 , the latter of which is a generalization of Cohen's d to many different groups. To obtain Cohen's f^2 , we divide variance between groups (σ_B^2) by average variance within groups ($\bar{\sigma}_W^2$):

$$f^2 = \frac{\sigma_B^2}{\bar{\sigma}_W^2} \quad (45)$$

Like Cohen's d , the f^2 metric is a signal-to-noise ratio. The 'signal' is the variance between groups, while the 'noise' is the variance within groups. (See Fleishman (1980) and Steiger (2004) for a more detailed discussion of the f^2 metric). Comparing the form of f^2 and G_{BW} , we see that the two measures of effect size are very similar. Both are signal-to-noise ratios, consisting of a ratio of between-group dispersion to within-group dispersion. Given this similarity, there should be some relation between the two measures.

Rather than attempt to show this similarity analytically, I use simulated data. I build a model based on the following assumptions:

1. Income within groups is lognormally distributed.

2. Within-group income dispersion is the same for all groups (but can vary over different model iterations).
3. Mean income between groups is lognormally distributed (and can vary between iterations).
4. Total inequality is (roughly) constant for all iterations.
5. The number of groups varies (between iterations) from 2 to 100.

For each iteration of the model, we define the mean income of each group by drawing randomly from a lognormal distribution. We then simulate individuals within each group by drawing randomly from (a different) lognormal distribution. The model has 2 key parameters: the lognormal scale parameter that defines the dispersion *between* groups, and the lognormal scale parameter that determines dispersion *within* groups. Varying these parameters changes the size of the group-income effect. For consistency, I use only parameter combinations that produce roughly the same level of total inequality (a Gini index of 0.5).

For each set of simulated data, I calculate both G_{BW} and f^2 . Because analysis of variance typically assumes that within-group data is normally distributed, I calculate f^2 using the *logarithm* of income. The results are shown in Figure 20. As expected, there is an extremely strong relation between the two effect-size measures. This indicates that an f^2 test of the power-income effect would likely give very similar results to the G_{BW} findings shown in Figure 6 of the main paper. To reiterate, I do not conduct such an f^2 test in this paper because the relevant data is not available.

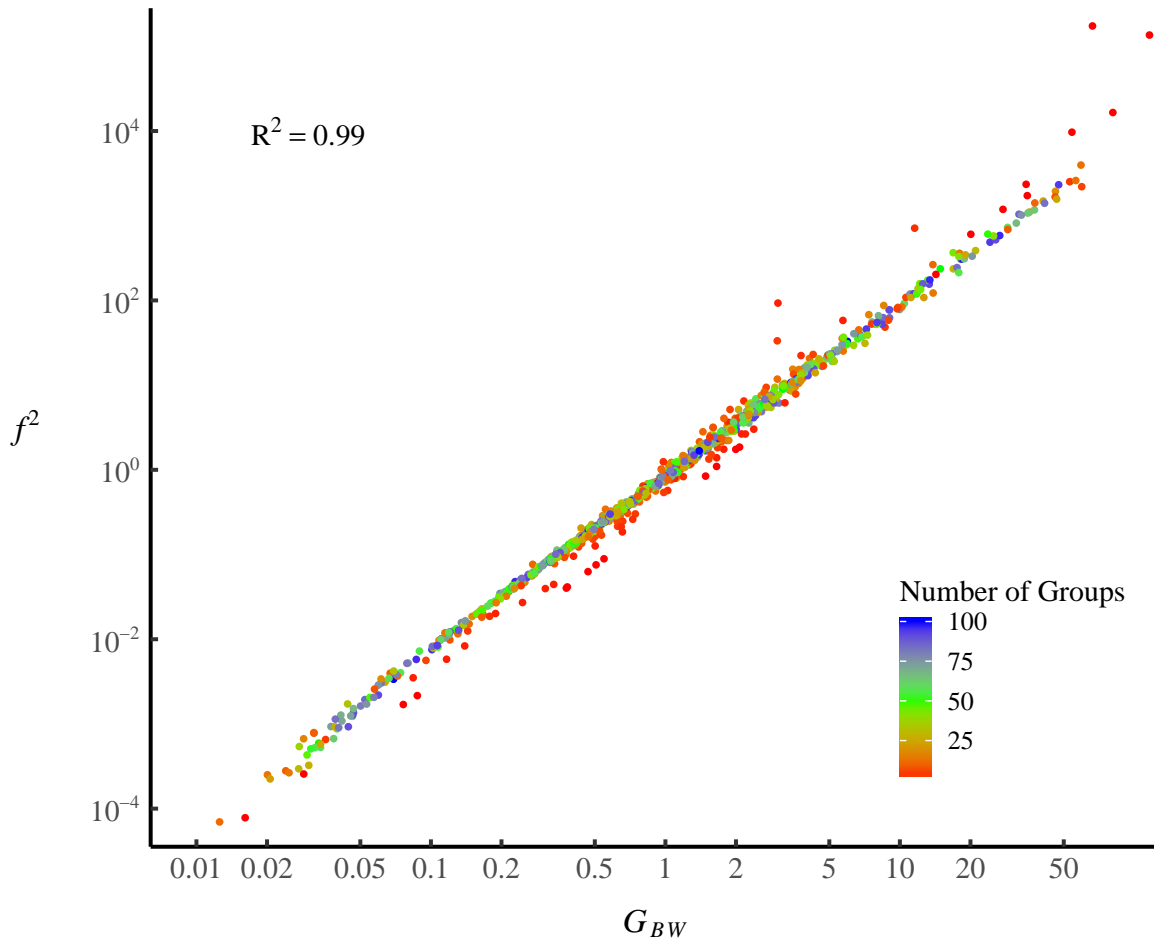


Figure 20: Cohen's f^2 vs. the Gini Metric G_{BW}

This figure compares Cohen's f^2 metric of effect size to my signal-to-noise Gini metric, G_{BW} . The comparison uses simulated data, and each data point represents different parameter combinations (see model assumptions above). Color indicates the number of groups used in each iteration.

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