# Beyond Theory of Mind: A formal interoperable framework for social inference and representation

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**Abstract**

Interpersonal relationships are a central feature of what it is to be human. Theory of Mind (ToM), or mentalising, is the ability to represent the hidden thoughts and beliefs of the self and others to navigate these relationships. However, to date there is no common set of formal principles that can be used to understand interpersonal relationships across context, disorder, and diversity, and in humans and artificial agents. Here we synthesise previously distinct components of social representation and inference into a common computational framework to allow translation across previously disparate processes. We discuss applications of the framework to problems regarding dysfunctional theory of mind, neural implementation, artificial intelligence, and discuss potential future directions for the field.

**Keywords:** Theory of Mind; Mentalising; Social Representation; Computational Psychology; Experimental Psychology; AI

**Open Scripts & Data:** https://github.com/josephmbarnby/Beyond-Theory-of-Mind

## 1. Refining Theory of Mind as a common framework

The formalisation of social cognitive processes is a nascent field. FeldmanHall & Nassar (2021) have called for social psychological theories to be stated in computational terms to reduce ambiguity and improve falsifiability. This is related to a broader urgency in psychological and psychiatric science to specify clear, transparent theories that can be tested and teased apart; able to be updated and changed (Haslbeck et al., 2022). In the case of Theory of Mind (ToM), this requires that computational models contain structures that define how representations of the mental states of self and others are generated (Barnby et al., 2023a). True to the original concept of ToM this presupposes that a self has some prior concept or representation of others to infer mental states, perspectives, attributes, and future observable actions, even if these are accessed intuitively (Conway et al., 2019; Premack & Woodruff, 1978; Tamir & Thornton, 2018).

Indeed, ToM has recently enjoyed rapid theoretical development with respect to formalised cognitive models, including how we use inverse reinforcement to learn about and predict others (Baker et al., 2011; Gmytrasiewicz & Doshi, 2004, 2005; Ng & Russell, 2000; Ray et al., 2008; Jara-Ettinger, 2019), form nested beliefs about others (Gmytrasiewicz & Doshi, 20 04, 2005), and select appropriate actions (Hula et al., 2015; 2018; Ho et al., 2022). The active inference framework has also been adapted to encompass multi-agent problems (Lehmann et al., 2023). These core algorithms have been subsumed, either directly or indirectly, into experimental designs which probe suffering (Crockett et al., 2014), planning under threat (Wise et al., 2023), intentional harm (Barnby et al., 2022), extreme beliefs (Story et al., 2023), self-esteem (Will et al., 2020), and equitable motivations (Fehr et al., 2006). Nevertheless, unifying these computational advances with social cognitive theory is non-trivial, and currently, there exists no common, interoperable, high-level structural framework which ties together specific psychological concepts and computational processes of interest into a common foundation for cognitive science and artificial intelligence (AI).

Here, we make a first attempt at drawing together social phenomena studied in the fields of experimental psychology, economics, clinical science, and computer science under a common framework. Using developments in computer science, we model how humans learn about and represent the self, in cases of social contagion, self-insertion, and metacognition, and about others using different depths of mentalising. This, we argue later, not only provides interoperability between methods and fields to facilitate cognitive theory construction, but also lays the foundation to make AI more social and efficient.

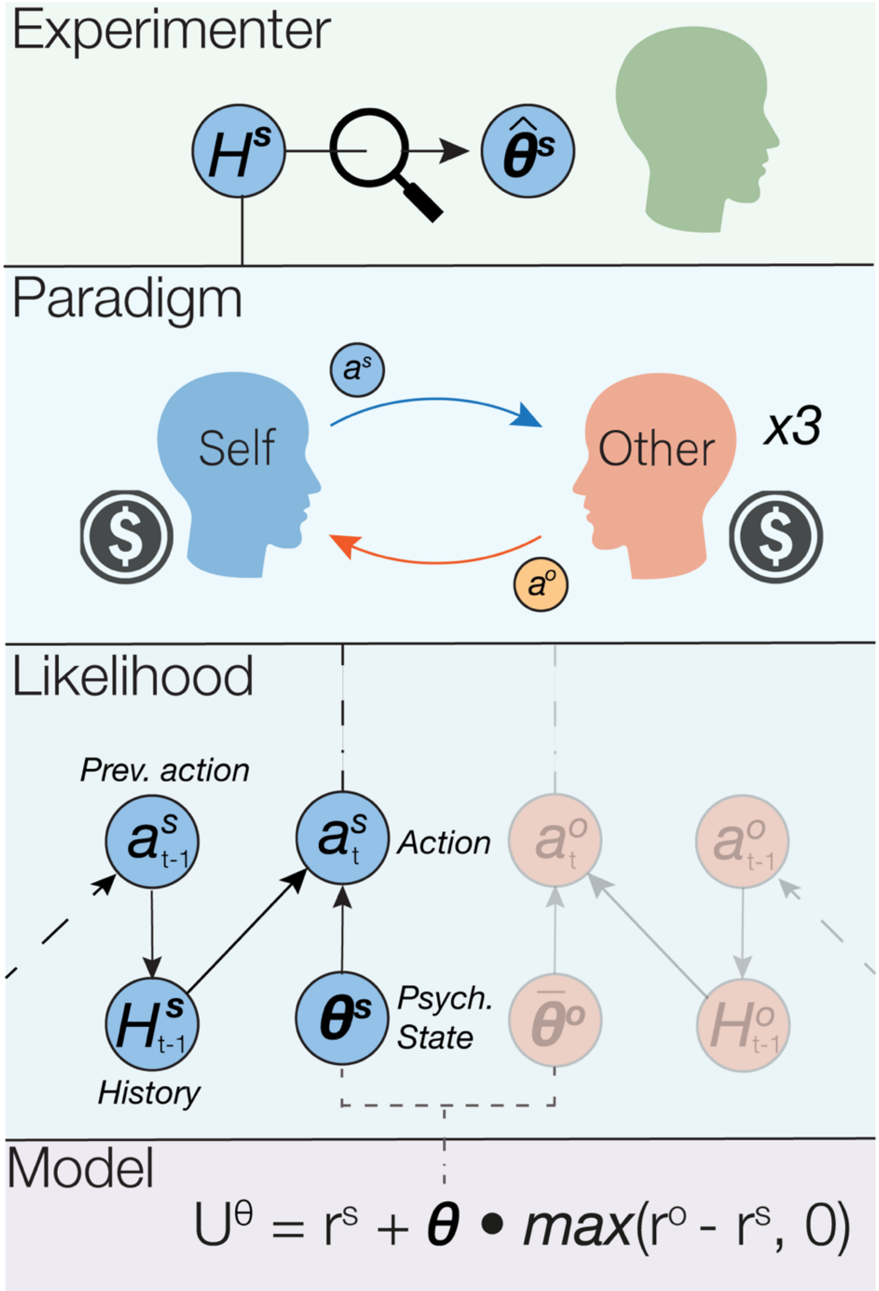


**Figure 1. Anatomy of notation used throughout our framework.** In this example, a self interacts with an other during the Trust Task. The combination of the participants action history, private information, and corresponding $\boldsymbol{\theta}$ are assumed to underlie the actions in the Trust Task. *Model:* For simplicity, $\boldsymbol{\theta}$ is assumed to be the relative payoff preferences that both a self and other use to make choices. *Likelihood:* A self's and other's actions are determined by their corresponding values of $\boldsymbol{\theta}^s$, $\boldsymbol{\theta}^o$ which converts their interaction history and private information into an action. From the self's perspective, the other is assumed to share the same generative principles, but the self only has a best guess about the other's $\boldsymbol{\theta}^o$, represented by a bar $\bar{\boldsymbol{\theta}}^o$. *Experimenter:* The experimenter modelling the self has only imperfect approximations of the self's characterization, represented by a hat $\hat{\boldsymbol{\theta}}^s$, and must use the history of actions of the self to create a best guess about the self's state. To note, this illustration is an example of shallow mentalizing, with no consideration for a partner's depth of mentalising. Nevertheless, this illustration serves to demonstrate how similar measurable quantities can be formally symmetric.

## 2. General principles

Here, we take a high-level approach to formalising Theory of Mind (ToM) to enable integration across disparate models and approaches within each domain. We will first address core principles that will appear throughout all formulations. We use a common interpersonal microeconomic task – the Trust Task - as an example of how these principles may be expressed in two-person exchange (Figure 1). We then use these principles within a series of formulations that demonstrate how humans learn about and represent themselves and others.

The Trust Task is a two-player, mixed-motive game. An investor (the self) is endowed with a sum of money (e.g. $10) and can send some to a trustee (the other). The amount sent by the investor is tripled by the experimenter and sent to the trustee who can then send some of this tripled amount back to the investor. Over several trials, both players can mutually gain reward from their actions if money is distributed fairly; detecting your partner to be competitive is essential to avoid deception and act to allay reward loss.

Modelling the Trust Task, and Theory of Mind in general, must consider at least three agents: self ('s'), other ('o') and the experimenter or observer. These are represented as superscripts in our notation. The self contains noisy private information about itself, as well as uncertainty over others. We introduce common terms throughout to tie together these tiered levels of uncertainty. These first- and third-person delineations are theoretically essential; the experimenter is otherwise modelling the participant as if they were an experimenter themselves (Daunizeu et al., 2014).

Let the psychological state that either a self or an experimenter wants to infer and represent be $\boldsymbol{\theta}$. While not central to this paper, $\boldsymbol{\theta}$ implicitly hides a hierarchical structure, containing both the structural description of the model/algorithm that the self uses to make choices ($\theta^{m}$) and any parameters within this model/algorithm ($\theta^{p_m}$). For an example of different models ($\theta^{m}$) in the Trust Task, a self might use one of two utility functions: one that includes only relative payoff preferences (did my partner get more, less or the same than me?) *or* one that considers both absolute (how can I maximise my reward?) and relative payoff preferences. Likewise, a self might use inverse reinforcement learning or nested planning to form policies and make choices. A self needs to learn the correct algorithm they or others are using, and then the corresponding set of parameters ($\theta^{p_m}$) to apply. The self is assumed to have some probability distribution to determine the *most likely model* given the task/partner at hand, and a probability distribution to determine *most likely parameters* to use given the model. Importantly, both can be subject of all the types of inference in the following sections (Figure 2).

We herein refer only to $\boldsymbol{\theta}$ in expressions for simplicity and to highlight the relations between the components of ToM. In the Trust Task example, $\boldsymbol{\theta}$ is only set as the relative payoff preference the self uses; we assume a simple utility function itself is the *most likely model* a self is using to make actions. The goal for the self is to then work out the *most likely parameter value* of the other under this model; $U^{\boldsymbol{\theta}^{o}} = r^{s} + \boldsymbol{\theta}^{o} \times \max(r^{o} - r^{s}, 0)$.

During an interactive exchange over more than one trial, it is necessary for each player to consider the history of actions, for example, what percentage of the investment the other sent back to the self. We write $\boldsymbol{\mathcal{H}}_t$ for the history of interactions in the environment up to time $t$. This includes non-interactive, private data that the self sees $\mathbf{x}_t$ (which might include direct input $\mathbf{x}_t^s$ to the self and/or stimuli that are available to an other from the perspective of a self, $\mathbf{x}_t^o$); actions made by the self or other, $a_t^s, a_t^o$, generically $a_t$; and outcomes such as rewards to the self or other, $r_t^s, r_t^o$, as above. We write $\boldsymbol{\mathcal{H}}_t^s$ to emphasise that the interaction history only pertains to joint actions made by the self (with equivalent quantities for the other).

We can put these terms together to broadly define how actions are selected. Importantly, components within the algorithm (such as action-values) can change with experience. The agent is then assumed to take actions stochastically based on $\boldsymbol{\theta}$ and $\boldsymbol{\mathcal{H}}_{t-1}$: $a_t \sim p(a|\boldsymbol{\theta}, \boldsymbol{\mathcal{H}}_{t-1})$. This implies that inferences about a person's action $a_t$ are dependent on both input history (which could further be public, private or a combination of the two) and their fixed parameters.

We can distinguish a self's internal knowledge and the knowledge an experimenter develops over the course of model estimation.

To allow for the possibility that the self has incomplete self- and other- knowledge we consider $\overline{\boldsymbol{\theta}}$ a random variable that is the self's subjective best-estimate. In principle, the self could misestimate its own algorithm, or be unsure about one or more of its own parameters: $\overline{\boldsymbol{\theta}}^s$ would then be the self's theory of its own mind – a key construct in metacognition – and $\overline{\boldsymbol{\theta}}^o$ is a theory of another's mind. This delineation distinguishes between the "true" and the "subjective perceptions" of psychological states. For example, someone's estimate of their own mental state ($\overline{\boldsymbol{\theta}}^s$) might be that they think they are more prosocial than they actually are ($\boldsymbol{\theta}^s$). The self can make inferences about themselves $p(\overline{\boldsymbol{\theta}}^s|\boldsymbol{\mathcal{H}}_t)$ and others $p(\overline{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t)$. As we will see, these can include the self's model of the other's model of the self, much like that has been used previously to explain behaviour in the Trust Task (Hula et al., 2015; 2018). We will later use a special notation for this, to make clear the Depth of Mentalising in such models where recursive properties are essential for strategic planning.

Equally, the experimenter maintains approximations $\widehat{\boldsymbol{\theta}}^s$ as its characterisation of a self and, if necessary, of the self's input $\hat{\mathbf{x}}_t^s$. As a window onto the complexities that arise with intentional modelling, this could include the experimenter's model of the self's potentially inaccurate subjective estimates of its own internal state or model. The experimenter can follow the same principles as the self in making inferences about $\widehat{\boldsymbol{\theta}}^s$ from $\boldsymbol{\mathcal{H}}_t$.

In the next section we set out how these terms can be arranged and phrased to formalise multiple components of ToM, and thus explain how the self infers and represents itself and others (Figure 2). To reiterate, we make a few assumptions in our framework to focus on the synergy of different theoretical strands of mentalising and to assist in formal clarity: We assume that both self and other(s) are engaged in a task or game that involves public information, and outcomes or rewards that jointly affect each player; $\boldsymbol{\theta}$ to infer by all parties is fixed (aside from the special case of social contagion and interpersonal convergence discussed later); the algorithm used by self and other are the same and time invariant; that learning and belief representation follows broadly Bayesian principles of probability, where uncertainty over quantities, especially relevant in social contexts, can be defined, measured, and has a causal role on the precision of belief integration. Of course, in natural interactions, one or more of these assumptions may break. A self or other can be uncertain over the algorithm that might be appropriate or may use non-Bayesian integration strategies. We suggest that the validity of these assumptions is an empirical question which can be tested given the formal implementation of our theory and address these concerns in our 'Future Directions' section.

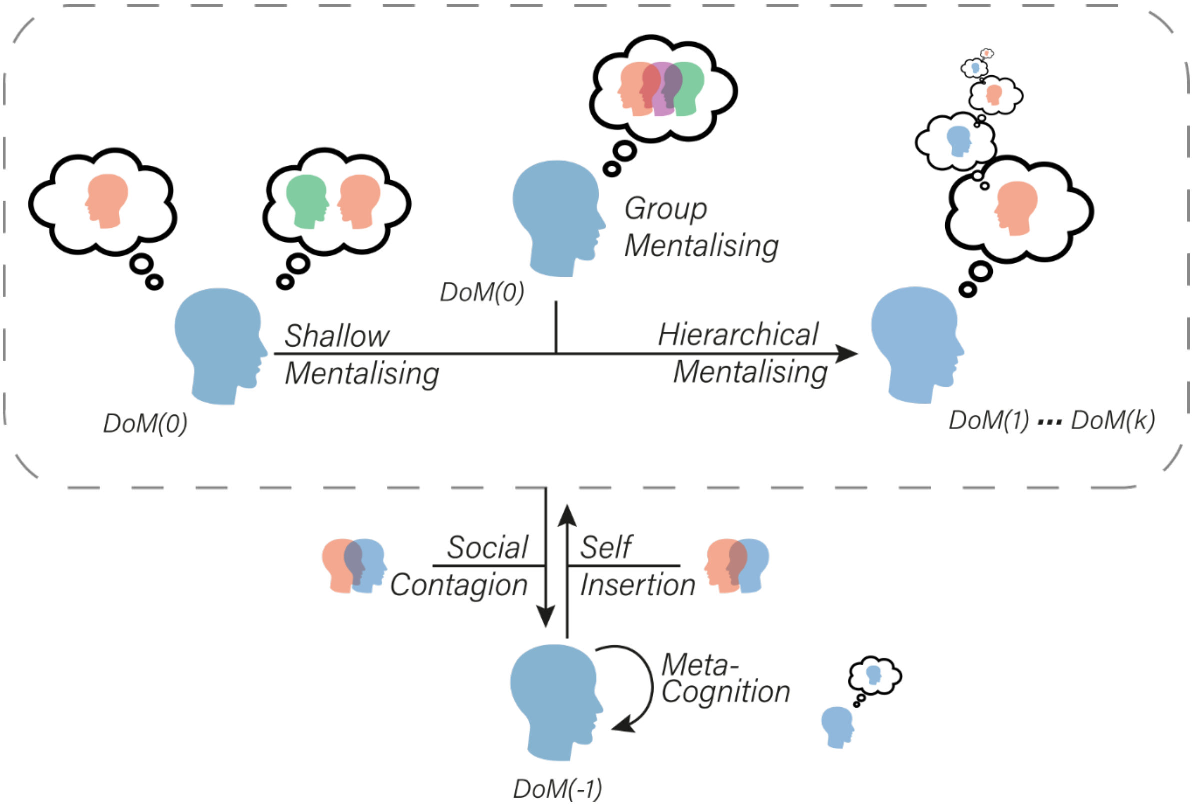


**Figure 2. Illustrated summary of Theory of Mind components within our framework.** Theory of Mind (ToM), or mentalising, is cast as a set of core algorithms furnished by social psychological theory that approximate social inference and representation. The object to approximate, $\boldsymbol{\theta}$, is treated as a non-trivial random variable imbued with qualitative dimensions relevant to the model and parameters appropriate to context. ToM is therefore a collection of interoperable algorithms and represented values which form a general-purpose framework to scaffold specific psychological and social dynamics. Metacognition is the self's process of approximating its own subjective model. Social contagion explains how humans come to shift their sense of self based on exposure, observation and interaction with others. Self-Insertion asserts that humans may implicitly assume others to be (at least initially) similar to their own representation of self. Shallow Mentalising involves the development of beliefs about others using a process of inverse reinforcement learning without recursion. Extending from this, Hierarchical Mentalising posits that a self can hold nested, interactive states about others. Group Mentalising develops this further to frame how representations of multiple others are held discretely or compressed into general heuristics.. Each dimension can be further subdivided by whether the state is relating to self, $\boldsymbol{\theta}^s$, or other, $\boldsymbol{\theta}^o$. DoM = Depth of Mentalising. For a formal summary see the Supplementary Materials.

## 3. Self

We first consider how an experimenter may construct beliefs about a self separately from an other, and the inferences a self would use to represent the self, e.g., how the self approximates itself. This involves inference using both input $\mathbf{x}_t^s$ and past interactive history $\mathcal{H}_{t-1}^s$ available to the self.

### a. An Experimenter's Estimation of a Self

$$P\left(\mathcal{H}_t^s|\widehat{\boldsymbol{\theta}}^s\right) \propto P(a_t^s|\widehat{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \mathcal{H}_{t-1}^s)P\left(\widehat{\boldsymbol{\theta}}^s|\mathcal{H}_{t-1}^s\right)$$

The experimenter infers $\widehat{\boldsymbol{\theta}}^s$ given a self's total history of interactions, $\mathcal{H}_t^s$, starting from a prior distribution $P\left(\widehat{\boldsymbol{\theta}}^s|\mathcal{H}_0^s\right)$ which is often broad. The prior is then updated and shifted when combined with the likelihood of actions made by participants, $P(a_t^s|\widehat{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \mathcal{H}_{t-1}^s)$; this explains that the probability an action was made is predicated on some notional value of $\widehat{\boldsymbol{\theta}}^s$ (usually chosen by an optimisation routine during fitting), the current private input participants received on a given trial, $\mathbf{x}_t^s$, and their interaction history up to the current trial $\mathcal{H}_{t-1}^s$. The resultant output $P\left(\mathcal{H}_t^s|\widehat{\boldsymbol{\theta}}^s\right)$ produces a set of actions that the optimisation routine has ejected to check against the real observed data recorded by the experimenter.

This general Bayesian approach has been implemented in a number of software packages, including those that build hierarchical Bayesian models of the values of $\widehat{\boldsymbol{\theta}}^s$ for many agents (Ahn et al., 2017; Piray et al., 2019). Although the experimenter's third person inferences are structurally different from those made by the participants in the exchange, the calculations are similar, as experimenters perform the same sort of learning that the participants do for themselves and their partners (see reviews of model construction, fitting and validation in Daw, 2012; Wilson & Collins, 2019).

The remaining high-level concepts we introduce involve characterizations of more sophisticated selves and/or the selves' models of their partners. Experimenters can extend their formalism to encompass these models by replacing the likelihood to make inferences about these other selves. In the interests of simplicity, we do not show these further possibilities.

### b. Self-Awareness and Metacognition

$$P(\overline{\boldsymbol{\theta}}^s|\mathcal{H}_t^s) \propto P(a_t^s|\overline{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \mathcal{H}_{t-1}^s)P(\overline{\boldsymbol{\theta}}^s|\mathcal{H}_{t-1}^s)$$

A self estimates $\overline{\boldsymbol{\theta}}^s$ based on their history of actions up to the present moment. This is calculated based on their prior knowledge of who they were $P(\overline{\boldsymbol{\theta}}^s|\mathcal{H}_{t-1}^s)$, coupled with the likelihood of their present self-generated action $P(a_t^s|\overline{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \mathcal{H}_{t-1}^s)$; action probabilities, like the section above, are predicated on $\overline{\boldsymbol{\theta}}^s$, their inputs up to the present moment, $\mathbf{x}_t^s$, and their history of interactions, $\mathcal{H}_{t-1}^s$.

If the self knows its own true characterization, $\boldsymbol{\theta}^s$, and knows everything about the input $\mathbf{x}_t^s$, then the likelihood $P(a_t^s|\boldsymbol{\theta}^s, \mathbf{x}_t^s, \mathcal{H}_{t-1}^s)$ (which, in common cases such as perceptual decision-making, is actually independent of the history) is related to a form of first-order model (Fleming & Daw, 2017) that it can have in its action. This would be conventional confidence, if the action is intended to be a correct report of a facet of the world (as in perceptual decision-making problems; Pouget et al., 2012). In more general cases, the self could still know how unusual its action was given the input.

If the self knows its own true characterization $\boldsymbol{\theta}^s$, but only has imperfect access to the input that actually led to the action $a_t^s$, then it has to do a further calculation to work out the likelihood (here, assuming no history dependence):

(3) $P(a_t^s|\boldsymbol{\theta}^s, \bar{\mathbf{x}}_t^s) = \int d\mathbf{x}_t^s \, P(a_t^s|\boldsymbol{\theta}^s, \mathbf{x}_t^s) \, p(\bar{\mathbf{x}}_t^s|\mathbf{x}_t^s)$

Here, the self also needs to estimate the input, $\bar{\mathbf{x}}_t^s$, conditioned on all available inputs, $\mathbf{x}_t^s$. This amounts to a restricted form of second order mode of confidence (Fleming & Daw, 2017) in the perceptual decision-making case, where the confidence judgement cannot avail itself of independent information about the true stimulus.

If the self additionally does not know its own characteristics perfectly, $\bar{\boldsymbol{\theta}}^s$, then it can refine its theory of its own mind by performing the same sort of Bayesian inference as the experimenter (see later for calculational limitations). In the case that the input is also incompletely uncertain, then the further integration of equation (3) is also required to calculate the effective likelihood, leading to the full term:

$$P(\bar{\boldsymbol{\theta}}^s|\bar{\boldsymbol{\mathcal{H}}}_t^s) \propto \int d\mathbf{x}_t^s \, P(a_t^s|\bar{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \bar{\boldsymbol{\mathcal{H}}}_{t-1}^s) P(\bar{\boldsymbol{\theta}}^s|\bar{\boldsymbol{\mathcal{H}}}_{t-1}^s) \, p(\bar{\mathbf{x}}_t^s|\mathbf{x}_t^s)$$

Metacognition is deeply intertwined with social interaction (Timmermans et al., 2012). These terms for first and second order confidence sets the stage for discriminating what information to broadcast to others (Heyes et al., 2020), and deciding which information to keep private and which public (Bang et al., 2020). This latter function is important for human social adaptation. It allow us to shape our expressed beliefs to facilitate social integration and strategy within groups, even at the cost of professing less accurate or inaccurate beliefs (Williams, 2021). This relates to social broadcasting we consider later when humans use hierarchical mentalising: how a self thinks an other will think about the self, given their choices or utterances, and thus how they should behave. This formulation is agnostic as to whether estimation noise (Shekhar & Rahnev, 2021) originates from external or internal sources and indeed may be a product of both. This latter notion is complimentary to local models, for example, hypotheses that formalise whether humans can identify sensory signals originating internally or externally (Dijkstra & Fleming, 2023).

For the remaining cases, inputs such as $\mathbf{x}_t^s$ may be present, but they do not change the nature of the interaction. For simplicity we drop them from formulae.

### c. Self-Insertion

$$P(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t^o; \boldsymbol{\theta}^s) \propto P(a_t^o|\bar{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}^o) P'(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_{t-1}^o; \boldsymbol{\theta}^s)$$

Normally, the self could make inferences about the other $\bar{\boldsymbol{\theta}}^o$ as if it was the experimenter (i.e., as in (a) above). However, when interacting with new people, we (at least initially) might expect others to be like us (although see Epley & Dunning, 2000). Here, in our formulation, a self interprets the actions of others as a function of their own model $\boldsymbol{\theta}^s$; this is contained in the prior, $P'(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_{t-1}^o; \boldsymbol{\theta}^s)$, that states the estimated characterisation of an other is based on the history of the other, $\boldsymbol{\mathcal{H}}_{t-1}^o$, and the self's own characterisation, $\boldsymbol{\theta}^s$. Given that the likelihood in this formulation, $P(a_t^o|\bar{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}^o)$, is only based on the other, the resultant posterior belief $P(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t^o; \boldsymbol{\theta}^s)$ gradually washes out the bias of the self. The stronger the prior, the harder is it to wash away.

Theories of the relational self (Anderson & Chen, 2002) posit that the self is the most extensive and well-grounded representation we have access to. The effect of this on social judgements, known in social psychology as the 'egocentric bias', has been cited as a fundamental, pervasive bias in social interaction (Kreuger & Clement, 1994). We interpret this as an instance of a more general self-insertion bias that places a form of form of easily accessible model selection and constraint on possible values of $\bar{\boldsymbol{\theta}}^o$. Indeed, being different from partners increases the reaction times of participants when they're tasked to learn and predict a partner's snack-food preferences (Tarantola et al., 2017). Even after learning, this self-insertion bias persisted. In the same study, the known popularity of items was also found to be integrated into priors suggesting some normative cultural representation was embedded into prior expectations. This work was expanded upon to incorporate social value preferences (Barnby et al., 2022): social similarity between participants and partners increased predictive accuracy over whether a partner would select a prosocial, selfish, or competitive option. Having more flexible priors overcame this initial bias. Another quirk of this social similarity was to reduce the perception of threat: partners were viewed on average as less intentionally harmful in their actions if interpersonal similarity was higher.

The simplest possibility is that the prior $P(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_0^o) = g(\bar{\boldsymbol{\theta}}^o, \boldsymbol{\theta}^s)$ is centred closely around $\boldsymbol{\theta}^s$ – the characterisations of self and other are approximately similar. However, this would generally not lead to limitations in inference as the prior would be integrated away given new information. Instead, it could be that the self continually mixes posterior estimate $P(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t^o; \boldsymbol{\theta}^s)$ with their prior $g(\bar{\boldsymbol{\theta}}^o; \boldsymbol{\theta}^s)$:

$$P'(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t^o; \boldsymbol{\theta}^s) = \omega \cdot P(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_{t-1}^o; \boldsymbol{\theta}^s) + (1-\omega) \cdot g(\bar{\boldsymbol{\theta}}^o; \boldsymbol{\theta}^s)$$

which would keep inserting $\boldsymbol{\theta}^s$ into the self's belief about the other with weight $1-\omega$. This would make the influence of the self-knowledge an even more persistent bias when learning about others.

We might also allow that the likelihood is influenced by $\boldsymbol{\theta}^s$ (making for a more complex expression than $P(a_t^o|\bar{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}^o)$. For instance, the self might consider their own algorithm as the only possibility for how the other chooses actions, even if they allow the other to have their own parameters within this algorithm: $P(a_t^o|\bar{\theta}^{o,Pm}, \bar{\theta}^{s,m}, \boldsymbol{\mathcal{H}}_{t-1}^o)$.

### d. Social Shaping and Contagion

$$P(\bar{\boldsymbol{\theta}}^s|\boldsymbol{\mathcal{H}}_t^o) \propto P(\boldsymbol{\mathcal{H}}_t^s|\bar{\boldsymbol{\theta}}^s) \int d\bar{\boldsymbol{\theta}}^c d\bar{\boldsymbol{\theta}}^o \, P(\boldsymbol{\mathcal{H}}_t^o|\bar{\boldsymbol{\theta}}^o) P(\bar{\boldsymbol{\theta}}^s|\bar{\boldsymbol{\theta}}^c) P(\bar{\boldsymbol{\theta}}^o|\bar{\boldsymbol{\theta}}^c) P(\bar{\boldsymbol{\theta}}^c)$$

We not only imagine ourselves into others but are influenced by those around us. From birth, being exposed to others alters our sense of self and shapes our early models of the world (Schilbach, 2013; Ciaunica, 2021). Even as adults, social exposure, whether consciously or not, shapes health behaviour (Christakis & Fowler, 2012), political affiliations (Bond et al., 2012), and decision-making (Schilbach et al. 2013; Rollwage et al., 2020; Schulz et al., 2020).

The shifting of a self's belief given toward a belief it has been exposed to has been explicated in formal models. For example, in paradigmatic cases of intertemporal discounting, being exposed to information about a partner's discounting preferences is enough to elicit a shift in one's own preferences (Garvert et al., 2015; Moutoussis et al., 2016; Thomas et al., 2022). In one model of this, the self is uncertain about its own preferences, $\boldsymbol{\theta}^s$ and thinks that the other's actual preferences $\boldsymbol{\theta}^o$ are correlated with $\boldsymbol{\theta}^s$. Therefore, the self combines the actions of the other $\boldsymbol{\mathcal{H}}_t^o$ with its own actions $\boldsymbol{\mathcal{H}}_t^s$ to make inferences $\bar{\boldsymbol{\theta}}^s$ about $\boldsymbol{\theta}^s$. In one version of this, the correlation arises since $\boldsymbol{\theta}^s$ and $\boldsymbol{\theta}^o$ are both assumed to be generated from a common class $\boldsymbol{\theta}^c$ preference, leading to the formula above. The resultant belief is the transference of an other's type, transmitted through their actions, onto the self's estimate of itself.

An alternative to this would be an other-insertion mirror image of the self-insertion of part (c). For instance, consider the case of a sufficiently large group of others that the self can calculate an approximation of $\overline{\boldsymbol{\theta}}^{o*}$ from $p(\overline{\boldsymbol{\theta}}^{o*}|\boldsymbol{\mathcal{H}}_t^o)$, where $\overline{\boldsymbol{\theta}}^{o*}$ is a special case where the preferences of multiple others are treated as a single group belief. The self could then use this inference:

$$P(\overline{\boldsymbol{\theta}}^s|\boldsymbol{\mathcal{H}}_t^s;\overline{\boldsymbol{\theta}}^{o*}) \propto P(a_t^s|\overline{\boldsymbol{\theta}}^s,\boldsymbol{\mathcal{H}}_{t-1}^s)P'(\overline{\boldsymbol{\theta}}^s|\boldsymbol{\mathcal{H}}_{t-1}^s;\overline{\boldsymbol{\theta}}^{o*})$$

If all group members operate in this way, inferring something about themselves from the actions of others in the group, then there may be a process of contagion and shaping that may contribute to a 'herding' steady state (Raafat et al., 2009). This process has been formulated as an arbitration between one's own experience and the consensus of others (Zhang & Glascher, 2020), where consensus of those around the self are essential in calibrating external influences on one's own knowledge. Importantly, our general formulation allows for the two types of contagion that arises both from dyadic interaction, and group consensus (Bikhchandi et al., 1992; Toelch & Dolan, 2015): in dyadic interaction only one other is considered, $\overline{\boldsymbol{\theta}}^o$, and in group interaction others are treated as a whole., $\overline{\boldsymbol{\theta}}^{o*}$. We deal with this in more detail later.

Over time, this herding process can form equilibrium states and facilitates group influence in the absence of a central coordinator; individuals within a group may naturally converge to similar beliefs. This can be through implicit contagion – the simple transfer of beliefs by exposure – or through explicit mentalising, using information cascades to deliberately consider the information of others (Raafat et al., 2009). This herding behaviour is not always optimal: groups can fail to make good decisions when information is not signalled correctly (Bang et al., 2017), or accuracy is not the target (Williams & Miyazono, 2023). Interestingly, persons with autism appear to be less susceptible to this form of social influence and make more 'rational' and sometimes even more prosocial decisions because of it (Forbes et al. 2023).

We might also consider the continual process of insertion and contagion as an ongoing feedback loop (Schilbach et al., 2013). Much as on a cultural level, whereby inter-personal beliefs form a steady state over time (Raafat et al., 2009), the back and forth of contagion and self-insertion may naturally lead to an inter-personal steady-state: representations of others become indistinguishable from oneself, or rather, oneself becomes indistinguishable from others. Movement toward a steady-state equilibrium of inter-personal self and other representations can be framed as a recursive process between self-insertion and social contagion over an infinite time horizon, leading to synchronous action (Figure 3). This can be imagined as a biased process, consistent with attractor state models of social cognition (Vallacher & Nowak, 2007; Vallacher et al., 2015), where beliefs may carry certain inertia, or weight, meaning contagion and insertion may converge at unequal rates.

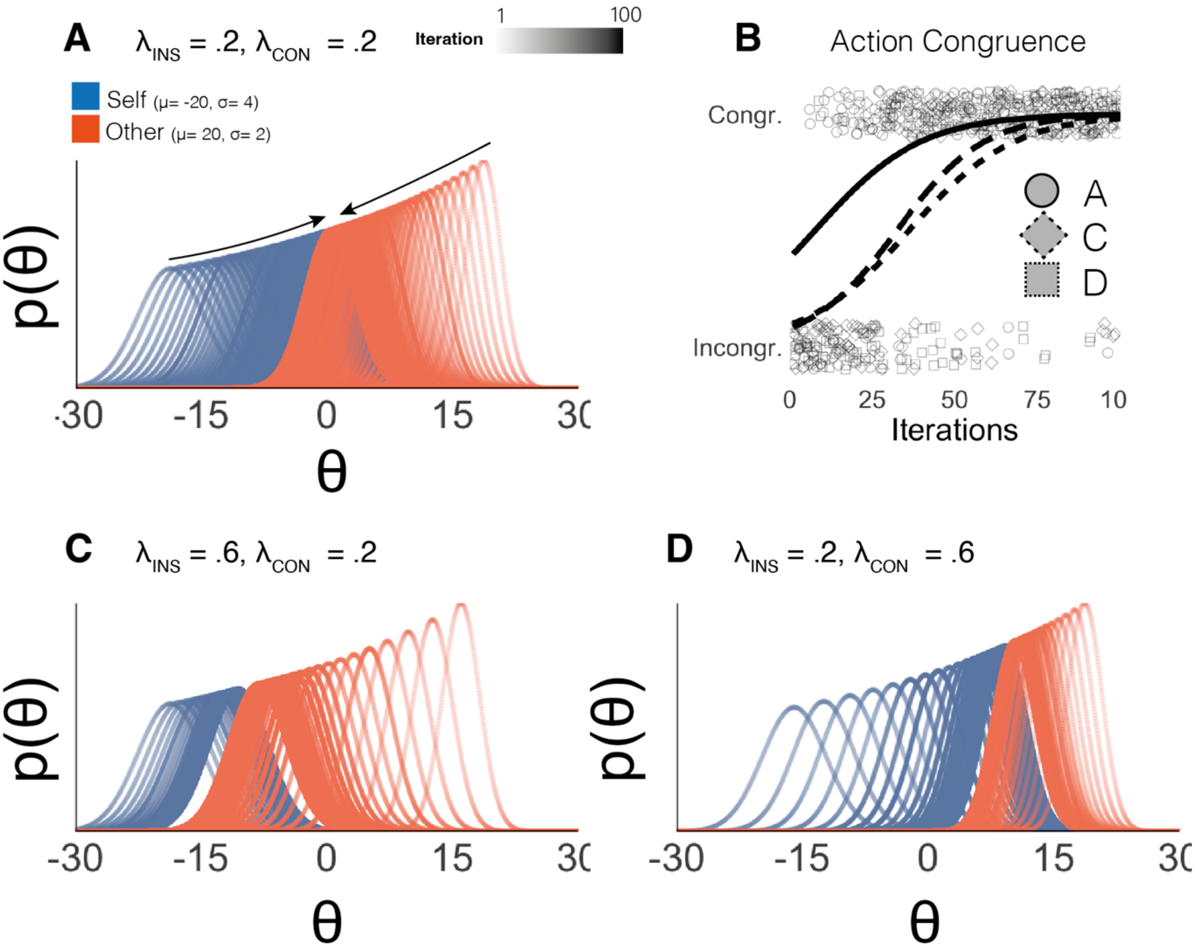


**Figure 3. Toy model of interpersonal convergence. (A)** Two prior beliefs of self $P(\boldsymbol{\theta}^s|\mu^s = -20, \sigma^s = 4)$ and other, $P(\boldsymbol{\theta}^o|\mu^o = 20, \sigma^o = 2)$. Over a number of iterations (here, 1-100) the expected probability of an other's actions are conditioned on the value of the self, $P(\boldsymbol{\mathcal{H}}_t^o|\boldsymbol{\theta}^s)$, and begin to converge with the expected probability of a self's actions given the other $P(\boldsymbol{\mathcal{H}}_t^s|\boldsymbol{\theta}^o)$. **(B)** Updating of representations occur when actions do not match (incongruence; $a^s \neq a^o$); this occurs at different rates ($\lambda_{INS}$ = rate for movement of other representations toward the self; $\lambda_{CON}$ = rate for self-representations toward the other). Here, the expectation over the difference in value of $\boldsymbol{\theta}^s$ and $\boldsymbol{\theta}^o$ approaches 0 at different rates, leading to increased frequency of congruent actions (where $a^s = a^o$). The logistic curves represent the probability that actions sampled from each distribution become congruent over each iteration. **(C & D)** When contagion and insertion become unevenly applied, for example in the case of a particularly resistant self-representation or undue influence from the preferences of others, respectively, the resultant convergence is either biased toward the original representation of self (C) or other (D). For the generative model that created these simulations see the Supplementary Materials. See Text S1 for the formalism.

## 4. Other

We next consider inferences a self makes about an other at different levels of the cognitive hierarchy. The goal of the following set of relationships is to infer and represent a single object, $\boldsymbol{\theta}^o$. Similar to past work, we consider shallow, group, and hierarchical mentalising (Barnby et al., 2023a). This involves inference using only the observed history of an other, $\boldsymbol{\mathcal{H}}_t^o$.

### a. Shallow Mentalising

$$P(\overline{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t^o) \propto P(a_t^o|\overline{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}^o)P(\overline{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_{t-1}^o)$$

At the bottom of the cognitive hierarchy, shallow Depth-of-Mentalisation (DoM 0) allow the self to represent an other's state given their total interaction history, $\boldsymbol{\mathcal{H}}_t^o$, and with a model of the other, $\overline{\boldsymbol{\theta}}^o$ (Figure 4). This latter term is essential to ToM: beliefs emanate from some internal representation about others and is the mirror-image to our general formulation for meta-cognition, the difference being the target of inference ($\overline{\boldsymbol{\theta}}^o$ vs $\overline{\boldsymbol{\theta}}^s$). This is a parsimonious explanation in light of empirical evidence that demonstrates social evaluations are generated early in interactions (Bone et al., 2023; Moutoussis et al., 2023), or the inversion of another's model following observation (Ng & Russell, 2000; Jara-Ettinger, 2019). This representation may be refined later using approximate Bayesian inference models, e.g. in attributional tasks (Barnby et al., 2022) or emotion recognition (Houlihan et al., 2023).

This form of learning is shallow in the sense of not including recursive thinking (the other is assumed not to model the self), and so may be important in cases in which hierarchical social planning is restricted (Hula et al., 2015) or when selves are simply observing others, rather than interacting with them (in a game-theoretic sense), or more realistically when an AI agent is programmed to ignore recursive beliefs. This class of mentalising formalisations can be useful for creating representations for a number of psychological processes, such as rational action planning (Baker et al., 2017), reward discounting (Chong et al., 2017), or modelling the value of an other's cumulative reward (Zhang & Glascher, 2020).

As mentioned before, this prior model of the other may arise from a model of the self (as considered above) or be developed from the observation of others in different contexts, for example, when generalising beliefs about the actions of a partner in the game Fire, Grass, Water after observing them in the analogous game Rock, Paper, Scissors (Guennouni & Speekenbrink, 2022; Robalino & Robson, 2012). We might also consider that a self's prior beliefs are about the population in general. This has been integrated into models of paranoia (Barnby et al., 2020; 2022) and borderline personality disorder (Siegel et al., 2020), where beliefs about an immediate partner may be influenced by prior beliefs about the untrustworthiness or threatening nature of others in general.

### b. Group Mentalising

Although most models aim to understand social interaction on a one-to-one level, understanding and strategising based on group characteristics, such as collective beliefs, actions, and group identity, are also a key part of navigating the social world. This form of belief is essential when considering the dynamics of stereotyping (Stewart & Raihani, 2023), herding behaviours (Raafat et al., 2009), and intergroup reciprocity or threat (Boyer et al., 2015). We consider two types of formulation which have been used to infer and represent the structure of groups.

In-group estimation approximates how a self represents whether an other is part of the same group. Computational models of social structures have formulated this as the probability of actions being congruent given past observation history (Gershman et al., 2017; Gershman & Cikara, 2020; Lau et al., 2018; 2021). Here, the self is required to infer the group membership by observing patterns of preferences from others, which include socially benign characteristics (e.g., movie choices) or more affectively charged characteristics, such as political and moral beliefs. These models explain the statistical process through which selves assign others to social clusters, and how influential group members may distort social categorisation (e.g., social similarity).

We can also consider how a self may entertain estimates and inferences about members of another group as a whole. For small groups, it may be possible individuate members and carry out separate inferences as in figure 4. Here, shallow mentalising is used to individually hold representations of others are separate, non-interactive distributions. As the group grows in size, this becomes less possible. One strategy might be to coarse-grain the characteristics considered (Jackson et al., 2023), and generalize across a large range of contexts.

As the group grows even further, individuation might become computationally infeasible. In this case a self might compute the state of a group or sub-group (Rojek-Giffin et al., 2023) based on the special case of $\bar{\boldsymbol{\theta}}^{o*}$ which generalises a single distribution across all members:

$$P(\bar{\boldsymbol{\theta}}^{o*}|\boldsymbol{\mathcal{H}}_t^o) \propto \prod_{i=1}^{N} P(a_t^{o,i}|\bar{\boldsymbol{\theta}}^{o,i}, \boldsymbol{\mathcal{H}}_{t-1}^{o,i})P(\bar{\boldsymbol{\theta}}^{o,i}|\boldsymbol{\mathcal{H}}_{t-1}^{o,i})$$

Such beliefs have been examined in the special case of the Public Goods Game (PGG), in which selves secretly decide how much money to contribute to the group. The total amount is multiplied by a factor (between 1 and the total number of players) and is then distributed evenly among all players, including any who declined to contribute themselves. Modelling of the PGG has considered that selves statistically represent a group as a single entity (Khalvati et al., 2021) – the fixed effect structure shown in the formula. The advantage of this heuristic is avoiding the computational complexity of fractionating individual members, although it leads to stereotype biases (Boyer et al., 2015). In this biased case, the inferred characteristics of a group becomes a prior $P(\bar{\boldsymbol{\theta}}^{o*}|\boldsymbol{\mathcal{H}}_t^o)$ that biases the likelihood when inferring about individual members: $P(\bar{\boldsymbol{\theta}}^{o}|\boldsymbol{\mathcal{H}}_t^o) \propto P(a_t^o|\bar{\boldsymbol{\theta}}^{o}, \boldsymbol{\mathcal{H}}_{t-1}^o)P(\bar{\boldsymbol{\theta}}^{o*}|\boldsymbol{\mathcal{H}}_t^o)$.

This heuristic consideration has been extended to use a blend of individual utility and group utility functions, estimating the joint utility of a self's own investment in the group, and their belief that others in the group would free-ride (Park et al., 2019).

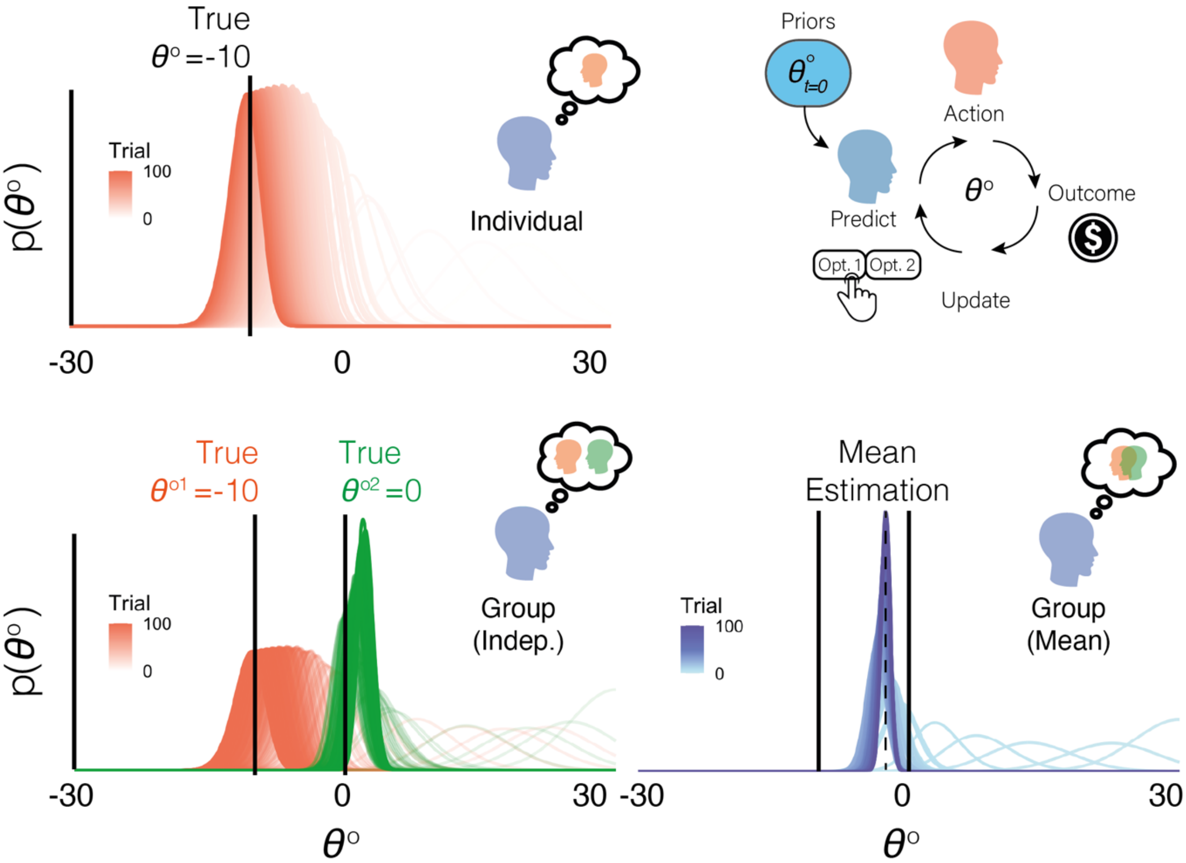


**Figure 4. Illustrations of shallow mentalising.** Here a self begins with some prior model about the other $p(\bar{\boldsymbol{\theta}}^o) \sim \mathcal{N}(\bar{\boldsymbol{\theta}}^o; \mu = 30, \sigma = 3)$, and updates them following observed decisions, $\boldsymbol{\mathcal{H}}_t^o$, of an other(s) over 100 trials. This begins to build a representation(s) of a single partner (top left) or multiple partners (bottom-left); the latter may be examples where others are observed interacting (third person) rather than the self interacting themselves (first or second person). We might also consider a process where, in line with prior work (Khalvati et al., 2021) that a self estimates the multiple partners as the fixed representation of the group's beliefs given their combined actions (bottom-right). See Text S2 for the formalism.

### c. Hierarchical Mentalising

$$b_k(\bar{\boldsymbol{\theta}}^o) = P_k(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t) \propto P_{k-1}(a_t^o|\bar{\boldsymbol{\theta}}^o,\boldsymbol{\mathcal{H}}_{t-1})P_k(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_{t-1})$$

We can also consider hierarchical depth of mentalising (DoM > 0) about a self's belief of an other's model at level $k \geq 0$ (Costa-Gomes et al., 2001; Camerer et al., 2004; Stahl, 1993). In the recursion, the level $k = 0$ self is the shallow mentaliser above in section 4a; a level $k = 2$ self considers that it is playing with a level $k = 1$ other, which itself models the self as the shallow mentaliser ($k = 0$), and so forth. In this sense, every $k$ level contains the nested beliefs of all $k$ levels below it.

Cases of hierarchical mentalising are typically employed in strategic planning, for example, when deciding how much to invest in a partner within micro-economic games (Hula et al., 2015), or how best to communicate language to match the internal concepts of an other (Goodman & Frank, 2016; Scontras et al., 2021).

We note $b_k(\bar{\boldsymbol{\theta}}^o|\boldsymbol{\mathcal{H}}_t)$ as a self's belief about an other's $\bar{\boldsymbol{\theta}}^o$ at recursive level $\mathrm{DoM}(k)$, and therefore how a self should act considering the possible behaviours of their partner in the future. In particular, the likelihood in the above equation (i.e., $P_{k-1}(a_t^o|\bar{\boldsymbol{\theta}}^o,\boldsymbol{\mathcal{H}}_{t-1})$) is computed based on an other's psychological state and model at a level lower than the one a self is currently planning (i.e., $k-1$). Depending on the DoM, this might entail that the other decides on their course of action $a_t^o$ based on their beliefs about a self's psychological state and model at a level yet lower than the one the agent is currently using to reason (i.e., $P_{k-2}(\bar{\boldsymbol{\theta}}^s|\boldsymbol{\mathcal{H}}_{t-1})$), and so on until an unintentional model ($\mathrm{DoM} = -1$) is hit (Gmytrasiewicz & Doshi, 2004, 2005). Unintentional models are absent of Theory of Mind and simply act based on a policy, disregarding the beliefs of others, and being incapable of forming any such belief. For an illustration see Figure 5.

Crucially, a self may apply different DoM levels under different environmental pressures e.g. in cooperative versus competitive settings. For example, deeper mentalisation could be beneficial for manipulating and outsmarting the opponent, and to predict the opponent's actions (Alon et al., 2022; Devaine, et al., 2014; De Weerd et al., 2017). This shows that higher DoM in competitive contexts is advantageous. On the other hand, shallow mentalisation may be more appropriate in cooperative or prosocial situations, where simple social value orientation biases may be a computationally efficient solution (Yoshida et al., 2008; Devaine, et al., 2014). In fact, miscalibration of mentalising is detrimental, leading to erroneous inferences. The over-estimation of intentional depth has also been termed 'hypermentalistion' that has been cited as the basis of the over-attribution of negative intentions to others in psychopathology (McClaren et al., 2022; Sharp, et al., 2015). This has recently been formalised in synthetic environments: reward-maximising selves that are imbued with overly high DoM assume random outcomes are being produced by a deceptive others (Alon et al., 2024a), demonstrating that simple miscalibrations in Bayes-optimal systems can lead to false beliefs.

An important limitation when formulating these models are the logical constraints on inference within the theory. In these models, a self is only able to make inferences about others lower in the hierarchy. Being able to truly model and infer about others above oneself in the hierarchy violates key principles in game theory (Pacuit & Roy, 2017), although this has recently been addressed by proposing that those on lower levels may be able to use behavioural pattern matching to match expected beliefs of an opponent to observations, thereby being wary of any unexpected, and potentially deceptive markers (Alon et al., 2024b)

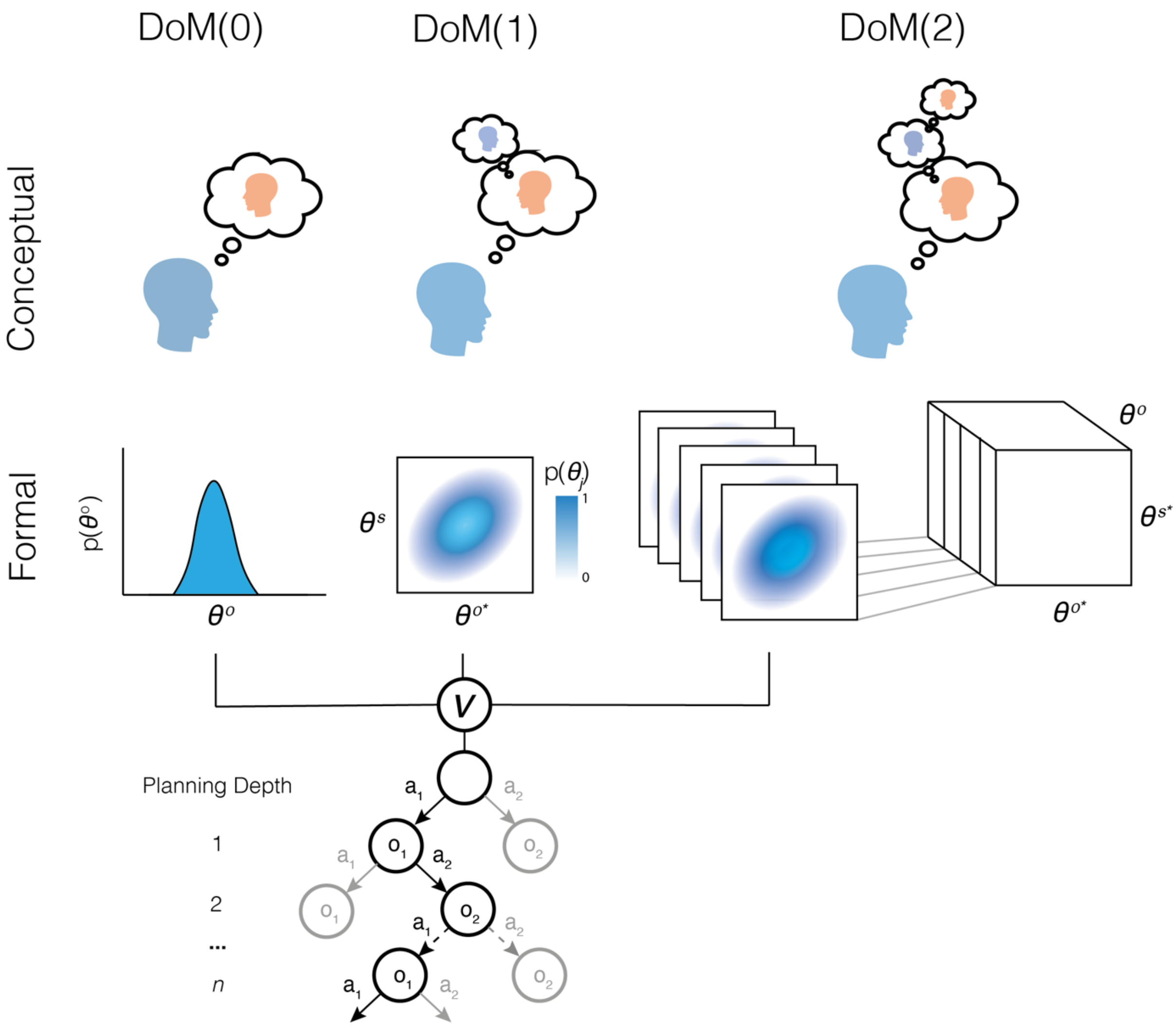


**Figure 5. Illustration of hierarchical mentalising.** Each level of DoM contains a model of *k* depth. This forms the basis of the expected utility given an other's policy, and thus a notional action $\boldsymbol{a} = \{a_1, a_2\}$ and corresponding predicted observation, $\boldsymbol{o} = \{o_1, o_2\}$, predicated on the beliefs about the other $b_k(\bar{\boldsymbol{\theta}}^o)$. Starred variables indicate that the variable is nested within the self's extrapolated model from the highest level. *V* = utility function. DoM = Depth of Mentalisation.

## 5. Neural Implementation of Theory of Mind

The neural basis of ToM has been studied extensively for nearly thirty years using a wide variety of tasks, including false belief tasks, strategic games, and social animations (Gilead and Ochsner, 2021). A meta-analysis of 73 such studies (Schurz *et al.*, 2014) identified robust neural correlates of ToM, consisting principally of mid-line brain regions (mPFC and precuneus), as well as the posterior TPJ. This system overlaps, to some extent, with the brain's so-called default network (Schilbach et al., 2008), which has been associated with solving problems involving social cognition (Jack et al., 2013). However, there is much less certainly about the precise roles of the component regions. Indeed, all these regions contain distinct subregions and are likely to have more than one role.

Activity in mPFC is elicited when thinking about the traits of self and others (i.e., their position in the state space (Denny et al., 2012; here $\overline{\boldsymbol{\theta}}^s$ and $\overline{\boldsymbol{\theta}}^o$) and is engaged when learning about the traits of others (shallow mentalising), such as their trustworthiness (Behrens et al., 2008; Fouragnan et al., 2013). mPFC also has a role in metacognition, i.e., reflecting on the mental states of self and others (Vaccaro and Fleming, 2018) and in hierarchical mentalising (Yoshida et al., 2010).

TPJ has a role in predicting behaviour based on mental states and other available information about a person (Saxe and Kanwisher, 2003). The form of this information can vary from a direct sensory interaction to a verbal description of an interaction. Enhanced activity is elicited in this region when people do not behave as expected (Behrens et al., 2008; Hampton et al., 2008).

Much less has been said about the role of the precuneus. An interesting suggestion is that it is concerned with navigation in social state space (Tavares et al., 2015).

## 6. Considerations for Artificial Agents

Our framework has implications for the development of Large Language Models (LLMs). LLMs have been cited as possessing some degree of ToM from performance tests in domains of complex language (Brown et al., 2020), reasoning (Hao et al., 2023) and image recognition (Esteva et al., 2017). However, what constitutes the necessary and sufficient conditions for ToM is an ongoing debate.

In 'simulated ToM' accounts (Sterck & Begeer, 2010) ToM is an acquired skill rather than a native trait: ToM may arise spontaneously given a sufficiently complex system based on generic algorithmic rules. This was recently tested by querying OpenAI's GPT-3 model with two sets of standard false-belief tasks (Kosinski, 2023). GPT-3 excelled, extending its response to include a mental description of the agent. Kosinski argues that these results may be explained by two competing hypotheses. The first, the "representation-negative" hypothesis, suggests that GPT-3 can correctly solve these tasks through some (unknown) statistical regularities without the need for a generalisable representation. The second hypothesis is "representation-positive": the use of a clear generalisable representation of the world.

A more critical issue, and one which may be used to assess the above, is model transparency and efficiency. Understanding *how* a model is working is essential to purposefully build more responsive human-AI interfaces, and to improve *in silico* models of human cognition. Performance estimates have been achieved so far using black-box inferences of representations following LLMs performance, requiring interpretation of specific beliefs *a priori* using some function-approximation mechanism (e.g. Oguntola et al., 2023). Likewise, allowing greater efficiency may reduce the expense of continually needing to scale models. With a form of model hierarchy we suggest, or with the use of user-modelling, LLMs can model context to constrain parameter weights to generalise across individual tasks/prompts.

## 7. Disordered and diverse Theory of Mind

A framework that allows theories to extend beyond the traditional focus on deficits in representing mental states of others has the potential to model how dynamic social processes are important in a number of psychiatric conditions.

Borderline personality disorder (BPD) describes difficulties with interpersonal relationships, with sometimes frantic efforts to avoid perceived abandonment, dissociation, and impulsivity. Despite clear challenges with perceiving others' intentions in relationships, meta-analytic studies have found no evidence for 'theory of mind' deficits (Bora, 2021; Németh, 2018). Computational approaches allow the modelling of dynamic interactions between representations of others, emotions and actions suggest social rupture in BPD may be explained by irritability during long term cooperation (Hula et al., 2018), greater focus on social cues (Henco et al., 2020), and less asymmetric adaptive updating (Siegel et al., 2020).

Paranoid delusions are inaccurate but fervently held beliefs that others are intending to harm the self. This may be explained by a difficulty in processing volatile environments (Sheffield et al., 2022) or uncertainty over the model of others (Barnby et al., 2022b). This may be underpinned by D2 dopamine, where blockade of D2 dopamine enables more flexible encoding of an other's model leading to greater trust (Mikus et al., 2022) and reduced attributions of harmful intent (Barnby et al., 2023b).

Addiction is the end result of a process that involves the neurobiological effect of the substance, maladaptive coping, and social influence. Traditional models focused on explaining reward learning and how decision-making is affected by context – including social context. Computational models of addiction have largely focused on reward-learning (Mollick and Kober, 2020). van den Ende et al (2022) note how the social environment is ignored or over-simplified in such models and we highlight the potential for a computational framework of social processes to allow an integration between reward learning and social context in the same model.

## 8. Moving Forward

We have synthesised a range of social processes in terms of a common computational framework that makes specific, falsifiable predictions about the dynamic interplay between representations of self and other. This forms a set of theories that can accommodate individual model formulations and at the same time allows bridging between different social processes formulated in the literature. However, we highlight some outstanding questions and future directions that may be useful next steps for cognitive science, neuroscience, computer science, and psychiatry.

The issue of how group size impacts on the ability for a self to hold separate representations of each member, and whether larger groups demand greater heuristic approximations (Dunbar & Schulz, 2021) remains outstanding. As mentioned, in competitive scenarios, individuals have been formulated to hold a unitary representation of a group (Khalvati et al., 2021). How this unfolds in larger scenarios, such as when estimating the generalisation of attributes across members of a larger group with stereotypes (Stewart & Raihani, 2023), or when generalising an attribute of a group member to other qualities of their person (Jackson et al., 2023) is still unclear. There are three hypotheses to consider: 1) computational capacities of groups scale with their size to accommodate discrete member representation (David-Barrett & Dunbar, 2013), 2) increasingly computationally 'cheap' heuristics are employed to enable efficient generalisation of traits and states across a group at the cost of detail (Jackson et al., 2023), 3) there is a hybrid model averaging process that, analogously to hierarchical Bayesian estimation (Huys et al., 2012), allows an internal constraint within the self of individual member representations under some group distribution. We suggest running experiments where participants are required to keep in mind the preferences or policies of several different group members, both coherent and incoherent in their decisions, and assessing the degree to which candidate heuristic models may outperform models with higher information fidelity or hierarchical group constraints.

It is also unclear how we hierarchically structure recursive representations. Typical models that employ DoM($k$>0) assume a truncated $k$-level hierarchy where one agent perceives the other to be at level $k$-1, that is, a DoM(1) self would assume an other to be DoM(0); a DoM(2) other would assume a self to be a DoM(1), and also on (Chong et al., 2016). This system has so far been used in contexts to assess the DoM optimal for competitive versus prosocial environments (Doshi et al., 2015), how DoM differs as a function of psychiatric diagnosis (Hula et al., 2015), or how mismatched DoM may produce suspiciousness (Alon et al., 2023). However, we might also consider a Poisson hierarchy of DoM, rather than k-level reasoning; that is, a self may hold a skewed distribution over the probability that an other is reasoning at a particular DoM level (Camerer, 2004). Assessing whether this model may be more amenable to flexible DoM change over time is essential to explain how (for example) a self may change their DoM level to match an others, or even to detect a greater probability that an other may be at a greater DoM, even if there is no ability for the self to adjust their own sophistication.

Relatedly, increasingly complicated hierarchical inference structures only partially help decisions under uncertainty. Instead of relying on such hierarchical inferences, more sophisticated utility functions can achieve similar behavioural sophistication with less computational resources. For example, while ToM-like hierarchical inferences about other agents' beliefs seem to help the development of cooperation between agents with overly simplistic utility functions, similar sophisticated behaviours can more parsimoniously be achieved with agents that do not engage in inferences but have adaptive better value functions (Yoshida et al., 2008). Research so far has not devoted much effort to optimize value functions. Moreover, our Bayesian approach assumes that the whole history of observations (e.g., $\boldsymbol{\mathcal{H}}_t^o$) has the same weight on inferences on particular quantities (e.g., $\overline{\boldsymbol{\theta}}^o$), equally contributing to a person's beliefs about an other. Indeed, observations may be differentially weighted. For instance, research on impression formation has shown temporal (primacy and recency) effects, as well as contextual and valence effects (Asch 1946; Bellucci 2023; Matovic & Forgas, 2018; Hecler et al., 2016).

Is it not only the rather complicated problems that we have been considering that risk intractability, but inference is also often complicated by a vast hypothesis and action space. To improve noise robustness, human might discretize the upcoming information into the bins that matter for hidden state inference (Lisi et al., 2021); this simplifies Bayesian inference whilst also introducing (local) quantization biases. Testing across all three axes of the cognitive hierarchy - Depth of Mentalisation, planning depth, and social bias – may further refine multiple routes to similar phenomena and get closer to when and where recursive abilities may be useful or a hindrance to successful social outcomes.

Finally, we point out that the current state of the art of empirical paradigms – microeconomic games in our context – are not the only way to tackle the computational dynamics underlying ToM. In particular, current paradigms might not be variable and complex enough to necessitate ToM processes, which may only be engaged for adaptive behaviour in more uncertain and volatile environments. Combining well controlled, interactive paradigms with real-world social measurements (e.g. Dumas et al., 2011; Bolis et al., 2022) may allow triangulation of naturalistic and theoretically important markers of interaction.

**CRediT**

**JMB:** Conceptualisation, Data Curation, Formal Analysis, Methodology, Project Administration, Software, Visualisation, Writing – Original Draft, Writing – Review and Editing. **GB:** Writing – Original Draft, Writing – Review and Editing. **NA, LS, CDS & VB:** Writing – Review and Editing.

**Conflicts of Interest**

None to declare.

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**Supplementary Materials**

**Table 1. Formal summary of Theory of Mind components within our framework.**

| ToM Construct | Distribution | Proportional Calculation |
|---|---|---|
| Experimenter approximation of self | $P(\widehat{\boldsymbol{\theta}}^s \vert \boldsymbol{\mathcal{H}}_t^s)$ | $P(a_t^s \vert \widehat{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \boldsymbol{\mathcal{H}}_{t-1}^s) P(\widehat{\boldsymbol{\theta}}^s \vert \boldsymbol{\mathcal{H}}_{t-1}^s)$ |
| Metacognition without input noise | $P(\overline{\boldsymbol{\theta}}^s \vert \boldsymbol{\mathcal{H}}_t^s)$ | $P(a_t^s \vert \overline{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \boldsymbol{\mathcal{H}}_{t-1}^s) P(\overline{\boldsymbol{\theta}}^s \vert \boldsymbol{\mathcal{H}}_{t-1}^s)$ |
| Metacognition with input noise | $P(\overline{\boldsymbol{\theta}}^s \vert \overline{\boldsymbol{\mathcal{H}}}_t^s)$ | $\int d\mathbf{x}_t^s \, P(a_t^s \vert \overline{\boldsymbol{\theta}}^s, \mathbf{x}_t^s, \overline{\boldsymbol{\mathcal{H}}}_{t-1}^s) P(\overline{\boldsymbol{\theta}}^s \vert \overline{\boldsymbol{\mathcal{H}}}_{t-1}^s) \, p(\overline{\mathbf{x}}_t^s \vert \mathbf{x}_t^s)$ |
| Self-Insertion | $P(\overline{\boldsymbol{\theta}}^o \vert \boldsymbol{\mathcal{H}}_t^o; \boldsymbol{\theta}^s)$ | $P(a_t^o \vert \overline{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}^o) P'(\overline{\boldsymbol{\theta}}^o \vert \boldsymbol{\mathcal{H}}_{t-1}^o; \boldsymbol{\theta}^{\boldsymbol{s}})$ |
| Social contagion | $P(\overline{\boldsymbol{\theta}}^s \vert \boldsymbol{\mathcal{H}}_t^o)$ | $P(\boldsymbol{\mathcal{H}}_t^s \vert \overline{\boldsymbol{\theta}}^s) \int d\overline{\boldsymbol{\theta}}^c d\overline{\boldsymbol{\theta}}^o \, P(\boldsymbol{\mathcal{H}}_t^o \vert \overline{\boldsymbol{\theta}}^o) P(\overline{\boldsymbol{\theta}}^s \vert \overline{\boldsymbol{\theta}}^c) P(\overline{\boldsymbol{\theta}}^o \vert \overline{\boldsymbol{\theta}}^c) P(\overline{\boldsymbol{\theta}}^c)$ |
| Shallow mentalising | $P(\overline{\boldsymbol{\theta}}^o \vert \boldsymbol{\mathcal{H}}_t^o)$ | $P(a_t^o \vert \overline{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}^o) P(\overline{\boldsymbol{\theta}}^o \vert \boldsymbol{\mathcal{H}}_{t-1}^o)$ |
| Group mentalising | $P(\overline{\boldsymbol{\theta}}^{o*} \vert \boldsymbol{\mathcal{H}}_t^o)$ | $\prod_{i=1}^{N} P(a_t^{o,i} \vert \overline{\boldsymbol{\theta}}^{o,i}, \boldsymbol{\mathcal{H}}_{t-1}^{o,i}) P(\overline{\boldsymbol{\theta}}^{o,i} \vert \boldsymbol{\mathcal{H}}_{t-1}^{o,i})$ |
| Hierarchical Depth of Mentalising (DoM) | $P_k(\overline{\boldsymbol{\theta}}^o \vert \boldsymbol{\mathcal{H}}_t)$ | $P_{k-1}(a_t^o \vert \overline{\boldsymbol{\theta}}^o, \boldsymbol{\mathcal{H}}_{t-1}) P_k(\overline{\boldsymbol{\theta}}^o \vert \boldsymbol{\mathcal{H}}_{t-1})$ |

**Text S1. Formalism for intra-personal steady state beliefs.**

Both interaction partners evaluate options using the following utility function:

$$U_t^{\theta} = r_t^s + \theta \cdot \max(r_t^s - r_t^o, 0)$$

The intra-personal representations of the self and the other are drawn from normal distributions for simplicity.

$$p(\theta_{t0}^s) \sim \mathcal{N}(\theta_{t0}^s; \mu_{t0}^s, \Sigma^s)$$

$$p(\theta_{t0}^o) \sim \mathcal{N}(\theta_{t0}^o; \mu_{t0}^o, \Sigma^o)$$

At each time point, *t*, each partner makes an action, choosing between two alternative forced-choice option pairs **R = {R1, R2}**.

$$p(a_t^s = 1 | \boldsymbol{R}, \boldsymbol{\mathcal{A}}_t^s) = \sum_{\theta^s} p(\theta_t^s) \cdot \sigma\left(U_t^{\theta s}[\boldsymbol{R1}] - U_t^{\theta s}[\boldsymbol{R2}]\right)$$

$$p(a_t^o = 1 | \boldsymbol{R}, \boldsymbol{\mathcal{A}}_t^o) = \sum_{\theta^o} p(\theta_t^o) \cdot \sigma\left(U_t^{\theta o}[\boldsymbol{R1}] - U_t^{\theta o}[\boldsymbol{R2}]\right)$$

Each partner is able to observe the actions of the other after each trial. The expectations over each representation are premised on the parameters that govern the opposing representation, so a self expects the other to make the same action, and vice versa.

In the case of incongruent actions ($a^s \neq a^o$), the parameters that govern each representation are then updated in line with the discrepancy of the expectation and the actual value of the distribution. For social contagion, the self's representation is updated based on the representation of the other at rate, $\lambda_{CON}$. For social insertion, the other's representation is updated based on the self at rate $\lambda_{INS}$.

$$\hat{\mu}_t^s = \mu_t^s - \lambda_{CON} \frac{\mathbb{E}_t^{\mu^s}[\mu_t^o] - \mu_t^o}{\Sigma^s + \Sigma^o}$$

$$\hat{\Sigma}_t^s = \Sigma_t^s - \lambda_{CON} \frac{\mathbb{E}_t^{\Sigma^s}[\Sigma_t^o] - \Sigma_t^o}{\Sigma_t^s + \Sigma_t^o}$$

$$\hat{\mu}_t^o = \mu_t^o - \lambda_{INS} \frac{\mathbb{E}_t^{\mu^o}[\mu_t^s] - \mu_t^s}{\Sigma^s + \Sigma^o}$$

$$\hat{\Sigma}_t^o = \Sigma_t^o - \lambda_{CON} \frac{\mathbb{E}_t^{\Sigma^o}[\Sigma_t^s] - \Sigma_t^s}{\Sigma_t^o + \Sigma_t^s}$$

This update forms the prior representation taken into the next time point where social insertion and contagion will continue: $p(\theta_{t+1}^s) = p(\theta^s | \hat{\mu}_t^s, \hat{\Sigma}_t^s); p(\theta_{t+1}^o) = p(\theta^o | \hat{\mu}_t^o, \hat{\Sigma}_t^o)$

**Text S2. Formalism for individual and group mentalisation (shallow)**

The partner evaluates options using the following utility function:

$$U_t^\theta = r_t^s + \theta^o \cdot \max(r_t^s - r_t^o, 0)$$

$$\theta^o = -10$$

Using a sigmoid function $\sigma(.)$, the probability that option 1 would be used as the partner's decision was like so, $p(\boldsymbol{\mathcal{A}}_t^o = Opt1|\theta^o) = \sigma(U_t^\theta\{Opt1\} - U_t^\theta\{Opt2\})$, where outcomes $Opt1$, and $Opt2$ were emitted based on a stochastic sampling process using $p\left(\boldsymbol{\mathcal{A}}_t^o = Opt1\middle|\theta_j^o\right)$ and 1-$p(\boldsymbol{\mathcal{A}}_t^o = Opt1|\theta^o)$ on each trial, respectively.

On each trial, the participant then observed actions from the partner and calculated the full probability of each option given the utility function described above by substituting the single value the partner used to generate actions in the above equation with a full vector of values, $\theta_j^o$:

$$p(\boldsymbol{\mathcal{A}}_t^o = Opt1|\theta^o) = \sigma(U_t^\theta\{Opt1\} - U_t^\theta\{Opt2\}) \cdot p(\theta^o)$$

$$p(\boldsymbol{\mathcal{A}}_t^o = Opt2|\theta^o) = 1 - p(\boldsymbol{\mathcal{A}}_t^o = Opt1|\theta^o)$$

This formed a product with their prior beliefs about the partner, $p(\bar{\theta}^o) = p(\theta^o) \sim \mathcal{N}(\theta^o; \mu = 7.5, \Sigma = 3)$, to form an updated posterior for the next trial:

$$p(\bar{\theta}^o|\boldsymbol{\mathcal{A}}_{t-1}^o) = \frac{p(\boldsymbol{\mathcal{A}}_{t-1}^o|\theta^o)p(\bar{\theta}^o)}{\sum_{\theta^o} p(\boldsymbol{\mathcal{A}}_{t-1}^o|\theta^o)p(\bar{\theta}^o)}$$

In the case where more than one partner is being learned about, there are two generative models that might be used. One the one hand, participants may retain two different representations. In our example, this was following the observation of actions from two different partners; the first generated actions based on $\theta^{o1} = -7.5$, and the second with $\theta^{o2} = 0$. Given the same priors, $p(\bar{\theta}^o)$, the participant then updated two different distributions, where $i = \{o1, o2\}$.

$$p\left(\bar{\boldsymbol{\theta}}^{o,i}\middle|\boldsymbol{\mathcal{A}}_{t-1}^{o,i}\right) = \frac{p(a_t^{o,i}|\bar{\boldsymbol{\theta}}^{o,i}, \boldsymbol{\mathcal{A}}_{t-1}^{o,i})P\left(\bar{\boldsymbol{\theta}}^{o,i}\right)}{\sum_{\theta^o} p(a_t^{o,i}|\bar{\boldsymbol{\theta}}^{o,i}, \boldsymbol{\mathcal{A}}_{t-1}^{o,i})P(\bar{\boldsymbol{\theta}}^{o,i})}$$

The representation of partners may also be a mean representation, where the product of the numerators are used to update beliefs following normalisation:

$$P(\bar{\boldsymbol{\theta}}^{o*}|\boldsymbol{\mathcal{A}}_t^o) \propto \prod_{i=1}^{N} P(a_t^{o,i}|\bar{\boldsymbol{\theta}}^{o,i}, \boldsymbol{\mathcal{A}}_{t-1}^{o,i})P\left(\bar{\boldsymbol{\theta}}^{o,i}\right)$$