STRONGLY NP-HARDNESS OF SQUARE-FREE SUBSET PRODUCT

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Definition 1. (SQUARE-FREE SUBSET PRODUCT problem): We are given $3N$ integers; find N of them whose product is square free.

Claim 2. *We have that $X3C \leq_p$ SQUARE-FREE SUBSET PRODUCT*

In this claim, X3C denotes the well-known Exact Cover by 3 Set problem. Garey and Johnson [1] prove its strong **NP**-hardness. Now, we prove our claim.

Proof. Starting from a an Exact Cover by 3 sets (X3C) instance (strongly NPC) label each element of the universe with a distinct prime (you can generate $3|X|$ of them in polynomial time); then convert every triple (x, y, z) of the subsets to xyz . □

It obviously resembles the better known SUBSET PRODUCT (which is not strongly NPC due to the presence of the target product B , see [2]).

It can also be modified a little bit to get other variants, like:

- Given $3N$ integers, find N of them whose product is a perfect 21-th power;
- Given N integers, find a subset whose product is the $3N$ -th primorial.

CONCLUSION

As long as we study a mathematical conjecture, we should encourage ourselves of having enough labouring hours on Prasolov and Sharygin maths books. Then, reading some articles on theory of computing like [3] is a good practice. Only after that, could we think of the ultimate final for all mathematics sciences.

REFERENCES

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Key words and phrases. product, square-free, subset.
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