Trade Policy and the Marshall–Lerner Condition: Application of the Tobit Model

Kazuto Masuda

Public Relations Department, Bank of Japan

Secretariat of Central Council for Financial Services Information

June 2021

Author Note

Address: 2-1-1 Hongoku-cho, Nihonbashi, Chuo-ku, Tokyo 108-8660, Japan.

Tel: 81-3-3279-1111

E-mail: kazuto.masuda1@gmail.com

Abstract

This paper establishes the micro-foundation for the income-price approach to export and import functions from the firm's profit maximization problem. Following Boyd et al. (2000), we derive the Marshall–Lerner condition mathematically and analyze the effects of home and foreign trade policies, such as the minimum access and quantitative trade restriction, on the Marshall–Lerner condition. In conclusion, such trade policies make the condition theoretically difficult to hold since the marginal effects of the Tobit estimates under deterministic trade policies are always lower than the ordinary least squares (OLS) estimates under no trade policies in absolute values.

JEL classification: C24, F13, D21

Keywords: Marshall–Lerner Condition, Micro-foundation, Tobit model, Trade policy,

Trade balance

1. Introduction

In this paper, we treat the minimum access and the quantitative trade restriction as applications of the Tobit model (Tobin, 1958). Then, we theoretically examine the effects of such trade policies on the establishment of the Marshall–Lerner condition. Also, we provide the micro-foundation of the income–price approach in terms of its export and import functions with the static profit maximization problem of the firm.

The Marshall–Lerner condition is established when the sum of the absolute values of elasticity parameter estimates of relative export price and relative import price in export and import functions is greater than one. This mathematical condition indicates the condition of establishing the intuitive phenomenon that a country's trade balance improves when the exchange rate depreciates. Thus, the establishment of the Marshall–Lerner condition secures our intuition regarding the relationship between the exchange rate and current account.

In this paper, we make the following four assumptions. First, we define the trade policy as one in which the government places lower and/or upper threshold values

on aggregate export and import quantities. If an upper threshold value for aggregate trade volume exists, we call it the quantitative trade restriction; if a lower threshold value for aggregate trade volume exists, we call it the minimum access (the trade promotion in export). We assume that these threshold values are deterministic (the stochastic case is discussed in the Appendix).

This assumption naturally requires a Tobit estimator. In the above cases, the Tobit estimator is consistent. The ordinary least squares (OLS) estimator does not satisfy consistency in the face of censoring, and the probit estimator does not provide precise elasticity parameter estimates in such cases.

Second, we suppose that homogeneous firms producing with import production goods, labor inputs and without capital stock given factor prices, exist in the competitive market of domestic industries. Few domestic industries are without import production goods, such as raw materials, processed goods, etc. We assume the profit of the firm with the free employment and disposal of its production factors. Furthermore, we assume firms' free entries and exits in and out of the competitive market (see Jehle

& Reny, 2001). Moreover, the number of firms in the home and foreign domestic industries are assumed to be the same, chronologically constant, and sufficiently large.

Third, we assume no serial correlations in prices and quantities, such as the relative prices and productions, etc. We treat the static problem, though we relax this assumption in the Appendix to include a special trend-stationary case.

Fourth, we set the price of firm production as the numeraire and assume that it coincides with the general price level in the economy. Also, this price of firm production to normalize the factor prices are assumed to be exogenous and independent of the firm's behavior. We assume perfect competition to establish the micro-foundation of the income-price approach to export and import functions. However, we include a monopoly case in a later section.

With regard to export and import functions, the income-price approach is usually adopted, which explains export and import with incomes and relative prices. Numerous theoretical and empirical analyses have been conducted on such export and import functions. Among them, Clarida (1994) performed a study based on the

theoretical import function derived from the “household utility maximization problem” in which he aimed to estimate the import function structurally.

As will become evident in subsequent sections, different from Clarida (1994), we construct the static partial equilibrium model of the firm’s profit maximization problem without capital stock, given the general price level and factor prices such as the wage, and the trade prices determined by the Phillips curves, and the foreign exchange rate market which are outside of our model.

With this static, partial equilibrium model, we succeed in drawing ample and important policy implications similar to those that are obtained with the general equilibrium model. Although we extend our static analysis to a special trend-stationary case in the Appendix, our main conclusions are unchanged.

This paper discusses the theoretical effects of trade policies on the establishment of the Marshall–Lerner condition. For reference, we introduce part of the empirical literature on the establishment of this condition. The Marshall–Lerner condition itself is also theoretically explained in numerous studies, including Ogawa

and Tokutsu (2002). Boyd et al. (2000) developed the log-linear trade balance equation to represent the Marshall–Lerner condition. Using the eight Organization for Economic Co-operation and Development (OECD) countries’ data, and data between multiple countries and the U.S. for trades, national accounts, etc., Boyd et al. (2000) and Gruber et al. (2016) argued that the Marshall–Lerner condition holds. In contrast, Ogawa and Tokutsu (2002) used Japanese data to deny the establishment of the Marshall–Lerner condition.

Furthermore, for the Tobit application, Wu (1992) applies the Tobit model to the import function in the case of quantitative trade restrictions in the empirical analysis of the Japanese peanut import function with time-series data. He reports that in the case of the quantitative import restriction, the OLS estimates were biased compared to the estimates of the Tobit model. Note that his model decomposes the price variables in import functions into the exchange rate, import price, and domestic price. This formulation had a slightly different functional form from ours but still suggested the sound empirical foundation of our Tobit formulation.

As shown, some contradicting empirical evidence exists regarding the establishment of the Marshall–Lerner condition. Following the results of Ogawa and Tokutsu (2002) and Wu (1992), we focus on the empirical denial of the Marshall–Lerner condition in Japan and proceed to discuss one possible cause for this empirical denial in trade policy.

Finally, in the Tobit model, we treat the “censored” model and not the “truncation” model. For example, in the above case of limited export and import variables, we can observe the independent variables in question, such as relative export and import prices and gross domestic products (GDPs) even under the threshold value. Following Maddala (1983), the case in this paper is the “censored” case.

The remainder of this paper is organized in the following manner. In section 2, we provide the micro-foundations of our export and import functions, based on the firm’s static profit maximization problem. We also discuss the problem of aggregation. In section 3, we explain our Tobit setups of the import function. Furthermore, we consider the relationship between the magnitudes of marginal effects of the Tobit price

elasticity estimates of export and import functions and the establishment of the Marshall–Lerner condition. Following Boyd et al. (2000), we form the log-linear trade balance equation and derive the Marshall–Lerner condition mathematically in section 4. Then, we show how trade policies interfere with the establishment of the Marshall–Lerner condition. In section 5, we briefly summarize our conclusion. In the Appendix, we address the econometric problem of estimating our model, constructing the likelihood functions for the both-sides censored case with both the level stationary process and a special trend-stationary process for model variables.

2. The Micro-Foundation of Export and Import Functions with the Income–price

Approach

2.1. The Micro-Foundation of Export and Import Functions

In this section, we provide the micro-foundation of the export and import functions from the firm’s profit maximization problem, formulated as the static partial equilibrium model.

First, let the import production goods of the firm i , $i = 1, 2, \dots, N$ at time t

be $C_{M,i,t}$; the total production of the firm i , $i = 1, 2, \dots, N$ at time t be $O_{i,t}$; the relative price of the import production goods to the price of production at time t $O_{i,t}$ be $P_{M,t}$; the labor input of the firm i , $i = 1, 2, \dots, N$ at time t be $H_{i,t}$; its real wage at time t be W_t ; the technological parameter at time t be A_t ; and the elasticity parameters for import production goods and labor input in the production function be $1 - a$, and $a \in [0, 1]$, respectively. We treat the price of the production of the firm i , $i = 1, 2, \dots, N$ at time t $O_{i,t}$ as the numeraire at time t .

Second, we make the following four assumptions: (i) Each firm i , $i = 1, 2, \dots, N$ accepts factor prices and the price of production (they are exogenous) and is homogeneous (each firm i , $i = 1, 2, \dots, N$ faces the same error term, only the aggregate shock, even when we admit error terms). We assume a competitive market of domestic industries that use import production goods in this section. (ii) There are no serial correlations or dynamic changes in the firm's production function, input goods, factor prices, and the price of production. Thus, our problem is assumed to be static. Moreover, (iii) each firm produces its output, using not only labor inputs but also import

production goods under the Cobb–Douglas production function. We assume the profit of the firms with the free employment and disposal of their production factors and with free entries and exits in and out of the market (see Jehle & Reny, 2001). Finally, (iv) the price of production at time t $O_{i,t}$ of firm i , $i = 1, 2, \dots, N$ introduced below is safely assumed to coincide with the general price level, such as the consumer price index (Gruber et al., 2016) or the GDP deflator (Ogawa & Tokutsu, 2002). Thus, the price of production at time t $O_{i,t}$ of firm i , $i = 1, 2, \dots, N$ is assumed to be the numeraire. Note that we relax some assumptions in the later sections. We also assume that individual firms i , $i = 1, 2, \dots, N$ face common relative factor prices, import production goods elasticity, and labor elasticity, as well as a common technological parameter: $P_{M,t}$, W_t , $1 - a$, a , and A_t .

Consider the individual firm i . Then, under the Cobb–Douglas production function $O_{i,t}$, this firm i 's temporal profit at time t $\pi_{i,t}$ is stated in the following manner:

$$\pi_{i,t} = O_{i,t} - P_{M,t}C_{M,i,t} - W_tH_{i,t},$$

where $O_{i,t} = A_t C_{M,i,t}^{1-a} H_{i,t}^a$, $O_{i,t}, C_{M,i,t}, H_{i,t} > 0$, and $P_{M,t}, W_t, A_t > 0$ are exogenous.

Except for the $P_{M,t}$ and $C_{M,i,t}$ terms, this kind of formulation is fairly standard and is explained in numerous microeconomics textbooks, such as Jehle and Reny (2001).

However, this formulation is, at most, an individual firm's profit maximization problem. The aggregated firm's profit maximization problem is as follows. Consider the aggregation of firm i , $i = 1, 2, \dots, N$. Then, under the Cobb–Douglas production function O_t , this aggregated firm's temporal profit at time t π_t is stated as follows:

$$\pi_t = O_t - P_{M,t} C_{M,t} - W_t H_t,$$

where $O_t = A_t C_{M,t}^{1-a} H_t^a$, $O_t, C_{M,t}, H_t > 0$, and $P_{M,t}, W_t, A_t > 0$ are exogenous.

In the above problem, we impose the summing-up procedures for model variables $O_{i,t}$, $C_{M,i,t}$, and $H_{i,t}$ (and, therefore, $\pi_{i,t}$) to obtain the respective aggregate variables O_t , $C_{M,t}$, and H_t (and, therefore, π_t), defined as $O_t = \sum_{i=1}^N O_{i,t}$, $C_{M,t} = \sum_{i=1}^N C_{M,i,t}$, and $H_t = \sum_{i=1}^N H_{i,t}$ (and $\pi_t = \sum_{i=1}^N \pi_{i,t}$). This results in the aggregate model above.

Thus, under the assumption of perfect competition (like that of the individual firm problem above), as well as given price variables and maximization of the aggregated firm's profit π_t with $C_{M,t}$ and H_t , we arrive at the following equation:

$$H_t = \frac{a}{(1-a)W_t} P_{M,t} C_{M,t}.$$

Substituting this equation into the production function, taking its natural logarithm, and representing the irrelevant factors in it as $\ln\{B(W_t, A_t)\}$, we obtain

$$\ln(C_{M,t}) = -a\ln(P_{M,t}) + \ln(O_t) + \ln\{B(W_t, A_t)\},$$

$$\text{where } \ln\{B(W_t, A_t)\} = -\ln(A_t) - a\ln\left\{\frac{a}{(1-a)W_t}\right\}. \quad (*)$$

Here, the residual factor $\ln\{B(W_t, A_t)\}$ is (nonlinear since we take a natural logarithm, and) the increasing function of the real wage W_t and the decreasing function of the technological parameter A_t . We will now make the additional assumption that $W_t = W$ and $A_t = A$ for any t . Thus, we can treat $\ln\{B(W_t, A_t)\}$ as the constant term.

Equation (*) is the aggregated firm's import function. This aggregate import function, which is derived from the aggregated firm's profit maximization problem, has the same functional form as the individual import function.

Moreover, assuming the two-country model, we can symmetrically model the export function. This is because the home export is equal to the foreign country's import and the exchange rate is adjusted to be denominated in the local currency of each country.

By defining the upper right index “+” of the variables to represent the respective foreign variables, O_t^+ , $P_{M,t}^+$, H_t^+ , W_t^+ , A_t^+ , a^+ , $1 - a^+$, and π_t^+ are similarly defined, respective variables in the above maximization problem for the aggregated firm in the home country.

Let the equation $P_{M,t}^+ = P_{FM,t} \cdot ER_t^+ / P_{F,t}^+$ be the relative price in the foreign import function, where $P_{FM,t}$ is the import price for the foreign country in the home currency (i.e., the home export price $P_{HX,t}$), $P_{F,t}^+$ is the foreign domestic price level (the price of numeraire O_t^+ , the foreign production), and ER_t and ER_t^+ are the exchange rates in the home and foreign currencies.

Using the relationship between the home and foreign exchange rates at time t , $ER_t = 1/ER_t^+$, and taking the natural logarithm of the relative price of the foreign

import at time t , $\ln(P_{M,t}^+) = \ln(P_{FM,t}) + e_t^+ - \ln(P_{F,t}^+) = \ln(P_{HX,t}) - e_t - \ln(P_{F,t}^+) = \ln(P_{X,t})$, where $P_{X,t} = P_{HX,t}/(ER_t \cdot P_{F,t}^+)$ is defined as the relative price at time t in the home export function. Here, e_t and e_t^+ are the natural logarithms of the exchange rates ER_t and ER_t^+ .

Using the above result regarding the relative price and $C_{M,t}^+ = C_{X,t}$ (since we treat $C_{M,t}^+$ and $C_{X,t}$ as the quantity; see Dridi & Zieschang, 2004, for a detailed discussion of this kind of price index) from our two-country assumption, as well as the similar assumption that $W_t^+ = W^+$ and $A_t^+ = A^+$ for any t , $\ln(C_{X,t}) = -a^+ \ln(P_{X,t}) + \ln(O_t^+) + \ln\{B(W^+, A^+)\}$ is the home aggregate export function, where $C_{X,t}$ is the home aggregate export goods (foreign aggregate import production goods), and other variables with $+$ correspond to respective foreign variables.

Thus, we succeed in deriving the aggregate export and import functions with the income–price approach from the aggregated firm’s static profit maximization problem. Again, these functional forms are the same for both individual and aggregate

variables.

Therefore, these aggregate and individual export and import functions derived with our profit maximization problem are equivalent to the traditional export and import functions derived using the income–price approach. Furthermore, they have micro-foundations in the static case that our assumptions hold (we relax this static assumption in the Appendix).

Based on this derivation, the absolute value of the elasticity parameters of the relative price that are derived using the income–price approach is equal to the labor input’s elasticity parameter in the production function, regardless of individual or aggregate export and import functions. Thus, we mainly discuss aggregate export and import functions hereafter. When necessary, we utilize “individual” to categorize export and import functions, etc.

The existing empirical studies necessitated one more theoretical justification in our formulation of export and import functions. Many empirical studies, including those by Ogawa and Tokutsu (2002) and Gruber et al. (2016), have suggested that the

production elasticity parameter estimate γ includes the case in which γ is not equal to 1, such that $O_t^\gamma = A_t C_{M,t}^{1-a} H_t^a$, $\gamma > 0$. This is different from the above simple production function, where $\gamma = 1$. These empirical studies supposed that the quantity of import production goods $C_{M,t}$ needs more or fewer additional units to produce the additional unit of production O_t .

Thus, something like increasing returns to scale or decreasing returns to scale exists. Using Japanese data, Ogawa and Tokutsu (2002) suggested that the income elasticity parameter estimate is less than 1. We summarize our production function as follows: $O_t = (A_t C_{M,t}^{1-a} H_t^a)^{1/\gamma} = \Lambda_t C_{M,t}^{v(1-a)} H_t^{va}$, $\Lambda_t = A_t^{\frac{1}{\gamma}}$, $1 \leq v = \frac{1}{\gamma} < \infty$, if $0 < \gamma \leq 1$. Thus, our assumption that $0 < \gamma \leq 1$ includes increasing returns to scale.

In the case of increasing returns to scale, “monopoly” exists in the markets, which forces us to discard the perfect competition assumption in this section. Therefore, we must introduce the demand curve. However, even in this case, the firm sets its price (of the production O_t) depending on its derived marginal revenue using this demand curve. Even if we adjust the constant term to include 1 plus the inverse of the price

elasticity of demand, this change does not alter equation (*) in our static setting (see microeconomics textbooks, such as those by Binger & Hoffman, 1998, for a constant price elasticity of demand and other detailed discussions; Jehle & Reny, 2001).

Furthermore, the output price set by this monopoly firm is still the general price level. Thus, this change in our assumption ($\gamma \in (0,1]$) does not affect our main conclusions in this section. Similarly, the above argument holds for the export function; hereafter, the term $\gamma^+ \in (0,1]$ is included in our export function.

Gruber et al. (2016) suggested that the income elasticity parameter estimate is more than 1—that is, it shows decreasing returns to scale in our framework. However, the existing scarcity of unknown input resources is not included in our simple production function, as Mas-Collel et al. (1995) similarly suggested these missing parts of input resources (in import function) as the source of decreasing returns to scale (see section 5).

Finally, a and a^+ are deep parameters, and we must estimate them empirically. For the estimation of this equation (*), we assume that the aggregate import

function itself has also the stochastic parts like measurement errors, other demand disturbances due to import production goods, and so on as $u_t \sim i.i.d. N(0, \sigma_u^2)$. This assumption is the same as supposing that individual firms face the same aggregate error term, u_t/N . In this paper, this error is counted as the error term without serial correlations following the normal distribution for convenience.

2.2. The Aggregation Problem

Since we will add the stochastic term to our model in the next section, we must consider the effects of this stochastic term on the aggregation. Maddala and Nelson (1975) argue it with the micro-level Tobit model of the heterogeneous censoring. According to them, the most crucial aspect of their model is that compared with the individual model, we must consider the heteroscedasticity of the error term in the aggregate model.

Nevertheless, in this paper, we assume the static model with homogeneous firms (that is, even regarding the error term, firms are assumed to be faced with the same, aggregate, serially uncorrelated error terms) in domestic industries using import

production goods. When we aggregate the individual firm model, this assumption of the static model with homogeneous firms does not cause the heteroscedasticity of the error term due to different cross-sectional income levels, the serial correlation problem, or the heterogeneous censoring (see Greene, 2003 for the former two cases, for example. The last case is proposed by Maddala and Nelson, 1975).

Thus, we can safely avoid the aggregation problem in this paper, and the same argument holds for the export function. Again, we succeed in interpreting our model (*) in the previous section as either the individual time-series model or the aggregate time-series model.

As mentioned above, the stochastic term is addressed in the next section. By including the stochastic term in our model and discussing its relevance for the aggregate problem in advance, we treat the limit values placed by governmental trade policies, such as minimum access and quantitative trade restrictions, as aggregate values. This is for the sake of simplicity in our discussion.

Furthermore, we assume that individual firms face the same aggregate error

term, $\frac{u_t}{N}$. This assumption ensures that our conclusion does not depend on either the individual or aggregate setting. Thus, we treat only the aggregate export and import functions hereafter, even under our competitive market assumption.

Finally, in the Appendix, we treat a special trend-stationary case for model variables as our extension model. In this case, based on our assumption of firm homogeneity, the trends in the model variables are naturally common for all respective variables. Thus, this special assumption safely avoids the aggregation problem discussed in this section.

3. The Tobit Model of the Import Function and Its Marginal Effects

In this section, we investigate whether the Marshall–Lerner condition holds under trade policies by employing the Tobit model with the theoretical but heuristic discussions, compared with our formal discussions in the next section. The Marshall–Lerner condition implies that the trade balance improves when the exchange rate depreciates. This section clarifies what we are accomplishing with the simple Tobit model. In the next section, we specify our discussions from the current section more

rigorously, and we mathematically extend our theory to a more complicated case with coexistence of home and foreign trade policies.

For this purpose, in advance, we examine the micro-foundation of the income–price approach of export and import functions in the previous section. Given the results of this analysis, we formulate trade policies, such as minimum access and quantitative trade restrictions, as the censored distribution problem of export and import functions and investigate the marginal effects of the Tobit estimates for the relative price.

3.1.The Tobit Model and the Import Function (The Minimum Access and the Quantitative Trade Restriction: Both-Sides Censored Model)

In this section, we follow Maddala’s (1983) method for constructing the Tobit model, and Jehle and Reny’s (2001) method for constructing the following firm’s import function, as explained in the previous section.

First, we concentrate on the estimation equation of the both-sides censored model. The both-sides censored model is the case of both minimum access and quantitative trade restriction. This case includes the left censored (the minimum access

case) and the right censored (the quantitative trade restriction case) as the special cases.

In order to analyze the effects of minimum access and quantitative trade restriction with consistent Tobit estimates, we state our log-linear econometric model of the import function of the aggregated firms at time t ($t = 1, 2, \dots, T$) as given below (see the previous section for a detailed explanation of the formulation of the model and the variables therein).

$$\ln(C_{M,t}) = -a \ln(P_{M,t}) + \gamma \ln(O_t) + \ln\{B(W, A)\} + u_t,$$

$$\text{or its matrix expression is } y = y^* = x\beta' + u.$$

Similarly, we can express our export function as $y^+ = y^{+*} = x^+\beta^{+'} + u^+$ in the matrix form, where y^{+*} is the natural logarithm of the latent home aggregate export (i.e., the foreign aggregate import; see section 2).

Thus, using the income-price approach for this import (or, symmetrically, export) function, we define the variables and parameters as follows. Let L_1 , L_2 , x , y , and y^* in the above equations and the subsequently introduced matrix expression be the natural logarithm of the deterministic lower threshold value of aggregate trade

volume placed by minimum access; the natural logarithm of the deterministic upper threshold value of aggregate trade volume placed by quantitative trade restriction; the $T \times 3$ independent variable matrix including only three variables: the natural logarithm of the relative price $\{\ln(P_{M,t}), t = 1, 2, \dots, T\}$, the natural logarithm of the aggregate production $\{\ln(O_t), t = 1, 2, \dots, T\}$, and ones; the column vector of the natural logarithm of dependent variables for T observations; the observed aggregate trade volume—that is, aggregate import production goods $\{\ln(C_{M,t}), t = 1, 2, \dots, T\}$; the column vector of the natural logarithm of the latent aggregate trade volume for T observations (in the above case, we assume $y = y^*$), respectively.

Also, let β' , σ_u , and u in the above equations and the subsequently introduced matrix expression be the 3×1 parameter estimates vector $(\beta_1, \beta_2, \beta_3)' = [-a, \gamma, \ln\{B(W, A)\}]'$; the standard error of regression, and the column vector of the error term following $N(0, \sigma_u^2)$ for T observations, respectively. We assume that (i) x is full rank, (ii) y , y^* , and u are the T -dimensional vectors, and (iii) defining x_t , y_t , y_t^* , and u_t for any t row vector or t -th element, they have no serial correlations and

are level stationary (the latter assumption can be relaxed for a special trend-stationary case; see the Appendix).

Note that in the above equation, y is equal to y^* . However, from the existences of minimum access and quantitative trade restriction, the natural logarithms of the threshold values L_1 and L_2 separate the firm's observed aggregated trade volume at time t , y_t (the t -th element of column vector y), into the following cases, defining x_t and u_t as the t -th row vector of $T \times 3$ independent variable matrix x and the t -th element of the vector u again,

$$y_t = y_t^* = x_t\beta' + u_t \text{ if } L_1 < y_t^* < L_2, \quad y_t = L_1 \text{ if } y_t^* = x_t\beta' + u_t \leq L_1,$$

$$y_t = L_2 \text{ if } L_2 \leq y_t^* = x_t\beta' + u_t.$$

Moreover, note that, in the above case, we do not admit any heterogeneity of the firms even from changes in the error term (we assume the firms face the same aggregate errors), such that the heterogeneous censoring never exists, as we also explain in the previous section.

Following Maddala (1983) and Amemiya (1973), the cumulative distribution

functions and probability density functions of the normal distribution are defined as

$$G_t = G(u_t; 0, \sigma_u^2) = \int_{-\infty}^{u_t} \frac{\exp\left(-\frac{s^2}{2\sigma_u^2}\right)}{\sigma_u(2\pi)^{0.5}} ds = \int_{-\infty}^{u_t/\sigma_u} \frac{\exp\left(-\frac{s^2}{2}\right)}{(2\pi)^{0.5}} ds = \Phi\left(\frac{u_t}{\sigma_u}\right) = \Phi_t \quad (\text{standard}$$

$$\text{normal}), \quad g_t = g(u_t; 0, \sigma_u^2) = \frac{\exp\left(-\frac{u_t^2}{2\sigma_u^2}\right)}{\sigma_u(2\pi)^{0.5}}, \quad \text{and} \quad \phi_t = \sigma_u g_t = \frac{\exp\left(-\frac{u_t^2}{2\sigma_u^2}\right)}{(2\pi)^{0.5}} \quad (\text{standard}$$

normal).

Then, we define the probability of observations for $y_t = L_1$ (the time t -th

latent aggregate trade volume y_t^* is equal to or less than the natural logarithm of the

deterministic threshold value L_1 placed by minimum access) as $\text{Prob}(y_t|L_1 \geq y_t^*) =$

$$\int_{-\infty}^{L_1 - x_t \beta'} g(s; 0, \sigma_u^2) ds = \Phi\left(\frac{L_1 - x_t \beta'}{\sigma_u}\right). \text{ Similarly, the probability of observations for}$$

$y_t = L_2$ (the time t -th latent aggregate trade volume y_t^* is equal to or greater than the

natural logarithm of the deterministic threshold value L_2 placed by quantitative trade

restriction) is set as $\text{Prob}(y_t|y_t^* \geq L_2) = 1 - \Phi\left(\frac{L_2 - x_t \beta'}{\sigma_u}\right)$. Finally, the probability of

observations for $y_t = y_t^*$ (that is, $L_2 > y_t^* > L_1$) under the uncensored, normal case is

$$\text{set as } \text{Prob}(y_t|L_2 > y_t^* > L_1) = \Phi\left(\frac{L_2 - x_t \beta'}{\sigma_u}\right) - \Phi\left(\frac{L_1 - x_t \beta'}{\sigma_u}\right).$$

With these provisions, in the following sections, we discuss how large the

marginal effects of our Tobit estimates are.

3.2. Marginal Effects of the Tobit Estimator and the Marshall-Lerner Condition

It is difficult to compare the Tobit estimates with the OLS estimates. However, although Greene (2003) does not provide rigorous proof, he reports that the marginal effect of the Tobit estimates is comparable with the OLS estimates. In this section, we focus on the marginal effect of our Tobit model.

As Greene (2003) shows in theorem 22.4 of his book, in the case of the Tobit estimator, the marginal effect can be calculated as $\frac{\partial E(y|x)}{\partial x} = \beta' \cdot \text{Prob}(y|L_2 > y^* > L_1)$, denoting β' as in the population estimates explained in the above.

Note that we set L_1 and L_2 for the minimum access and quantitative trade restriction as the natural logarithm of the deterministic lower threshold value and the natural logarithm of the deterministic upper threshold value, respectively, in the case of both-sides censoring. Also, for our concrete calculations, we adopt the marginal effects of the time-averaged variables, $\bar{y}$, $\bar{y}^*$ and $\bar{x}$.

Before beginning our discussion on marginal effects, it is important to note that the probability $\text{Prob}(y|L_2 > y^* > L_1)$ always takes the values in $[0,1]$. Then, we

conclude that the marginal effects of the left-censored, right-censored, and both-sides censored models are always smaller than the consistent Tobit estimate $\hat{\beta}' = \beta'$.

Again, the definition of the Marshall–Lerner condition is that the sum of the absolute values of elasticity parameter estimates of relative export price and relative import price is greater than one. Consequently, without examining data and parameter estimates, in terms of marginal effects, we conclude that the Marshall–Lerner condition tends to be difficult to hold under minimum access, quantitative trade restriction, or both.

4. The Trade Balance Equation and Coexistence of Home and Foreign Trade Policies

With the above provisions, we can construct the trade balance equation mathematically. In this section, we use the log-linear trade balance equation proposed by Boyd et al. (2000). We also express our home trade policy, such as trade promotion (i.e., the minimum access in export) and quantitative trade restriction for aggregate exports, as L_3 and L_4 , the natural logarithm of the deterministic lower limit and the

natural logarithm of the deterministic upper limit, respectively. Furthermore, we derive the Marshall–Lerner condition following Boyd et al. (2000) and mathematically show the possible failure of the condition under home and foreign trade policies.

First, we assume that no foreign trade policy exists. We also assume that the number of home and foreign firms using import production goods in the respective domestic industries is the same (N). We also use the aggregate variables and model.

Following Boyd et al. (2000), the trade balance TB_t is defined as the ratio of nominal aggregate exports to nominal aggregate imports at time t . Thus, the natural logarithm of the trade balance TB at time t is expressed as $\ln(TB_t) = \ln(P_{HX,t} \cdot C_{X,t}) - \ln(P_{HM,t}^+ \cdot ER_t \cdot C_{M,t})$, where $P_{HX,t}$ is the home export price at time t .

Additionally, we use the following relationship: $\ln(P_{M,t}) = \ln(P_{HM,t}^+) + e_t - \ln(P_{H,t})$, where $P_{HM,t}^+$ is the import price for the home country in the foreign currency, and $P_{H,t}$ is the home domestic price level (i.e., the price of the numeraire, the home production O_t).

Next, we represent the trade balance equation $\ln(TB_t)$ under our trade policy.

Taking the expectation of the $\ln(TB_t)$ equation, collecting the irrelevant terms as $\ln(Z_{1,t})$, and rewriting this $E\{\ln(TB_t)\}$ equation as the function of the exchange rate e_t , we obtain the expected trade balance equation under our home trade policy as follows:

$$\begin{aligned} E\{\ln(TB_t)\} &= \ln(Z_{1,t}) + [\text{Prob}\{\ln(C_{X,t})|L_4 > \ln(C_{X,t}^*) > L_3\}a^+ \\ &\quad + \text{Prob}\{\ln(C_{M,t})|L_2 > \ln(C_{M,t}^*) > L_1\}a - 1]e_t, \end{aligned}$$

where $C_{X,t}^*$ and $C_{M,t}^*$ are the latent home aggregate export and import volumes at time t ; $C_{X,t}$ and $C_{M,t}$ are the observed home aggregate export and import volumes at time t ; σ_u^+ is the standard error of the error term u_t^+ , $u_t^+ \sim i.i.d.N(0, \sigma_u^{+2})$ for any t in the home export function (the foreign import function); and

$$\begin{aligned} \text{Prob}\{\ln(C_{X,t})|L_4 > \ln(C_{X,t}^*) > L_3\} &= \Phi\left(\frac{L_4 - x_t^+ \beta^{+'}}{\sigma_u^+}\right) - \Phi\left(\frac{L_3 - x_t^+ \beta^{+'}}{\sigma_u^+}\right), \\ \text{Prob}\{\ln(C_{M,t})|L_2 > \ln(C_{M,t}^*) > L_1\} &= \Phi\left(\frac{L_2 - x_t \beta'}{\sigma_u}\right) - \Phi\left(\frac{L_1 - x_t \beta'}{\sigma_u}\right), \quad \ln(Z_{1,t}) = \\ \ln(P_{HX,t}) &+ \Phi\left(\frac{L_3 - x_t^+ \beta^{+'}}{\sigma_u^+}\right)L_3 + \left\{\Phi\left(\frac{L_4 - x_t^+ \beta^{+'}}{\sigma_u^+}\right) - \Phi\left(\frac{L_3 - x_t^+ \beta^{+'}}{\sigma_u^+}\right)\right\}[-a^+ \{\ln(P_{HX,t}) - \\ \ln(P_{F,t}^+)\} &+ \gamma^+ \ln(O_t^+) + \ln\{B(W^+, A^+)\}] + \sigma_u^+ \left\{\Phi\left(\frac{L_3 - x_t^+ \beta^{+'}}{\sigma_u^+}\right) - \Phi\left(\frac{L_4 - x_t^+ \beta^{+'}}{\sigma_u^+}\right)\right\} + \\ \left\{1 - \Phi\left(\frac{L_4 - x_t^+ \beta^{+'}}{\sigma_u^+}\right)\right\} &L_4 - [\ln(P_{HM,t}^+) + \Phi\left(\frac{L_1 - x_t \beta'}{\sigma_u}\right)L_1 + \end{aligned}$$

$$\left\{ \Phi \left(\frac{L_2 - x_t \beta'}{\sigma_u} \right) - \Phi \left(\frac{L_1 - x_t \beta'}{\sigma_u} \right) \right\} [-a \{ \ln(P_{HM,t}^+) - \ln(P_{H,t}) \} + \gamma \ln(O_t) + \ln\{B(W, A)\}] + \\ \sigma_u \left\{ \Phi \left(\frac{L_1 - x_t \beta'}{\sigma_u} \right) - \Phi \left(\frac{L_2 - x_t \beta'}{\sigma_u} \right) \right\} + \left\{ 1 - \Phi \left(\frac{L_2 - x_t \beta'}{\sigma_u} \right) \right\} L_2].$$

To calculate the marginal effects, we use the time-averaged variables to calculate the probabilities attached for the elasticity parameters, other parameters, and variables in the aggregate export and import functions. This is because we assume level stationarity for model variables, as explained in the previous section. Thus, we succeed in obtaining a modified expected trade balance equation under our home trade policy.

In this formula, the establishment of the Marshall–Lerner condition is represented as the case in which the coefficient of the expected nominal exchange rate, $[\text{Prob}\{\ln(C_{X,t}) | L_4 > \ln(C_{X,t}^*) > L_3\}a^+ + \text{Prob}\{\ln(C_{M,t}) | L_2 > \ln(C_{M,t}^*) > L_1\}a - 1]$, is more than 0. This condition is the same as that derived by Boyd et al. (2000), Ogawa and Tokutsu (2002), etc., excluding the multiplied probabilities.

Based on this modified expected trade balance equation, the existence of home trade policies in the aggregate export and import functions offers lower marginal effects of relative price elasticities in the absolute values than do the OLS estimates without

home trade policies. Therefore, the Marshall–Lerner condition is theoretically difficult to hold, as has already been shown heuristically in section 3.2.

Next, we consider the trade policy, such as the minimum access and quantitative trade restriction set by the foreign government, L_1^+ and L_2^+ , as the natural logarithms of the deterministic lower and upper limits relevant to the home aggregate import. We consider the foreign trade policy L_3^+ and L_4^+ as the natural logarithms of the deterministic lower and upper limits relevant to the home aggregate export. As assumed in section 2, we treat these limit values as the “quantity.” When the foreign trade policy coexists with the home trade policy, the maximum and minimum of the respective limits placed by either the foreign or home trade policies are binding.

Furthermore, since the thresholds for the Tobit formation should be “deterministic,” the standard formulation should be Heckit for the stochastic thresholds (see Maddala, 1983; see Heckman, 1979, for Heckit). However, we consistently assume the “deterministic” thresholds as the starting assumptions in this paper. Thus, effective thresholds depend on the natural logarithms of the deterministic limits placed by either

the home or foreign trade policy. The deterministic limits enable us to apply the usual

Tobit model to our case.

Once we admit the existence of foreign trade policy with deterministic thresholds, we treat the log-linearized variables and generate the following alternate expected trade balance equation:

$$E\{\ln(TBA_t)\} =$$

$$\ln(Z_{2,t}) + [\text{Prob}\{\ln(C_{X,t}) \mid \min(L_4, L_4^+) > \ln(C_{X,t}^*) > \max(L_3, L_3^+)\}a^+ +$$

$$\text{Prob}\{\ln(C_{M,t}) \mid \min(L_2, L_2^+) > \ln(C_{M,t}^*) > \max(L_1, L_1^+)\}a - 1]e_t, \text{ where the variables}$$

with + indicate the respective foreign variables,

$$\begin{aligned} \ln(Z_{2,t}) = & \ln(P_{HX,t}) + \Phi\left\{\frac{\max(L_3, L_3^+) - x_t^+ \beta^{+'}}{\sigma_u^+}\right\} \max(L_3, L_3^+) + \left[\Phi\left\{\frac{\min(L_4, L_4^+) - x_t^+ \beta^{+'}}{\sigma_u^+}\right\} - \right. \\ & \left. \Phi\left\{\frac{\max(L_3, L_3^+) - x_t^+ \beta^{+'}}{\sigma_u^+}\right\}\right] [-a^+ \{\ln(P_{HX,t}) - \ln(P_{F,t}^+)\} + \gamma^+ \ln(O_t^+) + \ln\{B(W^+, A^+)\}] + \\ & \sigma_u^+ \left[\Phi\left\{\frac{\max(L_3, L_3^+) - x_t^+ \beta^{+'}}{\sigma_u^+}\right\} - \Phi\left\{\frac{\min(L_4, L_4^+) - x_t^+ \beta^{+'}}{\sigma_u^+}\right\}\right] + \\ & \left[1 - \Phi\left\{\frac{\min(L_4, L_4^+) - x_t^+ \beta^{+'}}{\sigma_u^+}\right\}\right] \min(L_4, L_4^+) - [\ln(P_{HM,t}^+) + \\ & \Phi\left\{\frac{\max(L_1, L_1^+) - x_t \beta'}{\sigma_u}\right\} \max(L_1, L_1^+) + \\ & \left[\Phi\left\{\frac{\min(L_2, L_2^+) - x_t \beta'}{\sigma_u}\right\} - \Phi\left\{\frac{\max(L_1, L_1^+) - x_t \beta'}{\sigma_u}\right\}\right] [-a \{\ln(P_{HM,t}^+) - \ln(P_{H,t})\} + \gamma \ln(O_t) + \end{aligned}$$

$$\ln\{B(W, A)\} + \sigma_u \left[\Phi \left\{ \frac{\max(L_1, L_1^+) - x_t \beta'}{\sigma_u} \right\} - \Phi \left\{ \frac{\min(L_2, L_2^+) - x_t \beta'}{\sigma_u} \right\} \right] +$$

$$\left[1 - \Phi \left\{ \frac{\min(L_2, L_2^+) - x_t \beta'}{\sigma_u} \right\} \right] \min(L_2, L_2^+).$$

Thus, the coefficient of e_t in this alternate expected trade balance equation $E\{\ln(TBA_t)\}$ is dominated by more effective limit values, such as $\max(L_1, L_1^+)$, $\min(L_2, L_2^+)$, $\max(L_3, L_3^+)$, and $\min(L_4, L_4^+)$, than L_1 , L_2 , L_3 , and L_4 . Therefore, the establishment of the Marshall–Lerner condition under this equation is more severe than under the former expected trade balance equation $E\{\ln(TB_t)\}$ with only home trade policies. This is due to the coexistence of foreign trade policies.

For some special cases, such as trade wars, home and foreign trade policies should be more interactive. Such a case is empirically important; however, the settings of our framework in such a case become so complicated that the settings of our analysis in this section would change completely. Thus, we leave such special cases for future study.

5. Conclusion

We see the effect of trade policies such as the minimum access, quantitative

trade restriction, and both on the establishment of the Marshall–Lerner condition. Our theoretical conclusion suggests one possible cause among many complicated and related causes and results.

However, we conclude that if such restrictions on aggregate trade volume exist, the Marshall–Lerner condition should be theoretically difficult to hold since the marginal effects of export and import price elasticity parameter estimates become smaller in absolute values. Thus, with our theory, we can explain the empirical denial of the establishment of the Marshall–Lerner condition suggested by Japanese data.

With regard to the export and import functions with the income–price approach required for judging the establishment of the Marshall–Lerner condition, we show that we derive their functional forms from the firm’s static profit maximization problem.

Furthermore, following Boyd et al. (2000), we define the log-linear trade balance equation and certify that we can derive the Marshall–Lerner condition mathematically. We use this equation theoretically and rigorously to show that the existence of home trade policies makes the Marshall–Lerner condition difficult to hold.

Moreover, this trade balance equation admits the coexistence of home and foreign trade policies to analyze the effects of these policies on the establishment of the Marshall–Lerner condition. The coexistence of home and foreign trade policies, such as minimum access and/or quantitative trade restriction, theoretically makes the Marshall–Lerner condition more difficult to hold than a case with only home trade policies.

In our Tobit formulation of the above trade policy cases, we assume that the thresholds set by trade policies are deterministic. Once these thresholds become stochastic, we should formulate the standard Heckit or a modified Heckit. The standard and modified Heckit may have a different marginal effect than our Tobit model. Therefore, our conclusions regarding the establishment of the Marshall–Lerner condition should depend on the covariance structure of the error terms of Heckit formulation (see the Appendix).

Finally, for our judgment regarding the establishment of the Marshall–Lerner condition, we must consider hysteresis effects like the J-curve effect and export and import substitutions by foreign direct investment. Moreover, in recent studies, Fontagné

et al. (2018) suggest that, with microdata of French export firms, the decomposition of relative price into the exchange rate, export price, and domestic price is significant, as also shown in Wu (1992).

In our future research, we must accumulate additional empirical knowledge so that we can model such phenomena into export and import functions and enrich the theory derived from this practical knowledge.

Acknowledgment

We appreciate comments from Akihiko Kaneko and Takashi Yamagata in the earlier Japanese version of this paper. We also thank editors in Scribendi for their proofreading of this paper. The opinions and views expressed in this paper and the remaining errors belong solely to the author and do not reflect those of Bank of Japan and/or Central Council for Financial Services Information.

Appendix. The Econometric Issue: Construction and Maximization of the Likelihood Function of Our Model

1. Consistency of the Tobit Estimates

The maximum likelihood estimator (calculated below) of the Tobit model is consistent, as shown in Amemiya (1973).

Nevertheless, the OLS estimate is biased in our case, since we omit the inverse mills ratio term in the usual OLS regression and the omitted variable problem for OLS exists (see, e.g., Heckman, 1976; Maddala, 1983).

With regard to the OLS estimation method for Tobit, we have another two-stage estimation method proposed by Heckman (1976), which is different from the following maximum likelihood method. However, note that, in this paper, we treat only the maximum likelihood estimator (MLE) for simplicity.

2. Construction of the Likelihood Function

The model explained in this paper can be estimated by maximum likelihood method. In this Appendix, we detail the likelihood function of our model and its solution algorithm. The variables used herein are same as those explained previously.

First, for the home aggregate import function, existence of both the minimum access and quantitative trade restriction as home trade policies indicates the both-sides

censored case. The log-likelihood function l for the above both-sides censored case is given below:

$$l = \sum_0 \ln \left\{ \Phi \left(\frac{L_1 - x_k \beta'}{\sigma_u} \right) \right\} - 0.5 \sum_1 \left(\frac{y_k - x_k \beta'}{\sigma_u} \right)^2 + \sum_1 \ln \left\{ \frac{1}{(2\pi\sigma_u^2)^{0.5}} \right\} \\ + \sum_2 \ln \left\{ 1 - \Phi \left(\frac{L_2 - x_k \beta'}{\sigma_u} \right) \right\}.$$

In summary, we use the right-bottom index k of variables to represent each censored case—that is, “0” as the left censored case, “2” as the right censored case, “1” as the uncensored case. Moreover, again, let the natural logarithm of the deterministic threshold value of the left censoring be represented as L_1 and the natural logarithm of the deterministic threshold value of the right censoring be represented as L_2 .

Using the above likelihood function, we have other Tobit estimators with only the left-censored or right-censored cases from the above equations, omitting respective unnecessary terms represented by the right-bottom indices “2” or “0.” Note that we can apply this formula to the export function case, symmetrically.

For constructing and obtaining our MLE, the time series property of our data series is important. In this paper, one of our starting assumptions is the level stationarity

of our dependent and independent variables, as well as the deterministic nature of the limit values. However, we can extend our analysis to the trend-stationary case without any serial correlations. In this special case, the trends in the model variables are assumed to be naturally common for all respective variables due to the assumption of firm homogeneity.

However, in this case, we cannot assume constant real wage and technological parameters. We should also exclude the time trend t from W_t and A_t to obtain the estimated detrended real wage and technological parameters $\underline{W}_t$ and $\underline{A}_t$, which represent $\ln\{B(\underline{W}_t, \underline{A}_t)\}$ in place of $\ln\{B(W, A)\}$ in the home aggregate import function.

For the calculation of the above MLE, the term $\ln\{B(\underline{W}_t, \underline{A}_t)\}$ should be used to estimate only the unknown parameter a , as long as equation (*) from section 2 is used. Thus, if we consider a kind of trend stationarity, our estimation equation should be transformed into the restricted parameter estimation equation (as the likelihood function is easy to establish, we leave it for the reader).

When we consider the trend stationarity for dependent and independent variables without any serial correlations, the time series properties of the natural logarithms of the limit values are important. For example, if we can assume the deterministic nature of the limit values after excluding the trend, even in this special trend-stationary case, we can apply the Tobit model with properly transformed variables, as shown above. Thus, the natural logarithms of the limit values have trends, but they should be deterministic. Furthermore, we should use the average of these transformed data empirically to obtain the marginal effect of our Tobit model.

Regarding the coexistence of the foreign trade policy with “deterministic” thresholds, we should substitute L_1 and L_2 in our MLE formulation with $\max(L_1, L_1^+)$ and $\min(L_2, L_2^+)$, when handling the data.

However, even in the level stationary case, if we cannot assume the deterministic limit values, we cannot apply our simple Tobit model to these stochastic limit values. In this case, we should use the standard or modified Heckit with the ordered probit selection equation in our both-sides censored case (see Maddala, 1983,

for the stochastic threshold case; see Idson & Daniel, 1990, for Heckit with the ordered probit selection equation). In some modelling cases, such as popular modelling of the exchange rate with the high frequency data, we should adopt the non-normal distribution for the error term in the ordered probit selection equation (see Lee, 1981, for a non-normal distribution case).

As stated in the conclusion, the standard and modified Heckit with the ordered probit selection equation may have a different marginal effect from that of our Tobit model. Whether the marginal effect is different solely depends on the covariance structure of error terms in the models (see Maddala, 1983, for Heckit; see Saha et al., 1997, for the general expression of the marginal effect of Heckit).

In this paper, we do not construct the MLE for a case of “stochastic” thresholds. This case is complicated and should be different from our Tobit model, due to the covariance structure of error terms. Thus, we leave the case for future study. Overall, regarding the home aggregate export function, all the above arguments hold.

3. The Numerical Method

In order to obtain the unique optimal solutions in the above likelihood function, we should utilize the numerical method. We introduce the quasi-Newton algorithm suggested in Bunch (1988) as the alternative of the modified expectation-maximization algorithm (EM algorithm) suggested by Fair (1977) (see Greene, 2003).

This quasi-Newton algorithm—generally represented by the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm—is the first to specify the nonlinear optimization problem, such as the likelihood function with the quadratic approximation. Furthermore, the second stage is to select the small additional value d to the m -th step argument $\bar{\beta}_m$ and $\bar{\sigma}_{u,m}^2$ for any m of the likelihood function in question as the trial step to update the Hessian matrix. Note that the initial values of $\bar{\beta}_0$ and $\bar{\sigma}_{u,0}^2$ are based on the estimates of OLS. Then, the third stage is to repeat this step until the partial derivatives given $\bar{\beta}_m$ and $\bar{\sigma}_{u,m}^2$ are equal to 0.

Finally, at the terminal step $m = K$, we have the consistent estimates $\bar{\beta}_K = \beta$ and $\bar{\sigma}_{u,K}^2 = \sigma_u^2$ (this heuristic explanation is from MathWorks, 2001. See Broyden, 1970; Fletcher 1970; Goldfarb 1970; Shanno, 1970 for further details). In MATLAB, for

example, this quasi-Newton algorithm uses numerical gradients and Hessians with finite differences in default (e.g., Hang, 2019 for MATLAB code).

References

Amemiya, T. (1973). Regression analysis when dependent variable is truncated normal.

Econometrica, 41, 997–1016.

Binger, B. R. & Hoffman, E. (1998). *Microeconomics with calculus* (2nd ed.).

Addison-Wesley Educational Publishers, Inc.

Boyd, D., Capolare, G. M., & Smith, R. (2000). Real exchange rate effects on the

balance of trade: cointegration and the Marshall–Lerner condition.

International Journal of Finance and Economics, 6, 187–200.

Broyden, C. G. (1970). The convergence of a class of double-rank minimization

algorithms. *Journal of the Institute of Mathematics and Its Applications*, 6,

76–90.

Bunch, D.S. (1988). A comparison of algorithms for maximum likelihood estimation of

choice models. *Journal of Econometrics*, 38, 145–167.

Clarida, R. H. (1994). Co-integration, aggregate consumption, and the demand for imports: a structural econometric investigation. *American Economic Review*, 84, 298–308.

Dridi, J. & Zieschang, K. (2004). Export and import price indices. *IMF Staff Papers*, 51, 164–194.

Fair, R. C. (1977). A note on the computation of the Tobit estimator. *Econometrica*, 45, 1723–1727.

Fletcher, R. (1970). A new approach to variable metric algorithms. *Computer Journal*, 13, 317–322.

Fontagné, L., Philippe, M., & Orefice, G. (2018). The international elasticity puzzle is worse than you think. *Journal of International Economics*, 115, 115–129.

Goldfarb, D. (1970). A family of variable metric updates derived by variational means. *Mathematics of Computing*, 24, 23–26.

Greene, W. H. (2003). *Econometric analysis* (5th ed.). Prentice-Hall.

Gruber, J., McCallum, A., & Vigfusson, R. (2016). The dollar in the U.S. international

transactions (USIT) model. *IFDP Note*, Federal Reserve Board.

Hang, Q. (2019). Toolkit on econometrics and economics teaching [Source code].

<https://www.mathworks.com>.

Heckman, J. J. (1976). The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models. *Annals of Economic and Social Measurement*, 5, 475–492.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica*, 47, 153–161.

Idson, T. L. & Daniel, J. F. (1990). A selectivity model of employer-size wage differentials. *Journal of Labor Economics*, 8, 99–122.

Jehle, G. A. & Reny, P. J. (2001). *Advanced microeconomic theory* (2nd ed.). Addison-Wesley.

Lee, L., F. (1981). *Generalized econometric models with selectivity* (Discussion Paper Series. 81-140). Center of Economic Research, Department of Economics, University of Minnesota.

Maddala, G. S. (1983). *Limited dependent and qualitative variables in econometrics econometric society monographs no.3*. Cambridge University Press.

Maddala, G. S. & Nelson, F. (1975). Switching regression models with exogenous and endogenous switching. *Proceedings of the American Statistical Association (Business and Economics Section)*, 423–426.

Mas-Collel, A., Whinston, M. D., & Green, J. D. (1995). *Microeconomic theory*. Oxford University Press.

Mathworks (2001). *Optimization tool box for use with Matlab. User's Guide version 2*. The Mathworks, Inc.

Ogawa, K. & Tokutsu, I. (2002). *The Japanese economy: an invitation to empirical analysis*. Yuhikaku. (in Japanese)

Saha, A., Capps Jr, O., & Byrne, P. J. (1997). Calculating marginal effects in models for zero expenditures in household budgets using a Heckman-type correction. *Applied Economics*, 29, 1311–1316.

Shanno, D. F. (1970). Conditioning of quasi - Newton methods for function

minimization. *Mathematics of Computing*, 24, 647–656.

Tobin, J. (1958). Estimation of relationships for limited dependent variables.

Econometrica, 26, 24–36.

Wu, X., L. (1992). A comparison of Tobit and OLS estimates of Japanese peanut import

demand. *Journal of Agricultural Economics*, 43, 38–42.