In detail explanation for the “Proof of the Twin Prime Conjecture” and its applications to thermal physics

K.H.K. Geerasee Wijesuriya

Research Scientist in Physics and Astronomy

PhD student in Astrophysics/Cosmology

BSc (Hons) in Physical Science, Faculty of Science, University of
Colombo, Sri Lanka

geeraseew@gmail.com

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Abstract

Twin prime numbers are two prime numbers which have the difference equals to exactly 2. In other words, twin primes is a pair of two prime numbers which have the value of the difference exactly two. Sometimes the word “twin prime” is used for a pair of twin primes; an another name for this is considered as “prime twin” or called as “prime pair”. Up to date there is no any exact proof/disproof for twin prime conjecture since roughly 200 years in the world. Through this research paper, my attempt is to provide a valid proof for twin prime conjecture. This new paper is the **detailed explanation** of my previous paper that I completed on mid of the year 2020 titled as ‘Proof of Twin Prime Conjecture that can be obtained by using Contradiction method in Mathematics’ **(WHICH IS WELL-RECONGNIZED ALL OVER THE WORLD through researchgate as well)**. And this proof of the existence of infinitely many twin primes can be applied to many subject areas in Physics, Chemistry and etc. And the proof of twin prime conjecture can be used to solve several unsolved problems in Physics, Chemistry and etc as well.

Also as an additional result, at the end of this research paper, it discusses about an application of the Proof of Twin Prime Conjecture to the Quantum and Thermal Physics. There, this research paper consider three space volumes symbolized as area A , B and C. Inside areas A and B there are microscopic particles separately. By applying the proof of the twin prime conjecture, finally this will try to conclude that although the areas A and B have separated by area C, there are some particles those have moved from the area B to area A (due to the high thermal pressure of area B).

Keywords

prime; contradiction; greater than ; integer ; thermal energy; difference ; eigen-function; value; penetration

List of Affiliations

Faculty of Science, University of Colombo, Sri Lanka.

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I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

Author's Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee is 32 years old and Geerasee graduated with BSc (Hons) in Physical Science from the University of Colombo, Sri Lanka in 2014. And now she is following her PhD in Astrophysics/Cosmology.

Geerasee has been received invitations from several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, Geerasee has already received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Department of Mathematics, Faculty of Engineering, University of Moratuwa, Sri Lanka. Now she is a research scientist in Physics as her career. Furthermore she has achieved several other scientific achievements already.

Introduction

A prime number is a natural number which divides only by 1 and it self. For an example, the number 2 divides by 1 and 2 only. That means 2 is divisible by 1 and itself only. But 3 is also a prime number. Because 3 divides by 1 and itself only. But 6 is not a prime number. Because 6 is divisible by 1, 2, 3 and itself. Therefore 6 is not a prime number. But any even number greater than 2, divisible by 1, 2 and itself at least. Therefore any even number greater than 2 is not a prime number. Let's consider the numbers 3 and 5. Then 3 is divisible by 1 and itself only. Also 5 is divisible by 1 and itself only. Therefore 3 and 5 both of them are prime numbers. But the difference between 5 and 3 is $5 - 3 = 2$. Therefore when we consider 5 and 3, both of them are prime numbers but the difference between those two prime numbers is 2. Therefore such couple of two prime numbers we called as 'Twin Prime' numbers.

With this mathematical proof, I use the contradiction method to prove that there are infinitely many twin prime numbers.

If we use the contradiction method in some mathematical result, we initially assume the negation of the requited result. Then I proceed with the mathematical steps with the usage of the well verified mathematical results. Also I should use the assumption that I assumed initially. Then after series of mathematical steps, I will ended up with a contradiction. But I know that in order to get the final contradiction, I didn't use any invalid mathematical result except our initial assumption. Therefore I can conclude that the only issue is our initial assumption. Therefore it is possible to conclude that our initial assumption is wrong. Therefore mathematically I can conclude that the negation of our initial assumption is correct. Because in mathematics, we use only the propositions. That means I can say we got the required result finally. That's how mathematical contradiction method works.

And with the 'result 1' content of this research paper, this will apply the result that 'there are infinitely many twin prime numbers', to experimental research frame with separate areas A, B and C to get a physical result regarding particle penetration from the area B to area A.

Literature Review

The question of whether there exist infinitely many twin primes is an unsolved mathematics problem in number theory since hundreds years. With this proof of twin prime conjecture, this states that there are infinitely many primes p_0 such that $p_0 + 2$ number is also prime number. In 1849, a reputed mathematician named as de Polignac made the better general conjecture that for all natural number q , there are infinitely many primes p such that $p + 2q$ is also prime number. But when the case, which is $q = 1$ of de Polignac's conjecture, is the twin prime conjecture actually.

On April 17, 2013, Yitang Zhang (a well known and reputed and famous mathematician) showed a proof that, for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by 70 million or less than that successfully at that time. And his paper was published with a mathematics journal named as “Annals of Mathematics” in early May 2013. Terence Tao who is also a well reputed mathematician , also proposed a Polymath Project collaborative in order to optimize Zhang's bound. On April 14, 2014, which is one year later exactly from the previously mentioned date, the bound has been decreased to 246. Moreover, by considering the Elliott–Halberstam conjecture and its generalized form, the Polymath project states that the bound has been deducted to 12 and 6, respectively finally. These developed bounds has been found using another different path that was simpler than the previously mentioned mathematician Zhang's important discovery, and has been discovered separately by the mathematicians named as James Maynard and Terence Tao.

Although there are lots of applications to thermal physics from the subject of ‘Quantum Mechanics’, still thermal physics hasn’t any application from the result of ‘Twin Prime Conjecture’ to it , since the twin prime conjecture hasn’t proved before. Therefore the application of twin prime conjecture to thermal physics will be innovative.

Assumption

Let's assume that there are finitely many twin prime numbers.....(1)

Therefore we proceed by considering that there are finitely many twin prime numbers. Then let the highest twin prime numbers are P_{R-1} and $(P_{R-1} + 2)$. Then for all prime numbers P_N greater than $(P_{R-1} + 2)$, $(P_N + 2)$ is not a prime number. Where N and n denote two distinct natural numbers. And P_N denotes the N^{th} prime number. P_{N-1} denotes $(N - 1)^{\text{th}}$ prime number. And P_{N+1} denotes $(N + 1)^{\text{th}}$ prime number. But $P_{N-1} > (P_{R-1} + 2)$. But in the below content, I use only the symbol 'N' to denote prime numbers. That means P_N , P_{N-1} , P_{N+1} are considered as prime numbers. **But in the below content, I don't put any particular value for 'N'.** That means P_3 is NOT generated as $N = 3$, P_L is NOT generated as $N = L$. That means in the below content I use only the symbol 'N' to denote the prime numbers. i.e. Only P_N , P_{N+1} , P_{N-1} are considered as prime numbers and I don't put $N = 3$ or $N = L$ or $N = Q$ or any particular symbol as N to symbolize prime numbers. And in the below proof, the symbol 'P' also uses to denote a prime number. That means I use only the symbol 'N' to denote the prime numbers P_N , P_{N+1} , P_{N-1} (I do not use any particular value for N to symbolize any prime number). But in the below proof, the symbol 'P' also uses to denote a prime number (In the below proof, $P = P_{N-1}$ or P_{N+1}).

Proposed Methodology

Here in the methodology, I consider some integers with some special properties as below. And after that, I use our above assumption (1) and apply the assumption (1) to those integer numbers with the chosen special properties. Then this will try to get a contradiction. But to get the contradiction, this does not use any unverified mathematical theories. Therefore if this is capable to get a contradiction finally, it is possible to conclude that 'the only issue is our initial assumption' (1). Since our assumption (1) is a proposition, if so, it is possible to conclude that the negation of assumption (1) is true. That's how this mathematics proof will be working.

Now let's consider some integers with the below mentioned properties and features.

Let P_n is an odd number greater than 1. But let P_3 is an odd natural number (greater than 1) which is divisible by x_3 . But x_3^2 does not divide P_3 . Where x_3 is also an odd natural number greater than 1. But should consider that that the chosen x_3 gives us that " $[(x_3 - 1) / 2]$ is an odd number".

And let we choose P_n such that $P_n = [(M + 4) / x_3] - (D_0.P_3 / x_3)$; for some odd natural number D_0 , which is not equal to zero. But $(D_0 + 2)$ is divisible by x_3 . But $(D_0 + 2)$ is not divisible by x_3^2 . But we must have chosen D_0 natural number such that $x_3^2 \mid (P_L.x_3 + (D_0 + 2))$ for the non-zero natural number P_L(2)

(Where, according to the below later content, $P_L = (P - 4) / P_3 =$ non zero natural number)

(To know the meaning of M , please refer the below content)

*** And we have chosen a natural number M such that $\{ [(M + 4) / x_3] - (D_0.P_3 / x_3) \}$ ($= P_n$) is not divisible by P_3 , but greater than 1(3)

*** But we must have non zero integer t such that $P_3 = (M + 4). [(x_3 - 1) / 2t]$; where t is an integer not equals to zero and also $x_3^2 \mid (t - 1)$ (4)

To see the meaning of t and M , please refer the below content.

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Let's show that the conditions in (3) and in (4) both are possible simultaneously in the proof as below:

Let we choose odd non zero integer 'a' such that 'a' is not divisible by x_3 . Let choose integer P_Q such that $P_Q = a + a.x_3^2$ (Then P_Q is not divisible by x_3 since 'a' is not divisible by x_3). And 'a' is an odd number. And x_3^2 is also an odd number. Thus $(a.x_3^2)$ is an odd number. But the addition of two odd numbers creates an even number. Thus $(a + a.x_3^2)$ is an even number. Then $P_Q (= a + a.x_3^2)$ is an even number. Then $x_3.(a + a.x_3^2) = (M + 4)$ is an even number. i.e. $(M + 4)$ is an even integer.

But we should know that we choose odd natural number D_0 , which as not equals to zero. But $(D_0 + 2)$ should be divisible by x_3 . But $(D_0 + 2)$ is not divisible by x_3^2 . But we have chosen D_0 natural number such that $x_3^2 \mid (P_L \cdot x_3 + (D_0 + 2))$ for the non-zero natural number P_L . Also we should get P_Q such that $[P_Q - D_0 \cdot (P_3 / x_3)]$ is not divisible by P_3 , for the already chosen D_0 . And $P_Q = (M + 4) / x_3$.

Let choose an integer $t \neq 0$ such that $t = (P_Q / a)$.

But then $(t - 1) = (P_Q / a) - 1 = [(P_Q - a) / a] = x_3^2$ (since $t = P_Q / a$). That means $(t - 1) = x_3^2$. Therefore $(t - 1)$ is divisible by x_3^2(5)

But $(M + 4) = P_Q \cdot x_3$. Then $[(M + 4) / 2 \cdot t] = P_Q \cdot x_3 / 2t = P_Q \cdot x_3 \cdot a / (2 \cdot P_Q) = a \cdot x_3 / 2$ (6)

(because $t = P_Q / a$).

And we can consider the integer $P_3 \neq 0$ as $P_3 = (M + 4) \cdot [(x_3 - 1) / 2 \cdot t] = (x_3 - 1) \cdot a \cdot x_3 / 2$ (= integer, since $(x_3 - 1)$ is an even number) (by using the result (6))(7)

(**Because** $(x_3 - 1) \cdot a \cdot x_3 / 2$ is divisible by x_3 . But since 'a' is not divisible by x_3 , $[(x_3 - 1) \cdot a / 2]$ is not divisible by x_3 . That means P_3 is not divisible by x_3^2 . And 'a' is an odd number. Also $[(x_3 - 1) / 2]$ is an odd number according to our initial choice of x_3 . And x_3 is also an odd integer. Thus $[(x_3 - 1) \cdot a \cdot x_3 / 2]$ is an odd integer. Then by considering all of those things, the term $(M + 4) \cdot [(x_3 - 1) / 2 \cdot t] = [(x_3 - 1) \cdot a \cdot x_3 / 2]$ satisfies the required conditions to be the value of P_3 . Thus we can say that we can write $P_3 = (M + 4) \cdot [(x_3 - 1) / 2 \cdot t] = (x_3 - 1) \cdot a \cdot x_3 / 2$).

But please remember that $2 \mid (x_3 - 1)$ and $[(x_3 - 1) / 2]$ is an odd integer. ***See the Proof 1 below to see the existence of $(P_N - 2) = P_3 \cdot x_3$ such that $P_3 = a \cdot x_3 \cdot [(x_3 - 1) / 2]$ for some odd integer 'a' ; where $[(x_3 - 1) / 2]$ is an odd integer.

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But we should get for the chosen P_Q , $[P_Q - D_0 \cdot (P_3 / x_3)]$ is not divisible by P_3 .

Let's prove that as below: $P_n = P_Q - D_0.(P_3 / x_3) = a + a.x_3^2 - D_0.(x_3 - 1).a/2$

$= (a.x_3/2). [2.x_3 + (D_0 + 2)/x_3 - D_0]$. Let $(D_0 + 2)/x_3 = d$; d is a natural number which is not divisible by x_3 .

Then $P_n = (a.x_3/2). [2.x_3 + d - D_0] \dots\dots\dots(7.1)$

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Let $[2.x_3 + d - D_0] = (x_3 - 1).K'$. Where assume K' as an odd integer number.

Then $(D_0 - K')$ is an even number. But $5.(D_0 + 2)$ is an odd number.

Then $5.(D_0 + 2) \neq a'$. $(D_0 - K')^2$ for any integer a' , (including $a' = (a_0^2 - 1)$ for a_0 an integer).

Then

$D_0(+/-) [(D_0 - K')^2 + 5.(D_0 + 2)]^{(1/2)} = D_0(+/-) (D_0 - K'). f^{(1/2)} \dots\dots\dots(7)'$; $f^{(1/2)}$ is not an integer. Here, f is an integer. But $f^{(1/2)}$ is an irrational number.

Let $g' = (D_0 - K'). f^{(1/2)} = g' = \text{an irrational number}$.

Then $[(D_0 - K')^2 + 4.(D_0 + 2)] = (D_0 - K')^2.f - (D_0 + 2)$

$= [(D_0 - K').f^{(1/2)} - (D_0 + 2)^{(1/2)}] . [(D_0 - K').f^{(1/2)} + (D_0 + 2)^{(1/2)}]$

Here $f^{(1/2)}$ is an irrational number.

$[(D_0 - K').f^{(1/2)} - (D_0 + 2)^{(1/2)}] . [(D_0 - K').f^{(1/2)} + (D_0 + 2)^{(1/2)}]$

$= (D_0 - K')^2.f - (D_0 + 2)$. Let $(D_0 - K')^2.f - (D_0 + 2) = E^2 \dots\dots\dots(7)''$

Then $f - (D_0 + 2) / (D_0 - K')^2 = [E / (D_0 - K')]^2 \dots\dots\dots(7)'''$

Since K' and D_0 are odd integers, $(D_0 - K')^2$, is an even number.

Thus, $(D_0 + 2) / (D_0 - K')^2$ is not an integer. Since f is an integer, by $(7)'''$:

$[E / (D_0 - K')]^2$ is not an integer. Thus $[E / (D_0 - K')]$ is not an integer..... $((7)''''$

By $(7)'$, since K' is an arbitrary odd integer, f is an arbitrary integer. Thus by $(7)''$: E is an arbitrary integer. Thus by $(7)''''$: for all such arbitrary E , $[E / (D_0 - K')]$ is not an integer

means, E is not an integer. Thus by (7)'': $[(D_0 - K')^2 \cdot f - (D_0 + 2)]$ is not a squared number.

Thus $[(D_0 - K') \cdot f^{(1/2)} - (D_0 + 2)^{(1/2)}] \cdot [(D_0 - K') \cdot f^{(1/2)} + (D_0 + 2)^{(1/2)}]$ is not a squared number. But $[(D_0 - K') \cdot f^{(1/2)} - (D_0 + 2)^{(1/2)}] \cdot [(D_0 - K') \cdot f^{(1/2)} + (D_0 + 2)^{(1/2)}] =$

$[(D_0 - K')^2 + 4 \cdot (D_0 + 2)]$. Thus $[(D_0 - K')^2 + 4 \cdot (D_0 + 2)]$ is not a squared number.

Thus $[(D_0 - K')^2 + 4 \cdot (D_0 + 2)]^{(1/2)}$ is an irrational number.

Let $z = D_0 (+/-) [(D_0 - K')^2 + 4 \cdot (D_0 + 2)]^{(1/2)}$; where z is an irrational number.

Then $z = D_0 (+/-) [(D_0^2 + K'^2 - 2 \cdot D_0 \cdot K' + 4 \cdot D_0 + 8)]^{(1/2)}$

$= \{ 2 \cdot D_0 (+/-) [(4 \cdot D_0^2 + 4 \cdot [K'^2 - 2 \cdot D_0 \cdot K' + 4 \cdot D_0 + 8])]^{(1/2)} \} / 2$

Then $z^2 - 2 \cdot D_0 \cdot z - [K'^2 - 2 \cdot D_0 \cdot K' + 4 \cdot D_0 + 8] = 0$

Then $z^2 = K'^2 - 2 \cdot D_0 \cdot K' + 2 \cdot D_0 \cdot (z + 2) + 8$

Then $z^2 - 2 \cdot D_0 \cdot z = 4 + 4 \cdot D_0 + 4 + K' \cdot (K' - 2 \cdot D_0)$

Then $D_0^2 + z^2 - 2 \cdot D_0 \cdot z = D_0^2 + 4 + 4 \cdot D_0 + 4 + K' \cdot (K' - 2 \cdot D_0)$

Then $(D_0 - z)^2 = (D_0 + 2)^2 + 4 + K' \cdot (K' - 2 \cdot D_0) \dots (7.1.1)$

Since z is not an integer number, $(D_0 - z)^2$ is not a square number.

Therefore, $[(D_0 + 2)^2 + 4 + K' \cdot (K' - 2 \cdot D_0)]^{(1/2)}$ is not an integer number.

By (7.1.1): $\{ (K' - D_0) (+/-) [(D_0 + 2)^2 + 4 + K' \cdot (K' - 2 \cdot D_0)]^{(1/2)} \} / [2 \cdot (K' - 2)]$ is not an integer.

If $x_3 = \{ (K' - D_0) (+/-) [(D_0 + 2)^2 + 4 + K' \cdot (K' - 2 \cdot D_0)]^{(1/2)} \} / [2 \cdot (K' - 2)]$:

$x_3^2 - [(K' - D_0) / (K' - 2)] \cdot x_3 - [(D_0 + 2) / (K' - 2)] = 0$. Then x_3 is not an integer number.

Then $x_3^2 \cdot (K' - 2) = K' \cdot x_3 + (D_0 + 2) - x_3 \cdot D_0$

Then, $x_3 \cdot (K' - 2) = K' + [(D_0 + 2) / x_3] - D_0$

Then $2 \cdot x_3 + d - D_0 = K' \cdot (x_3 - 1)$; $d = (D_0 + 2) / x_3 \dots (7.1.2)$

Then if K' is an (odd) integer, x_3 is not an integer. But x_3 is an integer. Therefore our assumption, which is: K' is an (odd) integer is not valid. Therefore it is possible to conclude that K' is not an (odd) integer.

(But since $P_n = (a \cdot x_3 / 2) \cdot [2 \cdot x_3 + d - D_0]$ and since $P_3 = a \cdot x_3 \cdot (x_3 - 1) / 2$ and since P_n is an odd integer, K' can't be an even integer).

Therefore K' is not an even/odd integer. Therefore K' is not an integer.

Therefore it is possible to conclude that by (7.1) and (7.1.2):

P_n is not divisible by P_3 (7.2)

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Then we have the non-zero integer number $P_Q (= a + a \cdot x_3^2)$ such that $[P_Q - D_0 \cdot (P_3 / x_3)] (= P_n)$ is not divisible by P_3 . By (8): the given P_n in (8) is still not divisible by P_3(08)

But we know that by (7), we have $P_3 (\neq 0)$ such that $P_3 = (M + 4) \cdot [(x_3 - 1) / 2 \cdot t]$.

That means $t = (M + 4) \cdot [(x_3 - 1) / 2 \cdot P_3]$.

That means by (5) , (7) and (8):

when we choose the integer t such that $t = (M + 4) \cdot [(x_3 - 1) / (2 \cdot P_3)]$ whenever $(t - 1) = x_3^2$, it is possible to have $P_Q (= a + a \cdot x_3^2)$ such that $[P_Q - D_0 \cdot (P_3 / x_3)]$ is not divisible by P_3

That means, when there is the integer t such that $t = (M + 4) \cdot [(x_3 - 1) / (2 \cdot P_3)]$ whenever $(t - 1) = x_3^2 \rightarrow$ we have $P_Q (= a + a \cdot x_3^2)$ such that $[P_Q - D_0 \cdot (P_3 / x_3)]$ is not divisible by P_3

Thus by considering the statements in (3) and (4), it is possible to say: the conditions in (3) and in (4) both are possible simultaneously in the proof

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But P_N is an arbitrary prime number greater than $(P_{R-1} + 2)$. Because there are infinitely many prime numbers. And here $(P_N - 2) > (P_{R-1} + 2)$. Thus $(P_N - 2)$ is not a prime number.

Where each arbitrary P_N prime number obey (that means we choose a set of $P_N (> (P_{R-1} + 2))$ arbitrarily such that each arbitrarily chosen P_N give us) :

“ P_3 is divisible by x_3 . But x_3^2 does not divide P_3 ; whenever $(P_N - 2) = P_3 \cdot x_3$ ”

And here $(P_N + 2) > (P_{R-1} + 2)$. Then according to our assumption, $(P_N + 2)$ is also not a prime number.....(9)

Since P_N is a prime number, we already know that $(P_N - 2)$ is also not a prime number (Since $P_N - 2 > P_{R-1} + 2$)(10)

Then for the integer P_3 greater than 1 such that $(P_N - 2) / P_3 = x_3$; where x_3 is an integer greater than 1. But here we have already considered that $x_3 \mid P_3$. But x_3^2 does not divide P_3 . Here either P_3 or x_3 not equals to $(P_N - 2)$(11)

Then $(P_N - 2) = P_3 * x_3$ (12)

But $(P_N + 2)$, P_n both are odd numbers. Thus $(P_N + 2) = P_n + 2 \cdot I$; for some I integer (where $I \neq 0$)(13)

Then $(P_N - 2) = P_n + 2 \cdot I - 4 = P_n + 2 \cdot (I - 2)$ (14)

And we know that $(P_N + 2) = P_n + 2 \cdot I \rightarrow P_N = P_n + 2 \cdot I - 2$ (15)

Thus by (15): $P_n + 2 \cdot I - 2 = P_N$. Thus by (12) and (15): $P_3 * x_3 + 2 = P_n + 2 \cdot I - 2$

Thus $P_3 * x_3 - 2 \cdot I + 4 = P_n$ (16)

Thus $P_3 * x_3 + 2 \cdot (I - 2) = P_n + 4$. $(I - 2) = P_n + 2 \cdot P_N - 4 - 2 \cdot P_n = 2 \cdot P_N - 4 - P_n$ (by (14))

Thus $P_3 * x_3 + 2 \cdot (I - 2) = 2 \cdot P_N - 4 - P_n = P_n''$.

Thus $P_3 * x_3 + 2 \cdot (I - 2) = P_n'' = 2 \cdot P_3 * x_3 - P_n$ (17)

Thus $P_3 * x_3 + 2 \cdot I = 4 + 2 \cdot P_3 * x_3 - P_n$

Then $P_3 * x_3 + (2 \cdot I + M) = (4 + M - P_n) + 2 \cdot P_3 * x_3$; Where M is an integer ($M \neq 0$)

Then $(2 \cdot I + M) = (4 + M - P_n) + P_3 * x_3$; Where M is an integer $\neq 0$(18)

But we chose M such that $(M + 4)$ is divisible by x_3 . But $(M + 4)$ is not divisible by x_3^2

But by (7) : $P_3 = (M + 4) \cdot [(x_3 - 1) / 2.t]$. If $x_3 = 3$, then $(M + 4) / P_3 = t$. If $x_3 = 3$, P_3 divides $(M + 4)$(19)

But we know that P_3 is divisible by x_3 . But x_3^2 does not divide P_3 . And we know that $(P_3 * x_3)$ is divisible by x_3 . And we know that P_n is not divisible by P_3(20)

Thus by (18), (19) and (20):

P_3 does not divide $(2.t + M)$ (22)

But P_N is an arbitrary prime greater than $(P_{R-1} + 2)$. Then let $\{ (P_N + A_1), P_N \}$ are two arbitrary consecutive primes set such that each primes are greater than $(P_{R-1} + 2)$.
Where each arbitrary P_N prime number obey:

“ P_3 is divisible by x_3 . But x_3^2 does not divide P_3 ; whenever $(P_N - 2) = P_3 \cdot x_3$ ”

Here since $P_N > (P_{R-1} + 2)$ and since $(P_N + A_1) > (P_{R-1} + 2)$, $A_1 \neq (+/-) 2$. Because we have assumed that there are finitely many twin prime numbers. Then there is a highest twin primes (which are considered as P_{R-1} and $P_{R-1} + 2$). Then for any arbitrarily chosen consecutive two prime numbers greater than $(P_{R-1} + 2)$, the difference between them is greater than 2.

But now choose two particular two consecutive primes (greater than $(P_{R-1} + 2)$) from the arbitrary prime number set $\{ (P_N + A_1), P_N \}$ such that those chosen two particular consecutive primes obey: $[P_3 | (A_1 - 2)]$ and $P_N > A_1$. i.e. where particularly we choose A_1 such that $P_L - x_3 = B_2$. Where $(A_1 - 2) / P_3 = B_2$. Where $P_L = (P - 4) / P_3$ (later I have proven that P_L is an integer). Here P = chosen particular prime $(P_N + A_1)$. **Since $A_1 \neq -2$ for all primes $\{P_N, (P_N + A_1)\}$ (Greater than $P_{R-1} + 2$), we can have such particularly chosen two primes $\{P_N, (P_N + A_1)\}$, which are obeying $[P_3 | (A_1 - 2)]$ (because P_3 does not divide -4 . Because if $A_1 = -2$, then the expression $[P_3 | (A_1 - 2)] \equiv [P_3 | -4]$. But it is not valid. But since $A_1 \neq -2$ for all primes $\{P_N, (P_N + A_1)\}$, there is no any issue with that expression $[P_3 | (A_1 - 2)]$. And also according to recent discoveries done by number theory mathematicians, they have proven that there are infinitely many consecutive prime number couples, with the difference between each two prime numbers (in each couple), which is less than the value of**

247. But since we know about twin primes greater than 247, when assuming the assumption (1) above, we know $P_N > 247$. But as mentioned above, there are infinitely many consecutive prime number couples with $A_1 < 247$. Therefore it is possible to conclude that there exists such a particular prime number couple in the prime number set $\{ (P_N + A_1), P_N \}$ such that $P_N > A_1$. And refer the 'Proof' below to see the existence of two consecutive primes $(P_N + A_1)$ and P_N such that $P_3 \mid (A_1 - 2)$. And refer 'Proof 1' to see the existence of an integer $(P_N - 2)$ such that $(P_N - 2) = P_3 \cdot x_3$ such that P_3 is divisible by x_3 . But x_3^2 does not divide P_3 .

And P_N and $(P_N + A_1)$ are two consecutive two primes numbers greater than $(P_{R-1} + 2)$. Thus here $A_1 \neq (+/-) 2$, since there are finite number of twin primes according to our assumption. Because the difference between two consecutive two primes (greater than $P_{R-1} + 2$) is greater than 2. BUT REMEMBER THAT P_N AND $(P_N + A_1)$ ARE CONSECUTIVE PRIMES greater than $(P_{R-1} + 2)$ which obey P_3 is divisible by x_3 . But x_3^2 does not divide P_3 ; whenever $(P_N - 2) = P_3 \cdot x_3$.

{

Here $(P_N - 2) = P_3 \cdot x_3$ and $(P_N + A_1) = P = \text{Prime}$. That means $P_3 \cdot x_3 + (A_1 + 2) = P \dots \dots (a)$

And $(A_1 - 2)$ is divisible by P_3 . But P_3 does not divide 4. Thus $(A_1 + 2)$ is not divisible by P_3 .

But since $P_3 \cdot x_3$ is divisible by P_3 , by (a): P is not divisible by $P_3 \dots \dots \dots (b)$

But $(A_1 - 2)$ is divisible by P_3 and since $(x_3 \mid P_3)$, $x_3 \mid (A_1 - 2)$. But x_3 does not divide 4 since x_3 is an odd number (since $(P_N - 2) = P_3 \cdot x_3$). Thus $(A_1 + 2)$ is not divisible by x_3 . But since $P_3 \cdot x_3$ is divisible by x_3 , by (a): P is not divisible by $x_3 \dots \dots \dots (c)$

Therefore according to above steps as in (b) and (c), we can write $P_3 \cdot x_3 + (A_1 + 2) = P$ as a prime

}

But $(2 \cdot I + M) = P_N - P_n + 2 + M = (P_N + A_1) + (M + 2 - A_1 - P_n) \dots \dots \dots (23)$

By (19): $x_3 \mid (M + 4)$. But $[P_3 \mid (A_1 - 2)]$(24)

But since $[P_3 \mid (P_N - 2)]$ and since P_3 does not divide $(A_1 + 2)$, $\{ (A_1 + 2) + (P_N - 2) \}$ does not divide by P_3 . i.e. $P (= (P_N + A_1))$ does not divide by P_3 . Thus our choice of A_1 such that $[P_3 \mid (A_1 - 2)]$ is okay (because the expression $P_3 \mid (P_N - 2)$ implies that P_3 does not divide P). Then still I can state that P is as a prime number.

But $[P_3 \mid (P_N - 2)]$ and $[P_3 \mid (A_1 - 2)]$. Thus $(P_N - 2) = P_3 \cdot x_3$ and $(A_1 - 2) = P_3 \cdot B_2$; where x_3 and B_2 are integers and each of them not equals to 0.

Thus $(P_N + A_1 - 4) = (P_N - 2) + (A_1 - 2) = P_3 \cdot x_3 + P_3 \cdot B_2 = (P - 4)$

i.e. $P_3 \mid (P - 4)$(25) Therefore, $P_L = (P - 4) / P_3 = \text{natural number}$.

Let's consider M integer such that $M = P - C$; for some integer ' C ' $\neq 0$ (26)

Because we can have relationship between any two integers by using another introducing integer.

But by (19): $x_3 \mid (M + 4)$ and $P_3 \mid (P - 4)$ by (25).

By (26): $P = (M + C)$. Thus by (25): $[(M + C) - 4] = P_3 \cdot P_L$ (27)

Where $P_L = [(P - 4) / P_3] = \text{integer}$, but not equals to 0. Because P is a prime number (which not equals to zero).

Then $(P_L / x_3) = [(P - 4) / (x_3 P_3)] = (P - 4) / (P_N - 2)$

$= [(P_N - 2) + (A_1 - 2)] / (P_N - 2) = 1 + [(A_1 - 2) / (P_N - 2)]$ (because $P = P_N + A_1$)

If $A_1 < 0$, then either $1 + [(A_1 - 2) / (P_N - 2)] < 0$ or $0 < 1 + [(A_1 - 2) / (P_N - 2)] < 1$.

But $P_L = [(P - 4) / P_3]$. Since $(P - 4) > 0$ and since $x_3 > 1$, $P_3 > 1$; $P_L > 0$. Thus it is impossible to have $1 + [(A_1 - 2) / (P_N - 2)] < 0$. Thus if $A_1 < 0$, we have $0 < 1 + [(A_1 - 2) / (P_N - 2)] < 1$. Then (P_L / x_3) is not an integer, since $(P_L / x_3) = 1 + [(A_1 - 2) / (P_N - 2)]$.

If $A_1 > 0$, then $A_1 > 2$. Because the difference between the considered two consecutive primes ($= A_1$) is greater than 2. Then $0 < (P_N - A_1) < (P_N - 2)$(28)
(because for the chosen particular primes $\{P_N, (P_N + A_1)\}$; $P_N > A_1$).

$$\begin{aligned} \text{Then } (P_L / x_3) &= 1 + [(A_1 - 2) / (P_N - 2)] = 1 + [(A_1 - P_N + P_N - 2) / (P_N - 2)] \\ &= 2 + [(A_1 - P_N) / (P_N - 2)] = 2 - [(P_N - A_1) / (P_N - 2)]. \end{aligned}$$

Since $A_1 \neq (+/-) 2$ and by (28): $[(P_N - A_1) / (P_N - 2)]$ is not an integer.

Then $2 - [(P_N - A_1) / (P_N - 2)] = (P_L / x_3)$ is not an integer.

Then for $A_1 > 0$ and for $A_1 < 0$: (P_L / x_3) is not an integer.

Thus here x_3 does not divide P_L(29)

$$\begin{aligned} \text{Then } (P_L / x_3) &= 1 + [(A_1 - 2) / (P_N - 2)] = 1 + [(A_1 - P_N + P_N - 2) / (P_N - 2)] \\ &= 2 + [(A_1 - P_N) / (P_N - 2)] = 2 - [(P_N - A_1) / (P_N - 2)]. \end{aligned}$$

Since $A_1 \neq (+/-) 2$ and by (28): $[(P_N - A_1) / (P_N - 2)]$ is not an integer.

Then $2 - [(P_N - A_1) / (P_N - 2)] = (P_L / x_3)$ is not an integer.

Then for $A_1 > 0$ and for $A_1 < 0$: (P_L / x_3) is not an integer.

Thus here x_3 does not divide P_L(29)

But $[(M + 4) / x_3] = P_Q = \text{integer}$, but not equals to 0.

Thus by (27): $[(x_3 \cdot P_Q - 4 + C)] - 4 = P_3 \cdot P_L$

$$\text{Thus } C - 4 = [(P_3 \cdot P_L + 4) - x_3 \cdot P_Q]. \text{ Thus } C = [(P_3 \cdot P_L + 8) - x_3 \cdot P_Q] \text{(30)}$$

$$\text{By (23): } (2 \cdot I + M) = (P_N + A_1) + (M + 2 - A_1 - P_N) = P + (M + 2 - A_1 - P_N)$$

$$= P + P - C + 2 - A_1 - P_N = 2 \cdot P - C - P_3 \cdot B_2 - P_N \text{(31) (because } M = P - C \text{ and } A_1 - 2 = P_3 \cdot B_2)$$

$$\text{By (30): } C = [(P_3 \cdot P_L + 8) - x_3 \cdot P_Q].$$

$$\text{Then } 2 \cdot P - C - P_N = 2 \cdot P + x_3 \cdot P_Q - (P_3 \cdot P_L + 8) - P_N$$

$$= 2.(P - 4) + x_3 .P_Q - P_n - (P_3 .P_L) = 2. P_3 .P_L + x_3 .P_Q - P_n - (P_3 .P_L)$$

$$= P_3 .P_L + [x_3 .P_Q - P_n] \text{ (because } P - 4 = P_3 .P_L \text{ ; for non-zero integer } P_L)$$

$$\text{Then } 2.P - C - P_n = x_3 .P_3 .\{ (P_L / x_3) + [(P_Q / P_3) - (P_n / (x_3 .P_3))] \} \dots\dots\dots(32)$$

But we chose P_n such that $(x_3 .P_n) = (M + 4) - D_0 .P_3$; for some integer D_0 .

*** But we have chosen D_0 integer such that $x_3^2 \mid (P_L .x_3 + (D_0 + 2))$. Where $D_0 \neq 0$ and $(D_0 + 2)$ is divisible by x_3 (it implies that D_0 is not divisible by x_3). But $(D_0 + 2)$ is not divisible by x_3^2 .

And therefore here $(P_L / x_3) + [(D_0 + 2) / x_3^2] = D'$ for some integer D' .

Since P_L is not divisible by x_3 , there exists such an integer number $(D_0 + 2)$ (not equals to zero), such that $x_3^2 \mid (P_L .x_3 + (D_0 + 2))$; whenever $(2 + D_0)$ is not divisible by x_3^2 .

$$\text{Then } P_n = [(M + 4) / x_3] - (D_0 .P_3 / x_3). \text{ Then } [(P_Q / P_3) - (P_n / P_3)] = (D_0 / x_3)$$

$$= [x_3 .(P_Q / P_3) - (P_n / P_3)] - (x_3 - 1) .(P_Q / P_3).$$

$$\text{Then } [x_3 .(P_Q / P_3) - (P_n / P_3)] = (D_0 / x_3) + (x_3 - 1) .(P_Q / P_3)$$

$$\text{Then } [(P_Q / P_3) - (P_n / (x_3 .P_3))] = (D_0 / x_3^2) + (x_3 - 1) .(P_Q / (x_3 .P_3)) =$$

$$(D_0 / x_3^2) + (x_3 - 1) .(P_Q / (x_3^2 .u)) = [D_0 .u + (x_3 - 1) .P_Q] / (u .x_3^2) \dots\dots\dots(33)$$

where $u .x_3 = P_3$, and u is a non-zero integer and then u is not divisible by x_3 , since x_3^2 does not divide P_3 .

But by (4): we chose $(t .P_3) = (M + 4) .[(x_3 - 1) / 2]$; t is an integer $\neq 0$ such that $x_3^2 \mid (t - 1)$.

$$\text{Then } (P_3 / x_3) = u = (M + 4) .(x_3 - 1) / (2 .t .x_3) = \text{integer}$$

$$\text{Then } u = P_Q .(x_3 - 1) / 2 .t$$

Thus by (33): $[(P_Q / P_3) - (P_n / (x_3 .P_3))] = [D_0 .u + 2 .t .u] / (u .x_3^2) = (D_0 + 2t) / x_3^2$; where P_Q is not divisible by x_3 . Then according to the formula for u , still ' u ' is not divisible by x_3 .

Then $\{ (P_L / x_3) + [(P_Q / P_3) - (P_n / (x_3 \cdot P_3))] \} = (P_L / x_3) + [(D_0 + 2t) / x_3^2]$
 $= (P_L / x_3) + [(D_0 + 2) / x_3^2] + [2 \cdot (t - 1) / x_3^2] = [[P_L \cdot x_3 + (D_0 + 2)] / x_3^2] + t' = D' =$
integer number (Because by (7): $t' = 2 \cdot (t - 1) / x_3^2 = \text{integer}$).

Since P_L is not divisible by x_3 , there exists such an integer number D_0 (not equals to zero), such that $x_3^2 \mid (P_L \cdot x_3 + (D_0 + 2))$. Whenever $(2 + D_0)$ is divisible by x_3 . Whenever $(2 + D_0)$ is not divisible by x_3^2 .

Then by (32): $2 \cdot P - C - P_n = P_3 \cdot x_3 [D']$; where (P_L / x_3) and $(D_0 + 2) / x_3^2$ are not integers (by 29). But we chose D_0 such that $x_3^2 \mid (P_L \cdot x_3 + (D_0 + 2))$. Then $2 \cdot P - C - P_n = P_3 \cdot x_3 \cdot D'$;

where $D' = (P_L \cdot x_3 + (D_0 + 2)) / x_3^2 + t' = \text{integer}$, but not equals to 0.

Then $P_3 \mid (2 \cdot P - C - P_n)$(34)

by (31), (34): $P_3 \mid (2 \cdot I + M)$(35) (since in (31), B_2 is a non-zero integer)

In (35), there is no any particular value for x_3 . But it is possible to state that $x_3 = 3$ in (35) with the same result : $P_3 \mid (2 \cdot I + M)$ (since $3 (= x_3)$ obeys the required conditions to be $x_3 = 3$, since $(3 - 1) / 2$ is an odd number.

But by (22) (when x_3 is 3): P_3 does not divide $(2 \cdot I + M)$(36).

Thus by (35) and (36): We have a contradiction.....(37)

But in order to get the contradiction as in (37), I used only the well-established mathematical results and equations and our initial assumption (1). Therefore the only possibility that may cause the final contradiction (37) is: our assumption (1) is false. Since the assumption (1) is a proposition, according to the definition of a 'proposition' in mathematics, we can say that the negation of the initial assumption (1) is true. But the negation of our assumption (1) is: There are infinitely many twin prime numbers. Therefore as the result of above mathematical contradiction method, it is possible to state that "there are infinitely many Twin Prime Numbers".

Proof

Let's prove that there exists consecutive primes P_N and $(P_N + A_1)$ such that $[P_3 \mid (A_1 - 2)]$ for some odd integer P_3 which is greater than 1 (when there exist consecutive prime numbers P_N and $(P_N + A_1)$ which both are greater than $[P_{N-1} + 2]$) through this 'Proof' as below.

By 2nd reference: $P_{N-1} = (P_N + A_1) = 2 + \sum_{j=1}^{N-2} h_j$, where $h_j = P_{j+1} - P_j$ for all $j \in \{1, 2, \dots, (N-2)\}$ or $P_{N+1} = (P_N + A_1) = 2 + \sum_{j=1}^N h_j$ when $j \in \{1, 2, \dots, N\}$. Here $(P_N + A_1) = P_{N+1}$ or P_{N-1} , depends on the sign of A_1 .

$P_{N-1} = P_N + A_1 = 2 + \sum_{j=1}^{N-2} h_j$ (when $A_1 < 0$). If $A_1 > 0$, $P_{N+1} = P_N + A_1 = 2 + \sum_{j=1}^N h_j$

Consider the case that $A_1 < 0$.

Then $(A_1 - 2) = -P_N + \sum_{j=1}^{N-2} h_j$.

Then $(2 - A_1) = P_N - \sum_{j=1}^{N-2} h_j$ (38)

But by 2nd reference: for all $\epsilon > 0$, there is a natural number 'm' such that for all $(N-2) > m$; $h_{N-2} < P_{N-2} \cdot \epsilon$

Let ϵ_s is a positive real number $\epsilon_s = [-B + C_s + k' + P_N + B_2 \cdot P_3] / P_s > 0$, such that $h_s < P_s \cdot \epsilon_s$ for all $s > (N-3)$. Let here the chosen ϵ_s implies that $m = (N-3)$ (Here s is going from 1 to $(N-2)$). Then "for all $s > (N-3)$ " means $s \equiv (N-2)$. Where k' is an integer number which not equals to 0 and we choose k' such that $k' / (N-2)$ is an integer. Here the chosen k' integer number is responsible for $h_s < P_s \cdot \epsilon_s$ for all $s > (N-3)$ (i.e. $s = N-2$) and k' is responsible for $\epsilon_s > 0$ as well. That means here the value of k' is responsible to say " ϵ_s is existing such that $h_s < P_s \cdot \epsilon_s$, for $s = (N-2)$ ".

Here $h_j = b_j - [k' / (N-2)]$ for all $j < (N-2) = s$. And where $\sum b_j = B$ for $j < (N-2) = s$. Then for some C_s , $h_s = P_s \cdot \epsilon_s - C_s$; here $s \equiv (N-2)$. *** the meaning of 'j' is the order number and h_j is the (prime) gap between P_{j+1} and P_j . Please refer the below content and the 2nd reference. But here we chose C_{N-2} such that $h_{N-2} = P_{N-2} \cdot \epsilon_{N-2} - C_{N-2}$

But $h_{N-2} = P_{N-2} * C_{N-2} - C_{N-2} = (-B + k' + P_N + B_2 \cdot P_3)$. Where k' is a non zero integer number. Now let's use the 2nd reference to proceed further. By (38):

$$(A_1 - 2) = -P_N + \sum_{j=1}^{N-2} h_j = -P_N + (-B + k' + P_N + B_2 \cdot P_3) + B - (N-2) \cdot [k' / (N-2)] \\ = B_2 \cdot P_3 \dots\dots\dots(39)$$

Thus by (39): $(A_1 - 2) = P_3 \cdot B_2$. Thus there exist consecutive prime numbers P_N and $(P_N + A_1)$ both greater than $(P_{N-1} + 2)$ such that $(A_1 - 2) = B_2 \cdot P_3$; for some integer B_2 ($\neq 0$).

Similar to above, if $A_1 > 0$, we can proceed with the similar steps to prove that $(A_1 - 2) = B_2 \cdot P_3$; for some integer B_2 ($\neq 0$) when $A_1 > 0$.

Proof 1

Let's prove the existence of an integer $(P_N - 2)$ ($> P_{N-1} + 2$) such that $(P_N - 2) = P_3 \cdot x_3$ such that P_3 is divisible by x_3 . But x_3^2 does not divide P_3 as below.

Where $P_3 = (a/2) \cdot x_3 \cdot (x_3 - 1)$ for some odd integer 'a'. And where $[(x_3 - 1) / 2]$ is an odd natural number. Here $x_3 = 3$.

By 2nd reference: $P_N = 2 + \sum_{j=1}^{N-1} g_j$, where $g_j = P_{j+1} - P_j$ for all $j \in \{1, 2, \dots, N-1\}$

$$\text{Then } (P_N - 2) = \sum_{j=1}^{N-1} g_j \dots\dots\dots(40)$$

But by 2nd reference: for all $\epsilon > 0$, there is a natural number ' m_0 ' such that for all $N > m_0$; $g_N < P_N \cdot \epsilon$.

Let ϵ_s is a positive real number $\epsilon_s = [-A + C_s + x_3^2 \cdot k_1] / P_s > 0$, such that $h_s < P_s * \epsilon_s$ for all $s > (N - 2)$. Let here the chosen ϵ_s implies that $m_0 = (N - 2)$ (Here s is going from 1 to $N-1$. Then "for all $s > (N - 2)$ " means $s \equiv (N - 1)$). Where k_1 is an integer number which is not divisible by x_3 . And let $k_1 = a \cdot (x_3 - 1) / 2$. Where 'a' is an odd integer which is not divisible by x_3 . And $[(x_3 - 1) / 2]$ is an odd natural number. Here the chosen P_Q and t integer numbers ($\neq 0$) are responsible for $g_s < P_s * \epsilon_s$ for all $s > (N - 2)$ (i.e. $s = N - 1$) and $\epsilon_s > 0$. OR in another words, the previously chosen integer M and t are responsible

to say that above fact. That means here the value of k_1 (or in another words, 'a') is responsible to say " C_s is existing such that $g_s < P_s^* C_s$, for $s = N - 1$ ". Here $g_j = a_j$ for all $j < (N - 1) = s$. And where $\sum a_j = A$ for $j < (N - 1) = s$. Then for some C_s , $g_s = P_s^* C_s - C_s$; here $s \equiv (N - 1)$. *** the meaning of 'j' is the order number and g_j is the prime gap between P_{j+1} and P_j . Please refer the below content and the 2nd reference. But here we chose C_{N-1} such that $g_{N-1} = P_{N-1}^* C_{N-1} - C_{N-1}$. And where $[(x_3 - 1) / 2]$ is an odd natural number.

But $g_{N-1} = (-A + x_3^2 \cdot k_1)$. Now let's use the 2nd reference to proceed further.

By (40): $(P_N - 2) = \sum_{j=1}^{N-1} g_j$. But $\sum_{j=1}^{N-1} g_j = A + (-A + x_3^2 \cdot k_1) = x_3^2 \cdot k_1 \dots\dots\dots(41)$

Thus by (40) and (41): $(P_N - 2) = x_3^2 \cdot k_1$; where k_1 is not divisible by x_3 . Then $(P_N - 2) = x_3 \cdot (x_3 \cdot k_1) = x_3 \cdot P_3$; where P_3 is divisible by x_3 . But P_3 is not divisible by x_3^2 . And where $[(x_3 - 1) / 2]$ is an odd natural number. Thus $(P_N - 2) = P_3 \cdot x_3$; where P_3 is divisible by x_3 . But P_3 is not divisible by x_3^2 . Where $[(x_3 - 1) / 2]$ is 1. Thus there exists an integer set $\{ (P_N - 2) = P_3 \cdot x_3 : \text{where } P_3 \text{ is divisible by } x_3 \text{ but } P_3 \text{ is not divisible by } x_3^2 \text{ and where } [(x_3 - 1) / 2] \text{ is an odd natural number equals to } 1 \}$ such that P_N is an arbitrary prime number greater than $(P_{N-1} + 2)$. Whenever $x_3 = 3$, $(x_3 - 1) / 2$ is an odd number. Thus applying $x_3 = 3$ is okay (since for the above proof x_3 is arbitrary, whenever $(x_3 - 1) / 2$ is an odd number).

Results

Result 01

Therefore I have used our assumption (1) to get the contradiction finally, as showed in (37). Therefore it is possible to conclude that our assumption (1) is false. But we know that our assumption (1) is a proposition. Thus it is possible to say that the negation of the assumption (1) is true. But the negation of our assumption (1) is: There are infinitely many twin prime numbers. **Thus there are infinitely many twin prime numbers.**

Result 02

With this result 02, I am intending to apply our above main result to the thermal physics researches. BUT PLEASE NOTE THAT THERE ARE NO ANY RELATIONSHIPS BETWEEN EACH OF THE EQUIVALENT SYMBOLS OF ABOVE METHODOLOGY AND THIS RESULT 02. That means there is no any relationship between the symbol 'N' in above methodology and in this result 02 and so on for other symbols.

By the above main result, it is possible to say that there are infinitely many twin prime numbers.....(42)

Let's consider two regions symbolized as A and B. And there are microscopic matter particles (with zero electric charge) inside the area A and B, which move inside the areas A and B with the individual energies (thermal). But there is a border region (C) between the areas of A and B. But let's consider that the region A can have comparably infinitely many quantum states separately. And at the border line of the system of areas A, B and C, the potential energy tends to infinity. Therefore particles inside A/B/C can't go outside of the system A/B/C. Also particles those are outside of A/B/C can't come inside of A/B/C. And the region A and B are comparably large separate areas. Let's figure out the situation as below.

Figure 01 (How the matter particles distributed)

Blue color area is the region A, green color area is the region B and the red color area is the region C. But the region A can have comparably infinitely many quantum states separately. And area C is an area which the matter particles can't cross that particular area C easily and go to the opposite area. Let at time $t = 0$, there are no any considerable thermal pressure of the area A and area B. Also at $t = 0$, we allow to tend the temperature of the area A and B to Kelvin 0. But during the time $t \in (0, t_1]$, we gradually increase the temperature of the region B. That means we increase the thermal pressure of the area B gradually during time $t \in (0, t_1]$. But we do not considerably change the temperature of the region A during time $t \in (0, t_1]$. But during the time which is before $t = 0$ of the system up to $t = 0$, there are no any considerable energy or the thermal pressure inside the area A or B that allows to cross the area A or B by the opposite side microscopic matter particles (which were inside A or B).

Let's consider the 3 dimensional Schrodinger equation for an arbitrarily chosen particle (P) from the region A (during $t \in (0, t_1]$) as below:

$$-(\hbar^2 / 2M) \nabla^2 \psi_i + V\psi_i = E_i \psi_i$$

Where M is the mass of the microscopic particle P. V is the total potential energy of P. E_i is the energy eigenvalue of the 'i'th eigen state of P at some time t (where $t \in (0, t_1]$). $\hbar$ is the Plank constant. And ψ_i is the wave function of P when it is in the 'i'th eigen state. But P is an arbitrarily chosen microscopic particle.

$$\text{Where } \nabla^2 = \frac{1}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \left(\frac{1}{\sin \theta} \right) \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right]$$

Where r is the distance coordinate to P from the origin of the research frame. θ is the polar coordinate to P from the origin of the research frame. φ is the azimuthal coordinate to P from the origin of the research frame.

But it is possible to find the energy orthonormal eigen function (ψ_i) of 'i'th quantum state of P , by considering the thermal energy of P, other kinetic and potential energies of P.

But as we consider, there are infinitely many quantum states in A. Thus i can go from 1 to infinity. Let $\Phi = c_1. \psi_1 + c_2. \psi_2 + \dots + c_n. \psi_n + \dots$

Where we consider ψ_i 's are as orthonormal eigen functions. $i \in \{1, 2, 3, \dots, n, \dots\}$

Where c_i are linearly independent constants ; $i \in \{1, 2, 3, \dots, n, \dots\}$

Then Probability (Q_i) of finding the particle P in the quantum state 'i' = $|c_i|^2$ |

Let's consider that $|c_i|^2 = \{ (1 / P_{i+1}) \{ 1 / [1 - (P_i / P_{i+1})] \} \}^{1/i}$

Where P_i and P_{i+1} are two arbitrary natural numbers (greater than some arbitrary natural number N and for the considered arbitrary natural number N, the minimum P_i and P_{i+1} should obey $P_{i+1} - P_i = 2$. i.e. for the chosen N, $[P_2 - P_1] = 2$. Since there are infinitely many twin prime numbers according to my above proof (i.e. infinitely many such P_1 and P_2 such that $[P_2 - P_1] = 2$) , there exists infinitely many such natural numbers N) and for all such P_{i+1} and P_i ; $[P_i / P_{i+1}]$ is a rational number for all natural numbers P_{i+1} and P_i .

Since $P_{i+1} > P_i$, $(P_i / P_{i+1}) < 1$.

But $[P_i / P_{i+1}]$ is a rational number for all natural numbers P_{i+1} and P_i . That means for any natural numbers P_{i+1} and P_i , P_{i+1} and P_i haven't any common factors. Then it is possible to understand and possible to consider that P_{i+1} and P_i as two consecutive prime numbers.

But then $|c_i|^2 = 1 / [P_{i+1} - P_i]^{1/i}$; for all 'i' th eigen state of P.

Then the Probability (Q_i) of finding the particle P in the quantum state 'i' = $|c_i|^2$ = $1 / [P_{i+1} - P_i]^{1/i}$ Then $Q_i = 1 / [P_{i+1} - P_i]^{1/i}$ for primes P_{i+1} and P_i . We know that the number of quantum states inside area A tends to infinity.

But $Q_i = 1 / [P_{i+1} - P_i]^{1/i}$; where P_{i+1} is the 'i+1' th prime number (among the group of primes greater than some arbitrary natural number N), P_i is the 'i' th prime number (among the group of primes greater than the arbitrary natural number N). But N is greater than 2.

But we know that there are infinitely many consecutive prime numbers couples P_{i+1} and P_i (greater than N some natural number) with $P_{i+1} > (P_i + 2)$. And by my

above proof twin prime conjecture, it is possible to state that there are infinitely many prime numbers P_{i+1} and P_i with $P_{i+1} = (P_i + 2)$ (each of them are greater than some natural number N)(42.1)

Let's show that there are infinitely many consecutive prime numbers couples P_{i+1} and P_i (each of them are greater than some arbitrary natural number N) with $P_{i+1} > (P_i + 2)$ as below, inside this below curly brackets.

{

We know that there are infinitely many twin prime numbers (by my initial proof in this research paper).

Let's denote those infinite number of twin primes (each of the twin primes are greater than 3 and also greater than the arbitrary natural number N) as $\{b_1, b_2\}, \{b_3, b_4\}, \{b_5, b_6\}, \{b_7, b_8\}, \dots, \{b_{n-1}, b_n\}, \{b_{n+1}, b_{n+2}\}, \{b_{n+3}, b_{n+4}\}, \dots$

Where above $\{b_{i-1}, b_i\}$ indicates twin primes couple. Symbol i is a natural number.

But let's show that among above indicated $\{b_{i-1}, b_i\}$, taken from primes set $\{b_1, b_2, b_3, b_4, \dots, b_{n-2}, b_{n-1}, b_n, b_{n+1}, b_{n+2}, b_{n+3}, b_{n+4}, \dots\}$, there are two primes $\{b_m, b_{m+1}\}$ such that $(b_{m+1} - b_m) > 2$.

We know that $\{b_{n-1}, b_n\}, \{b_{n+1}, b_{n+2}\}, \{b_{n+3}, b_{n+4}\}$ are 3 consecutive twin primes set those have taken arbitrarily from the infinite number of twin primes (But here it is possible to have $b_j = b_{j+1}$ for some j th prime).

i.e. $[b_{n+4} - b_{n+3}] = 2, [b_{n+2} - b_{n+1}] = 2, [b_n - b_{n-1}] = 2 \dots \dots \dots (42.1.1)$

(But here it is possible to have $b_j = b_{j+1}$ for some j th prime).

If $b_j \neq b_{j+1}$ for all b_j and b_{j+1} in (42.1.1), then for some b_j and b_{j+1} , $[b_{j+1} - b_j] \geq 4$ (since b_j and b_{j+1} are odd numbers).....(42.1.2)

Here $j \in \{(n+4), (n+3), (n+2), (n+1), n, (n-1), (n-2)\}$.

But if all $b_{n+3} = b_{n+2}, b_{n+1} = b_n, b_{n-1} = b_{n-2}$ in (42.1.1), then either b_{n+4} or b_{n+3} or b_{n+2} or b_{n+1} or b_n or b_{n-1} is not a prime number. Because if $b_k = b_{k-1}$ for some k natural number such that $k \in \{n+3, n+2, n+1, n, (n-1), (n-2)\}$ in (42.1.1), then at least one b_k is

divisible by 3 (because each b_j here is greater than 3). Because among every 3 consecutive odd numbers, there is one odd number which is divisible by 3 (because all considering prime numbers here are odd numbers). Therefore not all $b_{n+3} = b_{n+2}$, $b_{n+1} = b_n$, $b_{n-1} = b_{n-2}$ in (42.1.1).

Then for some prime numbers among b_{n+4} , b_{n+3} , b_{n+2} , b_{n+1} , b_n , b_{n-1} :

$$[b_{j+1} - b_j] \geq 4 \text{ (since } b_j \text{ and } b_{j+1} \text{ are odd numbers)} \dots \dots \dots (42.1.3)$$

By (42.1.2) and (42.1.3): for some b_j and b_{j+1} , $[b_{j+1} - b_j] \geq 4$. But b_j and b_{j+1} are arbitrary prime numbers.

Therefore for some primes b_{j+1} and b_j (arbitrary) : $[b_{j+1} - b_j] \geq 4$.

Since b_{j+1} and b_j are some arbitrary primes, we can say that there exists infinitely many primes P_{i+1} and P_i such that $[P_{i+1} - P_i] \geq 4$.

Thus there are infinitely many consecutive prime number couples P_{i+1} and P_i with $P_{i+1} > (P_i + 2)$. Thus there are infinitely many consecutive prime numbers couples P_{i+1} and P_i (which are greater than the arbitrary natural number N) such that $P_{i+1} > (P_i + 2)$.

}

Now let's return to the proof after above (42.1).

Since $i \in \{1, 2, 3, \dots, n, \dots\}$, we can put the symbol 'i' as below also:

$$Q_i^{\sqrt{i}} = 1 / [P_{i+1} - P_i]^{\sqrt{i} \cdot \sqrt{i}} = 1 / [P_{i+1} - P_i]^i ; \text{ where } i \in \{1, 2, 3, \dots, n, \dots\}.$$

Then let's separate $1 / (P_{i+1} - P_i)^i$ into two parts as below.

(Here by (42.1), it is possible to write $Q_i^{\sqrt{i}} = 1 / (P_{i+1} - P_i)^i$ for all $i \in \{1, 2, 3, \dots, n, \dots\}$)

$$\text{Then } Q_i^{\sqrt{i}} = [1 / (q_{i+1} - q_i)]^i \text{ OR } Q_i^{\sqrt{i}} = (1/2)^i \dots \dots \dots (42.1.4)$$

{ Where $(q_{i+1} - q_i) > 2$ in the term $\sum_{i=1}^{\infty} \frac{1}{(q_{i+1} - q_i)^i}$ and q_{i+1} and q_i are two consecutive primes with $(q_{i+1} - q_i) > 2$ }

Then by (42.1.4): $\sum_{i=1}^{\infty} (Qi)^{\sqrt{i}} = \sum_{i=1}^{\infty} \left(\frac{1}{(qi+1-qi)^i} * \delta_i \right) + \sum_{i=1}^{\infty} \left[\left(\frac{1}{2^i} \right) * \delta'_i \right] \dots\dots\dots(42.2)$

Where $\delta_i = 1$, $\delta'_i = 0$; ($P_{i+1} - P_i$) > 2

Where $\delta_i = 0$, $\delta'_i = 1$; ($P_{i+1} - P_i$) = 2

Because we have infinitely many primes q_{i+1} and q_i such that ($q_{i+1} - q_i$) > 2. Also we have infinitely many primes P_{i+1} and P_i such that ($P_{i+1} - P_i$) = 2 (by 42.1)

Where ($q_{i+1} - q_i$) > 2 in the term $\sum_{i=1}^{\infty} \frac{1}{(qi+1-qi)^i}$ and q_{i+1} and q_i are the two consecutive primes with ($q_{i+1} - q_i$) > 2.

By (42.2): $\sum_{i=1}^{\infty} (Qi)^{\sqrt{i}} < \sum_{i=1}^{\infty} \left[\frac{1}{(qi+1-qi)^i} * \delta_i \right] + \sum_{i=1}^{\infty} \left(\frac{1}{2^i} \right)$; because $\max \{ \delta'_i \} = 1$

But $Q_i = 1/[P_{i+1} - P_i]^{\sqrt{i}}$. Consider { $\sum_{i=1}^{\infty} Qi - \left[\sum_{i=1}^{\infty} \left[\delta_i * \frac{1}{(qi+1-qi)^i} \right] + \sum_{i=1}^{\infty} \left(\frac{1}{2^i} \right) \right]$ }

But $\sum_{i=1}^{\infty} Qi - \left[\sum_{i=1}^{\infty} \delta_i * \frac{1}{(qi+1-qi)^i} + \sum_{i=1}^{\infty} \left(\frac{1}{2^i} \right) \right]$

$= \sum_{i=1}^{\infty} \frac{1}{(Pi+1-Pi)^{\sqrt{i}}} - \left[\left[\sum_{i=1}^{\infty} \frac{1}{(qi+1-qi)^i} * \delta_i \right] + \sum_{i=1}^{\infty} \left(\frac{1}{2^i} \right) \right]$

$= \sum_{i=1}^{\infty} \frac{1}{(Pi+1-Pi)^{\sqrt{i}}} - \sum_{i=1}^{\infty} (Qi)^{\sqrt{i}} - \sum_{k=1}^{\infty} \left(\frac{1}{2^k} \right)$ (by 42.2).....(42.2.1)

Where each k is a natural number where each k generates $(P_{k+1} - P_k) \neq 2$.

Then $\sum_{i=1}^{\infty} \frac{1}{(Pi+1-Pi)^{\sqrt{i}}} - \sum_{i=1}^{\infty} (Qi)^{\sqrt{i}} - \sum_{k=1}^{\infty} \left(\frac{1}{2^k} \right)$

$> \sum_{i=1}^{\infty} \frac{1}{(Pi+1-Pi)^{\sqrt{i}}} - \sum_{i=1}^{\infty} (Qi)^{\sqrt{i}} - \sum_{k=1}^{\infty} \left(\frac{1}{2^{(k-k_0)}} \right)$

Where k_0 can equals to a natural number (where $k > k_0 > 0$) and k_0 is varying depending on the value of k in each term $(P_{k+1} - P_k) (\neq 2)$. Where, each and every k_0 generates each k such that ($P_{(k-k_0)+1} - P_{(k-k_0)}$) = 2.

Then

$$\sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^{\sqrt{i}}} - \sum_{i=1}^{\infty} (Q_i)^{\sqrt{i}} - \sum_{k=1}^{\infty} \left(\frac{1}{2^k}\right) >$$

$$\sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^{\sqrt{i}}} - \sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^i} - \sum_{k=1}^{\infty} \left(\frac{1}{2^{(k-k_0)}}\right) > \sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^{\sqrt{i}}} - \sum_{i=1}^{\infty} \frac{2}{(P_{i+1}-P_i)^i}$$

.....(42.3)

(Because, $\sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^i} > \sum_{k=1}^{\infty} \left(\frac{1}{2^{(k-k_0)}}\right)$, since there are additional terms in $\sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^i}$ compare to $\sum_{k=1}^{\infty} \left(\frac{1}{2^{(k-k_0)}}\right)$, according to the definition of $(P_{i+1} - P_i)$ and according to the above definition of $(k - k_0)$).

Then $\sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^{\sqrt{i}}} - \sum_{i=1}^{\infty} \frac{2}{(P_{i+1}-P_i)^i} = \sum_{i=1}^{\infty} \frac{(P_{i+1}-P_i)^{\sqrt{i}} - 2}{(P_{i+1}-P_i)^i}$. But $(P_{i+1} - P_i) \geq 2$ and $i \geq 1$.

Thus $(P_{i+1} - P_i)^{\sqrt{i}} \geq 2$. Thus $\sum_{i=1}^{\infty} \frac{1}{(P_{i+1}-P_i)^{\sqrt{i}}} - \sum_{i=1}^{\infty} \frac{2}{(P_{i+1}-P_i)^i} \geq 0$.

Thus by (42.3) and by (42.2.1): $\sum_{i=1}^{\infty} Q_i - \left[\sum_{i=1}^{\infty} \frac{1}{(q_{i+1}-q_i)^i} * \delta_i + \sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) \right] > 0$.

Then $\sum_{i=1}^{\infty} Q_i > \left[\sum_{i=1}^{\infty} \frac{1}{(q_{i+1}-q_i)^i} * \delta_i \right] + \left[\sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) \right] > \left[\sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) \right]$

(since $\delta_i = 1$ whenever $(q_{i+1} - q_i) > 2$ {and $\delta_i = 0$ whenever $(P_{i+1} - P_i) = 2$ } and $\frac{1}{(q_{i+1} - q_i)^i} > 0$)

Thus $\sum_{i=1}^{\infty} Q_i > \left[\sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) \right]$ (43)

(Because by the result (42), there are infinitely many twin prime numbers. Then there are infinitely many P_{i+1} and P_i such that $(P_{i+1} - P_i) = 2$ for infinitely many 'i' natural numbers. Then we can tend $i \rightarrow$ infinity in the expression $\sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right)$)

Then by (43): $\sum_{i=1}^{\infty} Q_i > \sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right)$ (43)

But we know that $\sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) = 1$(44)

By (43), (44) : $\sum_{i=1}^{\infty} Q_i > \sum_{i=1}^{\infty} \left(\frac{1}{2^i}\right) = 1$

Thus $\sum_{i=1}^{\infty} Q_i > 1$(45)

But we know that Q_i is the probability of finding the particle P in the quantum state 'i'. But we know that there are infinitely many quantum states in region A. That's why we could tend i to infinity. By (45), the total probability of finding the particle P, in all of the quantum states 'i'

$$= \sum_{i=1}^{\infty} Q_i > 1$$

But we know that although there are infinitely many quantum states in the region A, the total probability of the existence of P (arbitrary chosen microscopic particle) in all 'i'th quantum states of region A should be 1.

But the total probability of finding the arbitrarily chosen particle P in the region A = $\sum_{i=1}^{\infty} Q_i > 1$. Since I chose P arbitrarily, the total probability of finding P in the region A is greater than 1 means there should be some another arbitrary particles those have come from the other region B to the region A.

That means: If we can have $Q_i = 1 / [P_{i+1} - P_i]^{\sqrt{i}}$ for each 'i' eigen state 'i' (with two consecutive primes (with $P_{i+1} > P_i > N$; for arbitrary natural number N greater than 2 and for the considered arbitrary natural number N, the minimum P_i and P_{i+1} obeys $P_{i+1} - P_i = 2$. i.e. for the chosen N, $[P_2 - P_1] = 2$) and ' $i \in \{1,2,3,\dots, n, \dots\}$ '), for arbitrarily chosen microscopic particle P chosen from the region A, then there should be some arbitrary particles those have come from the other region B to the region A. Those arbitrarily chosen particles P came from the region B by penetrating the region C, due to the high thermal density inside the region B.

Therefore if there is such a **high thermal energy** density inside the region B which is increasing gradually during $t \in (0, t_i]$, then those thermal energy radiation will transmit to the region A little bit (though the region C) and then there should be some arbitrary particles those have come from the other region B to the region A, **if the energies of P allow it to has:**

Probability (Q_i) of finding the particle P in the quantum state 'i' = $|c_i|^2$ |

$= \{ 1 / [P_{i+1} - P_i] \}^{1/i}$; for P_i and P_{i+1} two arbitrary natural numbers (greater than some arbitrary natural number N and for the considered arbitrary natural number N , the minimum P_i and P_{i+1} obeys $P_{i+1} - P_i = 2$. i.e. for the chosen N , $[P_2 - P_1] = 2$)) and for all such P_{i+1} and P_i ; $[P_i / P_{i+1}]$ is a rational number for any natural number P_{i+1} and P_i , greater than the considered arbitrary natural number N .

Figure 02 (the variance of Q_i vs $[P_{i+1} - P_i]$) value of $[P_{i+1} - P_i]$ ----->

The x axis of above figure 02 indicates the value of $[P_{i+1} - P_i]$ for different values of 'i'. Y axis indicates the value of Q_i for different value of 'i'.

Conclusion

We assumed initially that there are finitely many twin primes. After proceeding with that, I finally received a contradiction. But in order to get the contradiction, I used that P_N as a prime number greater than $(P_{R-1} + 2)$. And also we chose P_n odd integer (> 1) and also we chose an integer A_1 such that $P_3 \mid (A_1 - 2)$. Also to get the contradiction, I used the fact that $(P_N - 2)$ as a non-prime since $(P_N - 2) > (P_{R-1} + 2)$. And also I have used that x_3 as a natural number not equals to 1 (since $(P_N - 2)$ is not a prime number). And also I have used the fact (to get the contradiction as in (37)): The difference between any two consecutive prime numbers (which equals to $[(P_N + A_1) - P_N]$) which is A_1 , is greater

than 2 (Since $[(P_N + A_1)$ and P_N are greater than $(P_{N-1} + 2)$). i.e. $A_1 \neq (+/- 2)$. Therefore to get the contradiction as in (37), I have used the facts got from our assumption (1). Then the only possibility is our assumption (1) is false. Since our assumption (1) is a proposition, it is possible to state that ‘there are infinitely many twin prime numbers’.

Also, if there is a **high thermal energy** density inside the region B which is increasing gradually during $t \in (0, t_i]$, then those thermal energy radiation will transmit to the region A little bit (though the region C) and then there should be some arbitrary particles those have come from the other region B to the region A, if the energies of P allow it to have $Q_i = 1/[P_{i+1} - P_i]^{\sqrt{i}}$ for each ‘i’ eigen state (with two consecutive primes (with $P_{i+1} > P_i$ greater than arbitrary natural number N and for the considered arbitrary natural number N, the minimum P_i and P_{i+1} obeys $P_{i+1} - P_i = 2$. i.e. for the chosen N, $[P_2 - P_1] = 2$) and ‘i’ $\in \{1, 2, 3, \dots, n, \dots\}$ for arbitrarily chosen microscopic particle

Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two consecutive prime numbers which have the difference exactly 2.

We denote ‘i’ th prime gap $g_i = P_{i+1} - P_i$

Then according to the 2nd reference; Prime number $P_N = 2 + \sum_{j=1}^{N-1} g_j$

where by 2nd reference: for all $\epsilon > 0$, there is a natural number ‘n’ such that for all $N - 1 > n$; $g_{N-1} < P_{N-1} \cdot \epsilon$

*** The Time-independent Schrodinger equation can be written as below:

$$-(\hbar^2 / 2m) \cdot \nabla^2 \Psi + V \cdot \Psi = E \cdot \Psi$$

Where $\hbar$ is the reduced plank constant, m is the mass of the matter particle ‘P’ .

V is the potential energy operator of the system, E is the energy term (eigenvalue).

And Ψ is the wave function of the matter particle (eigen function).

$$\text{Where } \nabla^2 = \frac{1}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \left(\frac{1}{\sin \theta} \right) \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right]$$

$$\text{Let } \Phi = c_1. \psi_1 + c_2. \psi_2 + \dots + c_n. \psi_n + \dots$$

Where c_i are linearly independent constants ; $i \in \{1, 2, 3, \dots, n, \dots\}$

θ is the polar angle of considered matter particle P with respect to the origin.

φ is the azimuthal angle to the particle P with respect to the origin. r is the distance coordinate to P with respect to the origin.

$\Phi = c_1. \psi_1 + c_2. \psi_2 + \dots + c_n. \psi_n + \dots$ called as a linear combination of ψ_i eigen functions.

If ψ_i are orthonormal eigen functions, then probability of finding the particle in the quantum state 'i' = $|c_i|^2$

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