Open Journal of Mathematics and Physics | Volume 2, Article 148, 2020 | ISSN: 2674-5747
https://doi.org/10.31219/osf.io/mwkaf | published: 26 Aug 2020 | https://ojmp.org

FT

[white paper]
Diamond Open Access

A unidimensional windmill in the plane with varying pivot points

Open Mathematics Collaboration*†

August 27, 2020

Abstract

We show that we can choose a point in the plane such that the resulting windmill process with varying pivot uses each point of the plane infinitely many times.

keywords: international mathematical olympiad, imo, logic, set theory

The most updated version of this paper is available at
https://osf.io/mwkaf/download

How to co-author/co-sign this paper?

1. Send us a complete review report and answer the following questions.
2. Which discussions would you add to this paper?
3. Did you find any mistakes or inconsistent arguments? Give a detailed justification.

*All authors with their affiliations appear at the end of this paper.
†Corresponding author: mplobo@uft.edu.br | Join the Open Mathematics Collaboration

Introduction

4. We *discuss* the *solution* of **Problem 2** of the **International Mathematical Olympiad** (IMO, 2011) [1, 2].

Definitions

5. $\mathcal{S}$ = a finite set of at least two points in the plane

6. No three points of $\mathcal{S}$ are collinear.

7. *windmill* = the rotation process that starts with a line ℓ going through a single *pivot* point $P \in \mathcal{S}$

8. ℓ rotates clockwise about P until the first time that ℓ meets some $Q \in \mathcal{S}$

9. Q = *new pivot*

10. ℓ rotates clockwise about Q until it next meets a point in $\mathcal{S}$

11. *This process continues indefinitely.*

Theorem

12. *We can choose $P \in \mathcal{S}$ and ℓ going through P such that the resulting windmill uses each point of $\mathcal{S}$ as a pivot infinitely many times* [1].

Proof

13. Let ℓ have an orientation (Fig. 1).

14. ℓ divides the plane into the *grey* and *white* sides.

15. Start with ℓ going through a point O such that all points in $\mathcal{S}$ are in the white side.

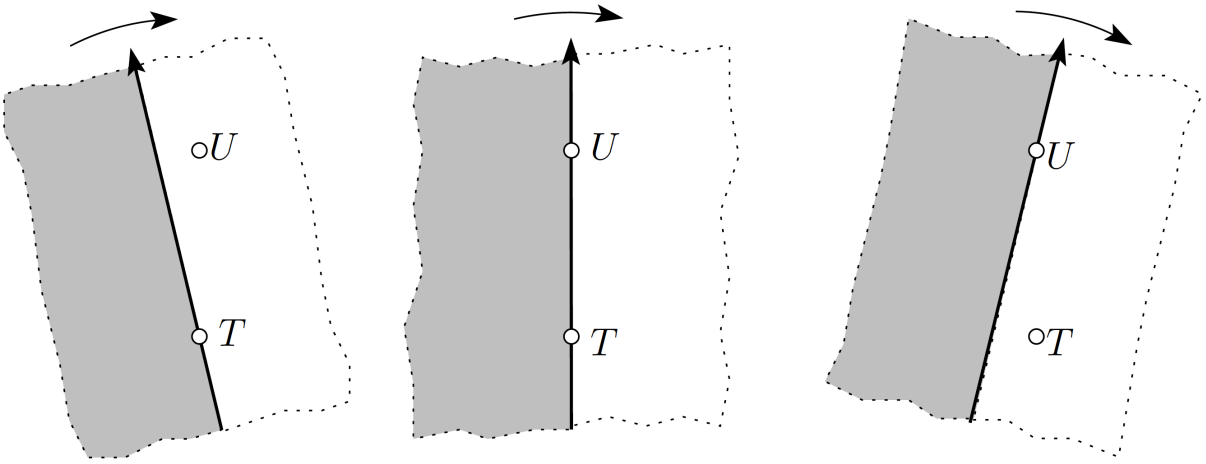

Figure 1: Two arbitrary points and the oriented rotating line are shown [1].

16. Fix the pivot in O.

17. Since (6), as ℓ moves clockwise (with the pivot fixed at O), it passes through *one point at a time*.

18. Move ℓ until there are

    (a) n points in each side if $|\mathcal{S}| = 2n + 1$.

    (b) $n - 1$ points in the grey side and n in the white if $|\mathcal{S}| = 2n$.

19. Now, start the *windmill process* as described in (7-11).

20. Note that whenever the pivot changes from T to U (Fig. 1), after the change, T is on the same side as U was before.

21. The *number of elements* of $\mathcal{S}$ on the *grey* side and the number of those on the *white* side *remain the same* throughout the whole process (except for those moments when the line contains two points).

22. Let the *configuration* in (18) be the **starting line**.

23. During the *rotation*, due to (20), *each point from the grey side becomes white and vice versa*.

24. Therefore, after having made a rotation of 180°, the *windmill should have passed through all the points.* □

Open Invitation

Review, *add* content, and **co-author** this paper [3, 4]. *Join* the **Open Mathematics Collaboration** (https://bit.ly/ojmp-slack). Send your contribution to mplobo@uft.edu.br.

Open Science

The **latex file** for this paper together with other *supplementary files* are available [5].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

Acknowledgement

+ **Center for Open Science**
  https://cos.io

+ **Open Science Framework**
  https://osf.io

References

[1] IMO. International Mathematical Olympiad (2011). https://www.imo-official.org/

[2] 3Blue1Brown (2019). https://youtu.be/M64HUIJFTZM

[3] Lobo, Matheus P. "Microarticles." *OSF Preprints*, 28 Oct. 2019. https://doi.org/10.31219/osf.io/ejrct

[4] Lobo, Matheus P. "Simple Guidelines for Authors: Open Journal of Mathematics and Physics." *OSF Preprints*, 15 Nov. 2019. https://doi.org/10.31219/osf.io/fk836

[5] Lobo, Matheus P. "Open Journal of Mathematics and Physics (OJMP)." *OSF*, 21 Apr. 2020. https://doi.org/10.17605/osf.io/6hzyp

[6] COS. *Open Science Framework*. https://osf.io

[7] Lobo, Matheus P. "A Unidimensional Windmill in the Plane with Varying Pivot Points." *OSF Preprints*, 26 Aug. 2020. https://doi.org/10.31219/osf.io/mwkaf

The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br)[1,2]
https://orcid.org/0000-0003-4554-1372

Hans Müller Silva Oliveira[1,3]
https://orcid.org/0000-0002-7137-9789

[1]Federal University of Tocantins (Brazil)
[2]Universidade Aberta (UAb, Portugal)
[3]Colégio Invictos (Araguaína, Tocantins, Brasil)