
GRAVITY IN FOURTH SPATIAL DIMENSION

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Arif Ali

M. Tech Structural Engineering
Jawaharlal Nehru Technological University
Kakinada, India.
arif068bce@ioepc.edu.np

Divya Raj Sapkota

Master of Business Administration
Kathmandu University
Kathmandu, Nepal.
17327_divya@kusom.edu.np

ABSTRACT

An interpretation of gravitational attraction as a consequence of expansion of bodies and distortion of space. For this interpretation, we formulate the acceleration due to gravity and orbital velocity based on expansion of bodies. It is also shown, how distortion in space counterbalances the effect of expansion.

1 Definitions and abbreviations

Volucity: Rate of change in volume per unit time. unit: m^3/s

Voluceleration: Rate of change of volucity per unit time or rate of change of volume per unit time squared. unit: m^3/s^2

Hyperbody: Any four-dimensional body lying in four-dimensional space.

Hyperfluid: The space we are accessible to is a three-dimensional cross-section of a four-dimensional hyperspace. This hyperspace is a four-dimensional incompressible fluid known as hyperfluid.

x-axis, y-axis and z-axis: Axis of three accessible dimensions.

w-axis: Axis of fourth spatial dimension.

2 Background and Introduction

Gravity was first explained by Sir Isaac Newton in the late 17th century in his publication, the Principia [1] that explains the observed motions of the planets and their moons, which Johannes Kepler previously formulated. Newton's equations predict a precession of mercury's orbit to be 5557 seconds of arc per century [2]. As measured from earth, the precession is 5600 seconds of arc per century [3]. Thus, there is a discrepancy of 43 seconds of arc per century.

Later, in the early 20th century, Albert Einstein came up with a theory of General Relativity (GR) [4], an extension to his earlier theory of Special Relativity. GR explained gravity as the result of space-time geometry. It got its early success by resolving the discrepancy of 43 seconds of arc per century in the precession of the perihelion of mercury.

2.1 Problem statement

In the present paper, we propose that gravitational attraction is the consequence of the dynamics of hyperbodies in four spatial dimensions. To prove this thesis, we derive the rate of expansion of three-dimensional bodies and explain how motion of hyperbodies in four dimensions leads to curvature in space that counterbalances the expansion.

2.2 Previous research

This paper is based on the existence of four spatial dimensions. Fourth spatial dimension was first introduced by Gunnar Nordstrom [5]. In 1921, Theodor Kaluza [6] came up with a theory to unify General Relativity and electromagnetism based on the idea of five dimensions, four dimension of space and one temporal dimension.

In our previous paper [7], we presented an interpretation of time dilation as the result of the dynamics of hyperbody in four dimensions. In it, we postulated that all the bodies we observe are a three-dimensional cross-section of a four-dimensional hyperbody. Hyperbodies are continuously in motion, and the four-dimensional resultant speed of a hyperbody is a constant, c , i.e., speed of light in a vacuum. Secondly, the time period of any physical change is directly

proportional to the speed of its hyperbody in the fourth dimension.

2.3 The fourth dimension

The fourth dimension is an extension of three-dimensional space [8]. We can perceive the fourth dimension just like a two-dimensional world would perceive the third dimension. The fourth dimension adds a degree of freedom other than forward-backward, left-right and up-down. A three-dimensional body has length, breadth and depth. Similarly, a hyperbody has an extension in the fourth dimension too. The extension is called hyper-length. Experiments like Quantum Hall Effect [9] and Bullet cluster experiment [10] depict the existence of the fourth spatial dimension.

3 Extension to the postulate of "Time dilation in fourth spatial dimension"

All the three-dimensional bodies expand at a constant rate of voluceleration.

The motion of a hyperbody through the hyperfluid creates drag at the surface of the hyperbody. This drag at the surface leads to disturbance in hyperfluid. The continuous disturbance distorts the three-dimensional space i.e., three-dimensional cross-section of a four-dimensional hyperfluid. The distortion is dynamic such that, at the surface of any body, the resultant distortion in space counterbalances the measurable effect of expansion.

4 What causes gravitational attraction?

Gravitational attraction is a conceptual attraction that results from the body's expansion, which is counterbalanced by the curvature of space. The curvature of space is formed due to the high-speed motion of hyperbody in the four-dimensional hyperfluid. Let us consider a three-dimensional symmetric body, i.e., a sphere; its radius increases with time at an accelerating rate. Figure 1 shows how the expansion is counterbalanced by the curvature in space formed due to the motion of the hyperbodies. Thus, surface gravity is the result of pushing of the surface in an outward direction due to voluceleration, and the fall of objects toward the celestial body is the result of current in the hyperfluid.

There are experiments that support the similarity of "gravitational red shifting & redshifting due to acceleration [11]", "Gravitational mass & inertial mass [12]", and "time dilation onset by a difference in relative velocity & Time dilation brought about by the effect of gravity [13]". These experiments support the concept of voluceleration and prove that the acceleration of the surface of the earth is causing the push rather than any force of attraction.

5 Voluceleration

All the bodies in this universe volucelerate at a constant rate. This voluceleration is the result of the dimension and dynamics of hyperbodies. The shape and size of the hyperbody and its position in the fourth dimension determine the shape of the three-dimensional section we observe. It is similar to, the shape of a three-dimensional body and its position on a two-dimensional plane determining the shape of a cross-section of that body on that two-dimensional plane.

5.1 Derivation of acceleration due to voluceleration

Let us consider a homogeneous three-dimensional symmetric body, i.e., a sphere. The body expands in space at a constant rate of voluceleration. Let the volume of the body be V , and the radius be r .

We know,

$$V = \frac{4}{3}\pi r^3 \quad (1)$$

If V changes with time,

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \quad (2)$$

$$= 4\pi r^2 \times \frac{dr}{dt} \quad (3)$$

Differentiating Equation 3 with respect to time.

$$\frac{d^2v}{dt^2} = \frac{d(4\pi r^2)}{dt} \cdot \frac{dr}{dt} + 4\pi r^2 \cdot \frac{d}{dt} \left(\frac{dr}{dt} \right) \quad (4)$$

$$\beta = \frac{d(4\pi r^2)}{dr} \cdot \frac{dr}{dt} \cdot \frac{dr}{dt} + 4\pi r^2 \cdot \frac{d^2r}{dt^2} \quad (5)$$

where, β is rate of voluceleration

$$\beta = 8\pi r \cdot \frac{dr}{dt} \cdot \frac{dr}{dt} + 4\pi r^2 \cdot \frac{d^2r}{dt^2} \quad (6)$$

$$\beta = 8\pi r \cdot \left(\frac{dr}{dt} \right)^2 + 4\pi r^2 \cdot \frac{d^2r}{dt^2} \quad (7)$$

Equation 7 is the expression for voluceleration.

From the postulate, the rate of voluceleration of a body is a constant. Let us assume that both parts, $8\pi r \cdot \left(\frac{dr}{dt} \right)^2$ and $4\pi r^2 \cdot \frac{d^2r}{dt^2}$ of Equation 7, are individually constant. Therefore,

$$8\pi r \cdot \left(\frac{dr}{dt} \right)^2 = \text{constant}, a \quad (8)$$

where a is a constant of variation

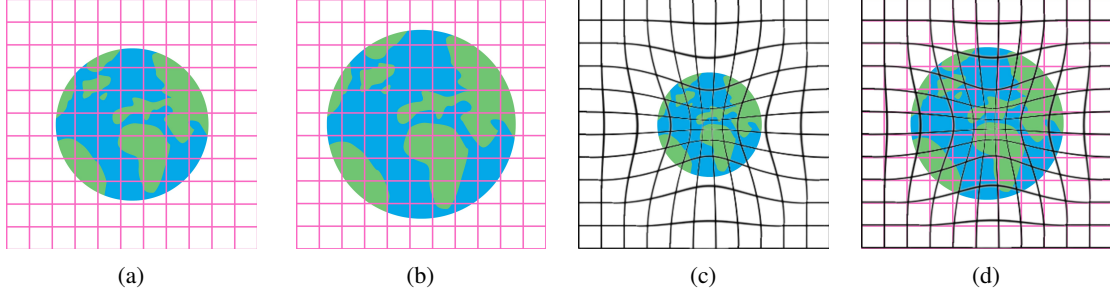


Figure 1: Voluceleration of earth's surface.

(a) initially, size of earth at time $t = 0$, (b) after time t , swelling of earth due to continuous voluceleration, (c) after time t , contraction of earth in our perception, due to curvature in space, (d) counterbalanced size when effect of voluceleration and space curvature is summed up.

$$\left(\frac{dr}{dt}\right)^2 = \frac{a}{8\pi r} \quad (9)$$

$$\left(\frac{dr}{dt}\right)^2 = \frac{k}{r} \quad (10)$$

where, k is a constant i.e., $\frac{a}{8\pi}$, and a is a constant of variation

$$\frac{dr}{dt} = \frac{\sqrt{k}}{\sqrt{r}} \quad (11)$$

Now, differentiating Equation 11 with respect to time where k is a constant. we get,

$$\frac{d^2r}{dt^2} = \frac{d}{dt} \left(\frac{\sqrt{k}}{\sqrt{r}} \right) \quad (12)$$

$$\frac{d^2r}{dt^2} = \frac{d}{dr} \left(\frac{\sqrt{k}}{\sqrt{r}} \right) \times \frac{dr}{dt} \quad (13)$$

$$\frac{d^2r}{dt^2} = \sqrt{k} \cdot \left(-\frac{1}{2} \right) r^{-\frac{1}{2}-1} \cdot \frac{dr}{dt} \quad (14)$$

$$\frac{d^2r}{dt^2} = \sqrt{k} \cdot \left(-\frac{1}{2} \right) \cdot r^{-1.5} \cdot \frac{dr}{dt} \quad (15)$$

Substituting value from Equation 11 in Equation 15,

$$\frac{d^2r}{dt^2} = \sqrt{k} \cdot \left(-\frac{1}{2} \right) \cdot r^{-1.5} \cdot \frac{\sqrt{k}}{\sqrt{r}} \quad (16)$$

$$\frac{d^2r}{dt^2} = -\frac{k}{2r^2} \quad (17)$$

checking if second part of Equation 7 remain constant after the substitution

$$\beta = 8\pi r \cdot \frac{k}{r} + 4\pi r^2 \cdot \left(-\frac{k}{2r^2} \right) \quad (18)$$

$$\beta = 8\pi k - 2\pi k \quad (19)$$

$$\beta = 6\pi k \quad (20)$$

now, substituting Equation 20 in Equation 17

$$\frac{d^2r}{dt^2} = -\frac{\beta}{12\pi r^2} \quad (21)$$

$$a_0 = -\frac{\beta}{12\pi r^2} \quad (22)$$

Where, a_0 is the acceleration of surface of any body observed from rest frame of center of the body.

Equation 22 is the expression for acceleration of the surface in terms of r and β .

From Equation 11 and Equation 20,

$$\frac{dr}{dt} = \frac{\sqrt{\frac{\beta}{6\pi}}}{\sqrt{r}} \quad (23)$$

$$v_r = \sqrt{\frac{\beta}{6\pi r}} \quad (24)$$

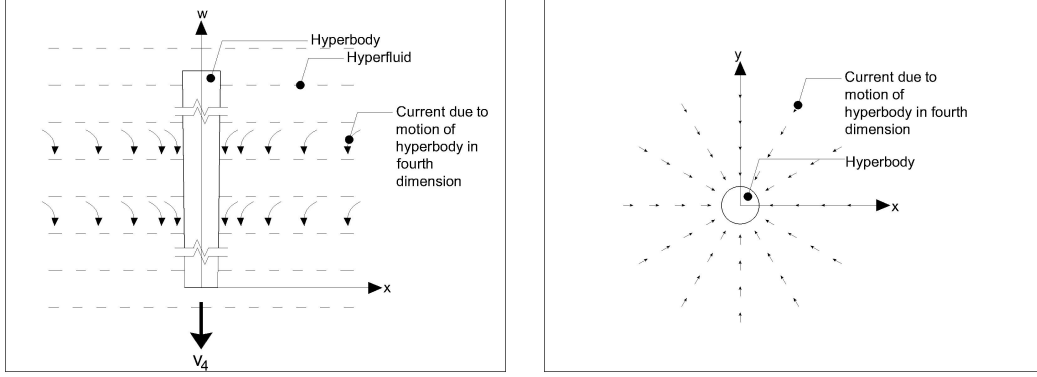
where, v_r is the rate of change of radius with respect to time.

Any system needs to be at an outward speed greater than v_r to escape a surface of a volucelerating body. Therefore, Equation 24 is the expression for escape velocity of a volucelerating body.

6 Orbiting and precession of perihelion of mercury

Space is a three-dimensional cross-section of a hyperfluid. The motion of hyperbody in the fourth dimension leads to dynamic distortion of three-dimensional space. Consequently, a current of space flow toward the surface of the hyperbody. The space-current toward the hyperbody loses its intensity in the outward direction, as shown in Figure 2.

We cannot interpret a four-dimensional body passing through three-dimensional space. Let us reduce one of our accessible three dimensions and add a fourth spatial dimension to our freedom. We will observe a three-dimensional body passing through a two-dimensional plane, creating



(a) Cross-section of current in hyperfluid, due to motion of hyperbody in fourth dimension.

(b) Top view of two-dimensional interpretation of hyperbody moving in fourth dimension.

Figure 2: Two-dimensional interpretation of hyperbody moving through hyperfluid

a dynamic curvature in the two-dimensional spatial plane. There is a current in the hyperfluid toward the center of the mass. This current cause the curvature in the two-dimensional spatial plane and counterbalances the measurable effect of voluceleration. The current loses its intensity as we move outward from the body's center, as in Figure 2. For a three-dimensional body moving through the two-dimensional plane, the intensity of the current, i.e., the number of lines of current per unit length passing through a given circumference, is inversely proportional to the rim through which the line of current passes.

$$\text{Intensity of current} \propto \frac{1}{2\pi r} \quad (25)$$

Interpreting the process of Figure 2 for four dimensions, the intensity of the current is inversely proportional to the surface area of the three-dimensional space through which the lines of current pass.

$$\text{Intensity of current} \propto \frac{1}{4\pi r^2} \quad (26)$$

Since the voluceleration of the surface is exactly counterbalanced on its surface,

$$\alpha_0 = \frac{\beta}{12\pi r^2} \quad (27)$$

where r is the radius of the volucelerating body. At distance R , acceleration due to distortion of space is as the combined result of voluceleration and distorting space is a_R .

From Equation 26,

$$\frac{a_R}{a_0} = \frac{4\pi r^2}{4\pi R^2} \quad (28)$$

$$a_R = \frac{\beta}{12\pi r^2} \times \frac{4\pi r^2}{4\pi R^2} \quad (29)$$

$$a_R = \frac{\beta}{12\pi R^2} \quad (30)$$

where, R is the distance from the center of volucelerating body and a_R is the acceleration of a body at a distance R from its central body due to curvature in space.

Any revolving body has outward acceleration called centripetal acceleration. For a body to be in a continuous circular motion, neither falling toward the central body nor moving away from the orbit, there must be a counterbalancing acceleration. This acceleration is provided by space-current resulting from the four-dimensional motion of the hyperbody.

$$a_c = a_R \quad (31)$$

$$\frac{v_o^2}{R} = \frac{\beta}{12\pi R^2} \quad (32)$$

where, v_o is orbital velocity,

$$v_o = \sqrt{\frac{\beta}{12\pi R}} \quad (33)$$

Equation 33 is the expression for the orbital velocity of a body orbiting around a central body of voluceleration β at a distance R from the central body.

The above formulation is an approximation for bodies orbiting a central body at a diminutive speed compared to the speed of light. However, considering the effect of time dilation, a minimal difference appears. This discrepancy occurs due to the difference in time elapsed for the heavenly body and the elapsed time recorded by a stationary observer on earth.

$$a = \frac{dv}{dt} \quad (34)$$

where, dt is an infinitesimal difference of time measured by the observer at earth, and dv is an infinitesimal difference of v

From our previous paper [7], Time dilation, due to the motion of hyperbody in four-dimensional space is given

by:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (35)$$

where, T_0 is the proper time of a system moving with velocity v and T be the time measured by observer at rest.

$$\frac{T}{T_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (36)$$

Equation 36 is the expression of time dilation due to motion of body in three dimensions.

From Equation 22, magnitude of voluceleration,

$$\beta = 12\pi r^2 \alpha_0 \quad (37)$$

Substituting Equation 37 in Equation 24,

$$v_r = \sqrt{\frac{12\pi r^2 \alpha_0}{6\pi r}} \quad (38)$$

$$v_r = \sqrt{2\alpha_0 r} \quad (39)$$

$$v^2 = 2\alpha_0 r \quad (40)$$

where v is the velocity at which a body is moving through space while standing on the surface of the volucelerating body.

Substituting Equation 40 in Equation 35,

$$T = \frac{T_0}{\sqrt{1 - \frac{2\alpha_0 r}{c^2}}} \quad (41)$$

$$\frac{T}{T_0} = \frac{1}{\sqrt{1 - \frac{2\alpha_0 r}{c^2}}} \quad (42)$$

Equation 42 is the expression of the relation between 'time in stationary clock' and 'time in moving clock' in terms of acceleration due to voluceleration.

equation

proper acceleration of a body is given by:

$$a_0 = \frac{dv}{dt_0} \quad (43)$$

where, dt_0 is the infinitesimal difference of proper time of the orbiting body.

Rewriting Equation 34 in terms of t_0 ,

$$a = \frac{dv}{dt_0} \times \frac{dt_0}{dt} \quad (44)$$

$$a = \frac{dx/dt}{dt_0} \times \frac{dt_0}{dt} \quad (45)$$

$$a = \frac{dx/dt_0}{dt_0} \times \left(\frac{dt_0}{dt}\right)^2 \quad (46)$$

$$a = a_0 \left(\frac{dt_0}{dt}\right)^2 \quad (47)$$

Substituting Equation 22 and 26 in Equation 47,

$$a = \frac{\beta}{12\pi R^2} \times \left(1 - \frac{2a_0 r}{c^2}\right) \quad (48)$$

$$a = \frac{\beta}{12\pi R^2} \times \left(1 - \frac{\beta}{6\pi R c^2}\right) \quad (49)$$

$$a = \left(\frac{\beta}{12\pi R^2} - \frac{\beta^2}{72\pi^2 R^3 c^2}\right) \quad (50)$$

Equation 50 is the expression for acceleration due to voluceleration for moving bodies. This expression predicts the precession of the perihelion of mercury precisely as Einstein's General Relativity does.

7 Conclusion

Initially, we stated that gravity is a conceptual force and can be explained as a consequence of the dynamics of hyperbodies in four spatial dimensions. This research was based on the hypothesis of the existence of a fourth spatial dimension. In this paper, we formulated the surface gravity due to constant voluceleration of a body, orbital velocity, and a celestial body's acceleration due to space distortion. Thus, this research provides an alternative way of explaining the gravitational force of attraction without considering the temporal dimension and the invariance of the speed of light in all reference frames.

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References

- [1] Newton I. The Principia: mathematical principles of natural philosophy. Univ of California Press; 1999.
- [2] Clemence GM. The relativity effect in planetary motions. Reviews of Modern Physics. 1947;19(4):361.
- [3] Brown K. Reflections on relativity. 2004.
- [4] Kenyon IR. General relativity.; 1990.
- [5] Nordström G. Über die Möglichkeit, das elektromagnetische Feld und das Gravitationsfeld zu vereinigen. 1914.
- [6] Salam A, Strathdee J. On Kaluza—Klein Theory. In: Selected Papers Of Abdus Salam: (With Commentary). World Scientific; 1994. p. 451-87.
- [7] Ali A, Sapkota DR. Time dilation in fourth spatial dimension. OSF Preprints; 2022. Available from: osf.io/czg2a.
- [8] Hinton CH. The Fourth Dimension. Health Research Books; 1993. 1-14, 61-75.

- [9] Cage ME, Klitzing K, Chang A, Duncan F, Haldane M, Laughlin RB, et al. The quantum Hall effect. Springer Science & Business Media; 2012.
- [10] Irani A. Dark Energy, Dark Matter, and the Multiverse. *Journal of High Energy Physics, Gravitation and Cosmology*. 2020;7(1):172-90.
- [11] Pound RV, Rebka Jr GA. Apparent weight of photons. *Physical review letters*. 1960;4(7):337.
- [12] Dicke R. The Eötvös Experiment. *Scientific American*. 1961;205(6):84-95.
- [13] Halliday D, Resnick R, Walker J. *Fundamentals of physics*. John Wiley & Sons; 2013. 1123–1124.