Open Journal of Mathematics and Physics | Volume 1, Article 51, 2019 | ISSN: 2674-5747
https://doi.org/10.31219/osf.io/mrv8t | published: 30 Oct 2019 | https://ojmp.wordpress.com

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A pedagogical proof for $\cos(\sin x) > \sin(\cos x)$

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January 21, 2020

Abstract

A pedagogical proof for $\cos(\sin x) > \sin(\cos x)$ is presented [1].

keywords: sine, cosine, trigonometry, inequality

Starting point

1.
$$\sin(x) + \cos(x) < \frac{\pi}{2}$$

2.
$$\cos(x) - \sin(x) < \frac{\pi}{2}$$

3. Both inequalities (1) and (2) are satisfied for $x \in \mathbb{R}$.

Notation

4. The symbol ⬚ means that one case of the proof is completed.

5. As usual, □ means QED (*quod erat demonstrandum*).

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Some basic trigonometric relations

6. $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

7. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

8. The analysis in the following sections becomes clear if one has in mind the trigonometric circle.

Quadrants I and IV

9. In quadrants I and IV: if $x > y$, then $\sin x > \sin y$.

10. From (1),
$$\frac{\pi}{2} - \sin x > \cos x.$$

11. Calculating the sine of (10) and using (9),
$$\sin\left(\frac{\pi}{2} - \sin x\right) > \sin(\cos x)$$

12. Using (6) in (11),
$$\sin\left(\frac{\pi}{2}\right)\cos(\sin x) - \sin(\sin x)\cos\left(\frac{\pi}{2}\right) > \sin(\cos x).$$

13.
$$\cos(\sin x) > \sin(\cos x) \qquad \square$$

Quadrant II

14. From (1),
$$\sin x < \frac{\pi}{2} - \cos x.$$

15. In quadrant II: if $x < y$, then $\cos x > \cos y$.

16. So, from (14) and (15),

$$\cos(\sin x) > \cos\left(\frac{\pi}{2} - \cos x\right).$$

17. Therefore, using (7) in (16),

$$\cos(\sin x) > \sin(\cos x). \qquad \square$$

Quadrant III

18. From (2),

$$\sin x > \cos x - \frac{\pi}{2}.$$

19. In quadrant III: if $x > y$, then $\cos x > \cos y$.

20. So, from (18) and (19),

$$\cos(\sin x) > \cos\left(\cos x - \frac{\pi}{2}\right).$$

21. Therefore, using (7) in (20),

$$\cos(\sin x) > \sin(\cos x). \qquad \square$$

Final Remarks

22. The quadrants I and IV could be analyzed together since the sine is an increasing function in the interval $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$.

23. The analysis of quadrants II and III, however, could not be done together.

24. In the quadrant II, the cosine is a decreasing function in the interval $(\frac{\pi}{2}, \pi)$.

25. In the quadrant III, the cosine is an increasing function in the interval $(\pi, \frac{3\pi}{2})$.

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Ethical conduct of research

This original work was pre-registered under the OSF Preprints [2], please cite it accordingly [3]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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