[white paper: pedagogical]

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The power set has 2^n elements

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April 24, 2021

Abstract

We prove that if A is a set consisting of n elements, then A has 2^n subsets.

keywords: power set, proof by induction

The most updated version of this white paper is available at

<https://osf.io/mptx7/download>

Introduction

1. This is a pedagogical white paper on *set theory*.
2. Our purpose is to discuss a result in [1] which is licensed under [2].
3. We use minimal notation but preserving all relevant mathematical information.

Meta-linguistic symbols

4. $:=$ means that what is on the left is defined by what is on the right.

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Proposition

5.

$$(|A| = n) \rightarrow (|\mathcal{P}(A)| = 2^n)$$

6. $|A|$:= cardinality of the set A

7. $\mathcal{P}(A)$:= power set of A

Proof of (5) by induction [1, 3]

Base case

8. Let $n = 0$.

9. $A = \emptyset \subseteq \emptyset$

10. $\mathcal{P}(\emptyset) = \{\emptyset\}$

11. $|\mathcal{P}(\emptyset)| = 1 = 2^0$

Induction step

12. Let $|A| = m$.

13. Assume $|\mathcal{P}(A)| = 2^m$ (induction hypothesis).

14. Let $B_1, \dots, B_k \subseteq A$ with $k = 2^m$.

15. Let $A' = A \cup \{a\}$.

16. Let $B'_i = B_i \cup \{a\}$ for $i \in \{1, 2, \dots, k\}$.

17. $B_1, \dots, B_k, B'_1, \dots, B'_k \subseteq A'$

18. In (17) you will find all the subsets of A' .

19. $|\mathcal{P}(A')| = k + k = 2^m + 2^m = 2 \cdot 2^m = 2^{m+1}$

□

Final Remarks

20.

$$(|A| = n) \rightarrow (|\mathcal{P}(A)| = 2^n)$$

21. (20) was proved by **induction**.

22. proof by induction = base case + induction step

Open Invitation

Review, add content, and co-author this white paper [4, 5].

*Join the **Open Mathematics Collaboration**.*

Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [6].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [7], please cite it accordingly [8]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

Acknowledgements

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Agreement

23. All authors agree with [5].

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