“A voice unknown and mysterious” :
Emmy Noether / Jean Cavailles, Exchanges
Cantor-Dedekind (1937) – a warning translated,
and the birth of set theory.

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Abstract

Translation of the forewords to Emmy Noether and Jean Cavailles’
1937 *Briefwechsel Cantor-Dedekind*, an edition of exchanges between Georg
Cantor and Richard Dedekind, the foundations for set theory.

Presentation

At the end of his introduction – a “warning” – for the Cantor-Dedekind correspondence, Cavailles gives clues as to its origins :

“The present edition was ready 4 years ago. Delayed by various circumstances, today it is published exactly as reviewed with ourselves by Miss Noether; memories leading back to those days in Goettingen where we got to know after many others her gay and gracious welcome, the intense radiance of her spirit.”

In this warning [“avertissement”], Cavailles wrote an essay on Cantor’s work and life, both appearing to have taken place against all odds, with few early supporters (Dedekind, of course, Mittag-Leffler, and Heine are mentioned).

Such figures are not missing in the history of the sciences, they do no paint science in a good light...

But, to younger people and new researchers, we want to assure that they should go about their business with no worries : “*Pay it no mind!*”, said loud and proud someone whose own life had sure overcome all odds. Conduct your science uncompromisingly.

Mistakes, of which lives are full, and become fuller, escape even into our sciences. Do plenty of them, and early on — as many as you can. Reflect on them, and grow strong and resilient.

In some way, the Cantor-Dedekind collaboration seems to mirror the Noether-Cavailles editorial project. Both of these legendary pairs of scientists had worked together with seemingly none of the barriers that make science hard, for reasons non-scientific.

We would like to report that the likes of Kronecker have disappeared; and, yet they continue to live happy lives and thrive in our universities. But, history always finds a way back to them – with a slap.

For what remains of Kronecker except a footnote in the life of Cantor...

WARNING

In 1872 Cantor had the good luck of meeting Dedekind, at random during a trip in Switzerland. From this encounter an exchange was born – lasting with interruptions until 1899 – out of which came about all fundamental ideas of set theory.

We know the at times dramatic destiny of Georg Cantor¹. There are few examples, notes Fraenkel²³, of lives as intimately tied to a work, of a theory as obviously work of a single person as that of Cantor.

In a letter he wrote at age 17, Cantor speaks of “a voice unknown and mysterious”, which, counter to the wishes of his own [family?], drew him to mathematics⁴.

In 1872, his colleague from Halle⁵, Heine encouraged him to study trigonometric series.

Here unfolded in just a few years astonishing discoveries – astonishing for the author first : “I see it, but I don’t believe it” he wrote to Dedekind in 1877 about one of them –, these radically new notions, the powers of abstract ensembles [sets], beginnings of topology, the arithmetic of transfinite numbers, edifice of a tenacity and beauty such it prompted Hilbert to say : “one of the most beautiful creations of the mathematical mind”, but whose novelty also attracted very soon defiance, in Weierstrass, and open hostility in Kronecker⁶.

In 1877 in letters (which we felt we needed to reproduce here) Cantor worries about the delay caused by *Crelle’s Journal* (managed by Kronecker) in the publication of his first article on set theory⁷.

We thought this was – Cantor first, at first – an attack on the classical notion of dimension. Worried, sensitive already to the generally unfavorable atmosphere, he considered withdrawing his article, and having it published as a separate brochure like Dedekind had done for *Stetigkeit u. irrationale Zahlen* [Continuity and irrational numbers].

¹**Trans. note** : we believe it was Cantor who was relentlessly pursued by a scientific opponent who sought no less than his professional destruction (we forgot the name of this particular gentle mathematician). Cantor had many critics, and reminds of this other scientist of a ‘tragic destiny’, Semmelweis, who faced considerable opposition from the medical institution. (Now that we have gotten further in translating this text : we meant Kronecker.)

²A. Fraenkel, *Georg Cantor. Jahresb. d. Deutsch. Math. Verein.* t. 39 (1930), p. 189-266, reprod. partially in *G. Cantor Ges. Abhandlungen*, hrsg. v. Zermelo. Berlin, Springer, 1932, p. 452. From him we take numerous information. In fact, it was A. Fraenkel who first made us aware of this correspondence.

³**Trans. note** : [Yearbook of the German mathematical association].

⁴*Ibid*, p. 453.

⁵**Trans. note** : German city located in the state of Saxony-Anhalt, about 150 km to the South of Berlin.

⁶**Trans. note** : needless to say this is not an exhaustive expose : in a list with seemingly no end, Poincaré among others thought lowly of Cantor and his new theory, which he called “a grave mathematical malady” (now on the wrong side of history).

⁷**Trans. note** : “sa démonstration sur l’identité de puissance des continus a nombre quelconque de dimensions” [his proof on the power identity of continuums of numbers of any dimensions?].

This time the worries were in vain, but it was the last time that Cantor published in *Crelle's Journal*.

The fundamental articles – where the main parts of his theory were already developed – appeared in the *Mathematische Annalen* [Mathematical annals] from 1875 to 1883, in the midst of growing hostility.

Even Weierstrass, according to the testimony of Schoenflies⁸, showed coldness.

The only support came from Mittag-Leffler who opened to Cantor the *Acta Mathematica* which he had just founded.

We know, as early as the second volume [“Tome II”], inside it appeared a French translation of the Memoirs of the *Mathematische Annalen*, a project undertaken, Mittag-Leffler tells⁹, by the group of young mathematicians gravitating around Hermite; the article *Quelques theoremes sur les ensembles infinis et lineaires de points*¹⁰ in particular is said to have been translated by Poincare.

Hence this was a partial recognition, which would be confirmed by the success of two memoirs of Mittag-Leffler, *Sur la representation analytique des fonctions d'une variable independante et Demonstration nouvelle du theoreme de Laurent*, where for the first Cantor's notions were used.

However, Cantor's irritability only grew; he knew Kronecker labeled him as “corrupter of youth”, publicly denigrated his works and barred access to Berlin from him. He complained of this, sometimes witty and funny – so for instance when he called Kronecker someone with a mind improper to face mathematical progress, sometimes with sickly impatience, so in his letters to Mittag-Leffler from around 1884¹¹.

It's in this same year [at this same date], that appears this other aspect, the most important of this drama.

The edifice of ensembles [or sets] was well outlined in the memoirs of the *Mathematische Annalen*, but it lacked the necessary crowning, the theorem on continuity.

“Of what has followed, we can with the help of the theorems established in N. 5 conclude that linear continuity has power of class II.”

And, so, came to an end in 1882 the last article On finite ensembles... [Sur les ensembles infinis lineaires de points].

(...)

As such the connection between the two branches of set theory, topology and trans-finite arithmetic was established.

The year of 1884 was the most intense. In a letter to Mittag-Leffler from October 25 he triumphed : “I am now in possession of an extremely simple proof of this most important theorem of ensembles.”

(...)

⁸A. Schoenflies, *Die Krisis in Cantors mathematischem Schaffen*. Acta Math. 50 (1928), p. 1-23.

⁹Mittag-Leffler, *ibid*, p. 25.

¹⁰**Trans. note** : this should correspond to “Beitrage zur Begrundung der transfiniten Mengenlehre” (1897). A more accurate translation of its title we propose is : Contributions to the foundation of the theory of infinite [trans-finite] sets.

¹¹Published by Schoenflies, *loc. cit.*.

The present edition was ready 4 years ago... (...)

J. C.

We publish here the mathematical parts of the Cantor-Dedekind correspondence, with the exception of those that E. Zermelo had already included in his edition of Cantor's work [*Cantorausgabe*]. Many things found in these letters can be found again almost unchanged in Cantor's later publications. The correspondence comes from the estate of Dedekind; the letters from Dedekind to Cantor are concepts.

[*die Brief von Dedekind an Cantor sind Konzepte.*]

E. N. — J. C.

Acknowledgements

Dedicated to Marsha P. Johnson.

Their memories and fights continue to inspire us.

Cryptographers remember Emmy, we too remember Marsha.