Interest Rate Swap Model

An interest rate swap is a financial contract between two parties to exchange future interest rate payments over a set period of time. There are two legs associated with each party.

Each leg of a swap can be based on fixed or floating interest rate payments or quanto payments. A swap can thus include any combination of these basic leg types.

Consider the floating rate leg of swap specified as follows:

- reset dates, $0 \leq T_0 < T_1 < \ldots < T_i < T_{i+1} < \ldots < T_{n-1}$,
- spot LIBOR rate, $L(T_i)$, which sets at time T_i for the accrual period $\Delta_i = T_{i+1} - T_i$.

For a swap *settled in arrears* the payment

$$L(T_i)\Delta_i$$

is made at T_{i+1}, i.e. at the end of the period.

Consider the fixed rate leg of swap specified by

- reset dates, $0 < \tau_0 < \tau_1 < \ldots < \tau_i < \tau_{i+1} < \ldots < \tau_{m-1}$,
- fixed rate, R.

At a reset date τ_{i+1} the fixed leg pays $R(\tau_{i+1} - \tau_i)$. In the above, the reset dates are not necessarily the same as for the floating leg.

Up to the rolling convention that specifies a day counting treatment for holidays, the intervals between reset dates (accrual periods) must be regular and one of the following: daily, weekly, biweekly, monthly, bimonthly, quarterly, semi-annual, annual, or bi-annual.

The present value of this payment is

$$B(T_{i+1})L_0(T_i)\Delta$$

where

- $B(t)$ is the discount factor for time t,
- $L_0(T_i) = \frac{1}{\Delta}\left[\frac{B(T_i)}{B(T_{i+1})} - 1\right]$ is the forward LIBOR rate.

Let $\delta_i = \tau_i - \tau_{i-1}$ be an accrual interval between two resets for the fixed rate leg.

A forward swap rate can be computed as

$$\kappa = \frac{\sum_{resets} \Delta_i L_0(T_{i-1}) B(T_i)}{\sum_{resets} \delta_i B(T_i)}$$

which makes the swap worthless at trade date.

The value of the floating side is,

$$\sum_{resets} \Delta_i L_0(T_{i-1}) B(T_i),$$

while the price of the fixed leg is

$$R \sum_{resets} \delta_i B(T_i) .$$

You can find more details at

https://finpricing.com/lib/EqVariance.html