Improving hierarchical models of individual differences: An extension of Goldberg's bass-ackward method

Miriam K. Forbes

*Centre for Emotional Health and School of Psychological Sciences,
Macquarie University, Sydney, Australia*

Correspondence to: Miriam K. Forbes, PhD
Room 701 Building 4 First Walk
Centre for Emotional Health,
Department of Psychology,
Macquarie University,
Sydney Australia 2109
Email: miri.forbes@mq.edu.au

Author Note

I would like to thank the Assessing Mental Health team (Philip J. Batterham, Matthew Sunderland, Natacha Carragher, and Alison L. Caley) for sharing their data for these analyses; Robert F. Krueger, Lewis R. Goldberg, and William R. Revelle for feedback on an earlier version of this manuscript; and the Matilda Centre's TTT group (the AMH team as well as Andrew J. Baillie, Samantha J. Lynch, Louise Mewton, and Tim Slade) for helping me think through this methodological approach. Feedback from the editor and anonymous reviewers on the larger project that included the extended bass-ackward analyses of the AMH data, and on the first submission of this manuscript and code, was also invaluable. Finally, the code for the ExtendedBassAckward function was modelled on the bassAckward function from the psych package in R, so thank you to William R. Revelle for developing and maintaining that fantastic resource.

The ideas underlying this manuscript were presented at the HiTOP2019 meeting in Buffalo, NY, and have been posted on OSF (preprint: <https://osf.io/ksxwv/>; data, code and supplementary materials: <https://osf.io/pcwm8/>).

Draft preprint version 5, November 24th 2022. This paper is now accepted for publication in *Psychological Methods*.

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Abstract

Goldberg's (2006) bass-ackward approach to elucidating the hierarchical structure of individual differences data has been used widely to improve our understanding of the relationships among constructs of varying levels of granularity. The traditional approach has been to extract a single component or factor on the first level of the hierarchy, two on the second level, and so on, treating the correlations between adjoining levels akin to path coefficients in a hierarchical structure. This article proposes three modifications to the traditional approach with a particular focus on examining associations among all levels of the hierarchy: 1) identify and remove redundant elements that perpetuate through multiple levels of the hierarchy; 2) (optionally) identify and remove artefactual elements; and 3) plot the strongest correlations among the remaining elements to identify their hierarchical associations. Together these steps can offer a simpler and more complete picture of the underlying hierarchical structure among a set of observed variables. The rationale for each step is described, illustrated in a hypothetical example and three basic simulations, and then applied in real data. The results are compared to the traditional bass-ackward approach together with agglomerative hierarchical cluster analysis, and a basic tutorial with code is provided to apply the extended bass-ackward approach in other data.

Keywords: individual differences research; top-down hierarchy; bass-ackward approach; tutorial

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Hierarchies represent a particularly useful framework for individual differences research because they can accommodate the multiple levels of construct granularity in fields spanning normative and maladaptive personality, cognitive ability, and common mental disorders, to name a few (e.g., Carroll, 2005; Kim & Eaton, 2015; Markon, 2009; Wright et al., 2012). For example, research on the empirical structure of psychopathology was recently summarised in a hierarchical and dimensional framework—the Hierarchical Taxonomy of Psychopathology (HiTOP; Kotov et al., 2017). This hierarchy integrates dimensions of varying breadth, from an overarching general factor of psychopathology down to narrow symptom components and maladaptive traits, offering a framework for understanding the key correlates of psychopathology—such as risks, outcomes, and treatment effects—at varying levels of specificity and generality (Conway et al., 2019; Krueger et al., 2018).

Existing Methods for Explicating Hierarchies in Individual Differences Data

There are several existing statistical methods can be used to explicate hierarchies in individual differences data. One longstanding example is agglomerative hierarchical cluster analysis (Ward, 1963). It works by forming a hierarchy of mutually exclusive groups of observed variables based on their similarity—iteratively combining the most closely related pairs of objects or clusters that result in the least loss of information. However, this approach is limited to a simple structure wherein each variable in the model can be in only a single cluster, so cannot map the more complex relationships that are likely to be ubiquitous for many psychological constructs (e.g., cross-loadings of interstitial constructs, or negative relationships; Goldberg, 2006).

To overcome these limitations of common approaches to variable-centred cluster analysis, Goldberg (2006) proposed the “bass-ackward” method to delineate hierarchies in individual differences data—so called because it takes a top-down approach, rather than the bottom-up approach from agglomerative cluster analysis. Goldberg’s bass-ackward method uses either a principal components analysis (PCA) or an exploratory factor analysis (EFA) framework, with orthogonal or oblique rotations.¹ In the traditional bass-ackward approach, a single component or

¹ Goldberg (2006) and Loehlin and Goldberg (2014) highlighted advantages of principal components over factors, including computational economy, avoidance of Heywood cases, and calculating factor scores directly instead of estimating them. These advantages are most noteworthy in complex hierarchies with many levels and components. I focus on orthogonal (varimax) PCA in the examples provided here, but note that it is also possible to use oblique rotations, and/or to use EFA (e.g., Kim & Eaton, 2015), which give closely similar results when examining large numbers of variables (e.g., Goldberg, 1990). The ExtendedBassAckward function described in the supplementary materials is also equipped to do orthogonal or oblique PCA or EFA.

factor is extracted at the first level of the hierarchy, and an additional component or factor is extracted in each subsequent level (see Figure 1). The Pearson correlations between sequential levels of the structure are then examined (i.e., between component scores or estimated factor scores) to characterise the relationships among the components. For example, Loehlin and Goldberg (2014) recommended considering correlations $|r| \geq .9$ as a component perpetuating from one level to the next (the solid lines in Figure 1), and interpreting correlations $.3 \leq |r| < .9$ as indicating the emergence of new components (e.g., a higher-order component splitting into two narrower components, shown as dashed lines in Figure 1).²

Figure 1. Example of a four-level bass-ackward structure where solid lines indicate the perpetuation of a component between levels ($|r| \geq .9$) and dashed lines indicate emergence of new components ($.3 \leq |r| < .9$).

The traditional bass-ackward method has some advantages over hierarchical factor models, such as higher-order and bifactor models. For example, in these other hierarchical factor models, each level of the hierarchy is based on different variance: First-order latent variables are based on the shared variance among the observed variables, whereas second-order latent variables are based on the shared variance among the first-order latent variables; and the bifactor general factor is estimated based on different variance than the specific factors. In these models, the different levels of the hierarchy are thus statistically and conceptually entangled, so cannot be estimated separately. By contrast, the levels of the hierarchy that emerge in a bass-ackward framework are conceptually distinct and estimated separately; the same variance is re-partitioned in different ways at each level of the hierarchy—into 1, 2, 3, ... n components or factors—essentially examining the structure of the data at varying levels of granularity.

² These correlations should not be confused with factor loadings in a hierarchical factor model. This distinction is particularly clear when extracting orthogonal components in a bass-ackward approach: there are no correlations among the lower-order components to reparameterise in the higher-order components.

These different levels of the hierarchy can then be used to pursue varied research questions without needing to pin down a single fixed number of dimensions to characterise the data. For example, we can elucidate a single hierarchical model to facilitate research on detailed mechanisms and processes—via a focus on narrow phenotypes—as well as research on overarching risk factors—via a focus on broad dimensions.

An Extended Bass-Ackward Approach

The traditional bass-ackward approach evidently has multiple strengths that offer improvements over some alternative methods for modelling hierarchies of individual differences. However, if the aim of a study is to characterise a single hierarchical model with broad dimensions that subsume narrower dimensions in a complex hierarchical structure, there are three aspects of the bass-ackward method that I propose could be further improved.

First, removing redundant components from the hierarchy could simplify the interpretation of multi-tiered hierarchies. As mentioned above, the traditional bass-ackward approach often includes a convention of interpreting components correlated $\geq .9$ as perpetuating through the hierarchy (i.e., redundant across levels). I propose we remove these redundant components from the hierarchy altogether. To be conservative, an optional additional criterion would be that the components or factors also have a Tucker's congruence coefficient $>.95$, indicating that they are equal (Lorenzo-Seva & Ten Berge, 2006). For constructs that perpetuate to the lowest level of the hierarchy, the version at the bottom of the hierarchy should be retained because it is most likely to be cleanly defined when other fine-grained constructs are also identified (i.e., there is greater specificity of the constructs in the model). Similarly, for constructs that perpetuate from the top or through the middle of the hierarchy, the version of the construct closest to the top should be retained (i.e., the broadest manifestation, most likely to fully subsume lower-level constructs in the hierarchy).

Second, identifying constructs that are likely to be statistical artefacts will further help to isolate the substantive hierarchical structure of the data. We would not usually expect all levels of the hierarchy (e.g., all PCA or EFA solutions with 1 to n components or factors) to be interpretable and meaningful; forcing an n -component structure may create artefactual combinations of narrower components, overextract components with multiple indicators, or simply represent a poor fit to the data and thus result in an uninterpretable solution (cf. Guilford, 1975). I propose that if there are clear artefactual components or factors in the structure, removing them—i.e., incorporating “theoretical psychological thinking” in the interpretation of the solutions (Guilford, 1975, p. 802)—may improve the validity of the resulting model. Notably, this proposed step will introduce researcher degrees of freedom and confirmation bias into the process, so should be undertaken with caution and transparency, as discussed further below (cf. Wicherts et al., 2016).

Third, and most importantly, examining the associations among the components across *all* levels of the hierarchy will allow us to better disentangle the key relationships between broad and narrow components. The strongest or most informative associations among components may span multiple levels of the hierarchy, rather than being carried in sequential (adjoining) levels. For example, a broad dimension that has three or more sub-components may not be identifiable when taking the traditional bass-ackward approach, as narrow components or factors that emerge in the lowest levels of the structure are only able to correlate with other similarly narrow elements on the preceding level (see examples below). I propose the associations among the components or factors across all levels of the hierarchy should be examined to provide a more complete picture of the relationships of interest.

Summary. These proposed modifications can be summarised as three simple steps constituting the *extended bass-ackward approach*:

- 1) identify and remove redundant components or factors that perpetuate through multiple levels of the hierarchy;
- 2) (optionally) identify and remove artefactual elements; and
- 3) plot the strongest component correlations among the remaining elements to identify their hierarchical structure.

Removing redundant elements becomes particularly important when following the last step, as the expanded focus on the associations among components or factors at all levels of the hierarchy results in a rapidly increasing number of relationships of interest (e.g., 85 correlations for a five-level hierarchy versus 1320 correlations for a 10-level hierarchy). If artefactual elements are also removed, additional analyses should be conducted to test the robustness of the solution (e.g., testing the resulting hierarchy in a confirmatory factor analysis framework, or examining convergence with an alternative approach to elucidating the hierarchical structure, such as agglomerative hierarchical cluster analysis and/or the traditional bass-ackward method, as described below).

A brief hypothetical example is presented below to give an overview of the proposed approach, followed by three basic simulations, and finally an applied example—each comparing the proposed extended bass-ackward approach to the traditional bass-ackward method together with agglomerative hierarchical cluster analysis. The hypothetical and applied examples draw on the HiTOP framework mentioned above, which represents a multi-tiered hierarchy of individual differences used in psychopathology research (Kotov et al., 2017). To apply the extended bass-ackward approach, I have written some rudimentary R functions to generate the required output as well as a brief standalone tutorial on how to apply the functions and interpret the output (see

Supplementary Materials; see also <https://osf.io/pcwm8/> for the code and data required to generate the results for the simulations and applied example).³

Hypothetical Example

Figure 2A represents a simple hypothetical set of results using a traditional bass-ackward approach to examine the structure that unfolds among indicators of the internalizing dimension in the HiTOP framework (Kotov et al., 2017). This broad dimension of psychopathology is related to a propensity to experience high levels of negative affect, and includes sub-dimensions of fear, distress, eating pathology, and sexual problems (Kotov et al., 2017). In Figure 2A, the first unrotated principal component represents a *broad internalizing* dimension, which splits into a *sexual problems* component and a narrower *internalizing* component (i.e., without sexual problems indicators). The *sexual problems* component subsequently perpetuates through the remaining levels of the hierarchy. The *internalizing* component splits into a *fear* component and a *distress (eating pathology)* component that has strong loadings from traditional indicators of the distress dimension (e.g., depression and generalized anxiety), and weak loadings from eating pathology symptoms. On the lowest level of the hierarchy, the *distress (eating pathology)* component splits into more clearly delineated *distress* and *eating pathology* components.

Figure 2. A hypothetical example of how the structure of the internalizing spectrum from the HiTOP model (Kotov et al., 2017) might emerge using (A) the traditional bass-ackward method where solid lines indicate the perpetuation of a component between levels ($|r| \geq .9$), dashed lines indicate emergence of new components ($.3 \leq |r| < .9$), (B) hierarchical agglomerative cluster analysis, and (C) the proposed extension to the bass-ackward approach where solid lines represent the strongest component correlation for each lower-order component with the higher-order components. INT = internalizing; Sex = sexual problems; Dist = distress; Eat = eating pathology.

³ An R package *ackwards* is in development to automate these analyses (<https://github.com/jmgirard/ackwards>) and an updated tutorial estimating extended bass-ackward models will be posted on the OSF page for this project (<https://osf.io/pcwm8/>) when the package is ready.

The four components at the lowest level of the hierarchy are familiar, interpretable, and well-defined, and the *broad internalizing* component reflects the overarching dimension we expected based on the HiTOP model. On the second level of the hierarchy, *sexual problems* diverged, which indicates that this component is less interrelated with the other three components. However, we might suspect that the *distress (eating pathology)* component reflects an artefact of forcing a 3-component structure (i.e., resulting in eating pathology indicators being weak indicators for a *distress* component). Figure 2B represents a hypothetical cluster analysis solution in the same data, closely mirroring the hierarchy in Figure 2A with the exception that eating pathology agglomerates with fear, rather than distress. This finding would reinforce our suspicion that the *distress (eating pathology)* component seems likely to be an artefact of forcing a 3-component structure.

Figure 2C represents the solution after using the extended bass-ackward approach. First, we simplify the results by removing the redundant *sexual problems* and *fear* components that repeat across multiple levels of the hierarchy. Second, supported by the finding that the *eating pathology* component is more strongly correlated with *internalizing* than with *distress (eating pathology)*—and that it agglomerates with *fear* in a cluster analytic hierarchy—we infer that the *distress (eating pathology)* component is an artefact rather than a substantive construct and remove it from our interpretation of the unfolding structure of the data. By moving away from a focus between only sequential levels of the bass-ackward results, we thus clarify that *eating pathology* is unlikely to be a sub-component of *distress*. Finally, we can plot the strongest component correlations among the remaining components. The resulting model is shown in Figure 2C, offering a simplified hierarchical structure (5 correlations among 6 components, versus 9 correlations among 10 components; cf. the index of similarity proposed by Goldberg, 2006). This structure could be tested for model fit in a confirmatory factor analysis—perhaps comparing it to a model that has a single internalizing latent variable with four indicator variables (sexual problems, fear, distress, and eating pathology) to test whether the divergence of the *sexual problems* and *internalizing* components was an artefact of forcing a 2-component structure.

Simulated Examples

We can also compare the extended bass-ackward approach with traditional bass-ackward and agglomerative hierarchical cluster analysis methods in simulated data with a known structure to examine under which circumstances their performance differs. For the following three examples, data were simulated using the `sim.structure()` command in the `psych` package (Revelle, 2020) in R version 4.1.1 (R Core Team, 2021; see <https://osf.io/pcwm8/> for the code). Panel A in Figures 3-5 show the hierarchical factor structures from which data ($n = 5000$) were simulated. These simulations were not preregistered and were selected to illustrate examples where the performance of

the three methods differ, when the proposed extensions improve on the traditional bass-ackward method, and how the cluster analysis solutions can clarify and strengthen our confidence in the hierarchical structure of the data.

Figure 3. A summary of the first simulation, including the structural model from which the data ($n = 5000$) were simulated, reflecting a hierarchy we might expect to ‘unfold’ over sequential levels (Panel A); and the recovered structures using the extended bass-ackward approach (B), Ward’s agglomerative hierarchical cluster analysis (C), and Goldberg’s original bass-ackward method (D).

Simulation 1: A hierarchy we might expect to ‘unfold’ over multiple levels. The first example shown in Figure 3A includes an overarching general factor (**G**), two closely related specific factors (**S1** and **S2**) identified by observed variables **v1-v4**, and **v5-v8**, respectively, and a third uncorrelated specific factor (**S3**) that has two items with strong loadings (**v9** and **v10**), and two items with weak loadings that more strongly load on the general factor (**v11** and **v12**). The structure of

these data unfolds similarly across the three methods (see Figures 3B-3D): The G component splits into $S1/S2$ and $S3$ components; $S1/S2$ splits into $S1$ and $S2$; and $S3$ splits into $v9/v10$ and $v11/v12$ components. Here, the proposed extended bass-ackward approach does not provide any real advantages over the traditional bass-ackward result, except for the simplification of removing the three redundant components on the third level (6 correlations among 7 components, versus 9 correlations among 10 components).

Figure 4. A summary of the second simulation, including the structural model from which the data ($n = 5000$) were simulated that includes cross-loadings indicators ($V4$ and $V5$) between $S1$ and $S2$ (Panel A); and the recovered structures using the extended bass-ackward approach (B), Ward's agglomerative hierarchical cluster analysis (C), and the original bass-ackward method (D).

Simulation 2: Item-level cross-loadings that cannot be captured by cluster analysis. The second example shown in Figure 4A is similar to the previous example, except $v4$ and $v5$ have cross-loadings on $S1$ and $S2$. The structure of these data are very similar between the traditional and

extended bass-ackward approaches (Figures 4B and 4D), which both allow v4 and v5 to cross-load, as expected. However, the cross-loadings cannot be captured in the mutually exclusive clusters (Figure 4C): v4 and v5 agglomerate before merging with the rest of S1 (v1-v3), and higher up in the hierarchy merge with the rest of S2 (v6-v8). Further, the extended bass-ackward approach again simplifies the structure compared to the traditional bass-ackward method (6 correlations among 7 components plotted, versus 9 correlations among 10 components, as above).

Simulation 3: A hierarchy we would expect to have two tiers. The last simulation example, shown in Figure 5A, includes four identical uncorrelated specific factors (S1-S4, each identified by three items) and a general factor (G).

Figure 5. A summary of the third simulation, including the structural model from which the data ($n = 5000$) were simulated, reflecting a hierarchy we would expect to have two tiers (Panel A); and the recovered structures using the extended bass-ackward approach (B), Ward's agglomerative hierarchical cluster analysis (C), and Goldberg's original bass-ackward method (D).

Using the extended bass-ackward approach (Figure 5B), we remove all three redundant components on the third level of the hierarchy, and one on the second level. The other component on the second level of the hierarchy appears to be an artefact of forcing a two-component structure. For example, five items cross-load $>.3$ on both components on this level, and similar structures are not evident in the cluster analysis. If we remove S4 (S1 S3) in addition to the redundant components, we have a clean and clear two-tier hierarchy that reflects the known structure of the data.

By contrast, the traditional bass-ackward result obscures and overcomplicates the underlying hierarchy (plotting 12 correlations among 10 components, compared to 4 correlations among 5 components in the extended bass-ackward result). For example, the items that identify S3 (v7-v9) emerge as a component at the bottom of the hierarchy that appears to be nested under S2 (S3) on the preceding level. By clarifying that the S3 component is most strongly associated with G, the extended bass-ackward approach clarifies the underlying hierarchical structure that is obscured using the traditional approach. Cluster analysis correctly identifies the four narrow clusters of symptoms. While it cannot technically collapse the three subsequent levels of hierarchical clustering into two tiers, the clearly delineated structure aids in clarifying and strengthening our confidence in the extended bass-ackward solution.

Of course, the examples described so far have been over-simplified illustrations. Let us turn now to explore a more complex applied example.

Applied Example

I recently led a study (Forbes et al., 2021) that applied the extended bass-ackward approach to delineate the hierarchical structure of symptom data from the Assessing Mental Health (AMH) study (Batterham et al., 2016). A detailed description of the AMH study methods is available elsewhere (Batterham et al., 2016) and more details on the analytic sample and pre-processing of the data are provided in Forbes et al. (2021). Briefly, the AMH study tested large self-report item banks of a variety of mental health problems in a population-based Australian adult sample with the aim of developing new brief mental health screeners. The analytic sample ($n = 3175$) overrepresented individuals with psychopathology, relative to population prevalence rates (see Batterham et al., 2016). In total, 562 symptom-level items were reduced to 155 highly homogeneous symptom clusters—including 74 individual items, and 81 multi-item clusters—based on the convergence of two clustering methods (ICLUST and Ward's hierarchical agglomerative clustering; Revelle, 1978; Ward, 1963). All symptom clusters had a minimum 90% of the variance in the constituent items associated with a shared general factor. These items and item clusters were used as the indicator variables in the hierarchical analyses. As described in Forbes et al. (2021), 1 to 10 components were extracted in the final hierarchical models. Given the large number of variables being analysed (155

indicator variables loading on 55 different components), we focused on loadings $\geq .4$ in interpreting each component.

The traditional bass-ackward model was estimated using the *bassAckward* function in the *psych* package (Revelle, 2019) based on a pairwise complete Spearman correlation matrix estimated among the 155 highly homogeneous symptom clusters. A summary of the traditional bass-ackward results is given in Figure 6A. We see familiar structures from the literature on the empirical structure of psychopathology, such as fear, distress, substance and alcohol use, and thought disorder (cf. Kotov et al., 2017). There are also some novel structures—a *disinhibited negative affect* component, for example. However, the redundancy and potential artefacts in the structure add complexity and noise to our interpretation of the underlying hierarchy; there are also several hierarchical associations that do not become clear until we look at component correlations across all levels of the hierarchy.

The *ExtendedBassAckward* function described in the Supplementary Materials generates the required output to construct an extended bass-ackward result, including the component loadings for each level of the hierarchy, the correlations across all levels of the resulting hierarchy, and the congruence coefficients. The first step in processing the output includes identifying chains of redundant components that are correlated $|r| \geq .9$ and, optionally, have congruence coefficients $> .95$. The *FindRedundantComp* function described in the Supplementary Materials returns summary output that includes the paths of redundant variables that can be chased through the structure based on correlations, and based on congruence coefficients. For example, in the AMH data we can see that *OCD* (J2) could be chased to *OCD* (F3), meaning J2 is correlated $\geq .9$ with F3 and with components on all intervening levels. Similarly, *OCD* (I2), *OCD* (H2), and *OCD* (G3) could all be chased to *OCD* (F3), telling us that the path *j2—i2—h2—g3—f3* is a string of potentially redundant variables perpetuating across multiple levels of the hierarchy. To take a conservative approach, we can also confirm that the congruence coefficient output highlights the same set of redundant components, as is the case here. We can thus infer that I2, H2, G3, and F3 are redundant with J2, and remove these components. Following this step for all components in the hierarchy illuminates the patterns of redundancy shown in green in Figure 6A. Specifically, as described above, for constructs that perpetuate to the lowest level of the hierarchy, the version at the bottom of the hierarchy is retained; for constructs that perpetuate from the top or through the middle of the hierarchy, the version of the construct closest to the top is retained.

The second step focuses on interpreting the remaining components based on their component loadings, identifying those without a substantive interpretation to (optionally) remove them. The four presumed artefactual components in this example are greyed in Figure 6A. Three of the four (C1-C3) are on the third level of the hierarchy where the indicators of *thought disorder* (D2) split across *internalizing* and *substance use*—apparently as a result of forcing a three-component structure. The

third component on this level *alcohol use* (C3) appears to represent a slight rotation of the lower-order *alcohol use* (J5) component that does not quite meet our threshold

Figure 6. (A) Traditional bass-ackward model estimated in real symptom-level data. Solid lines depict the perpetuation of a component between levels ($|r| \geq .9$), dashed lines depict emergence of new components ($.3 \leq |r| < .9$), dotted lines depict secondary correlations $.3 \leq |r| < .9$. The red line is a negative correlation. Green boxes depict redundant components and grey boxes depict potential artefacts (see further description in-text). (B) Hierarchical structure based on the proposed ‘extended bass-ackward’ approach. Solid lines represent the strongest component correlation for each lower-order component with the higher-order components and dashed lines are secondary component correlations $.3 \leq |r| < .9$. The dendrogram from Ward’s agglomerative hierarchical cluster analysis is shown in Figure S1.

for redundancy ($r = .89$, congruence coefficient = .93). Finally, in the six-component solution the suicidality indicators split across *psychosis and suicidality* (F4) and *disinhibited negative affect* (F2) components. This association between suicidality and psychosis indicators was not seen on any other level of the hierarchy—either through component correlations between levels, or symptom clusters (cross-)loading within levels—suggesting that *psychosis and suicidality* (F4) may also represent an artefact. To check whether any of these four structures are robust across methods (i.e., likely to be substantive constructs), we can also examine the cluster analytic hierarchy for these constructs. In this case, none of the apparent artefacts were evident in the cluster analysis hierarchy (see Figure S1), so we can be relatively confident in removing these four components from the hierarchy. Regardless, it is important to make this decision explicit and to consider any impacts on the interpretation of the results (e.g., compared to a traditional bass-ackward approach that includes all components).

The third and final step involves adding the strongest correlations from lower- to higher-order components for the remaining constructs, as well as any correlations $.3 \leq |r| < .9$ not accounted for hierarchically (i.e., that cannot be traced as an upwards path in the model). The *HierarchicalCorrs* and *NegativeCorrs* functions described in the Supplementary Materials isolate the candidate correlations for this step, which completes the hierarchical structure shown in Figure 6B. At the top of the hierarchy, *general psychopathology* split into three components: (1) *substance and alcohol use*, composed of the lower-order *substance use* and *alcohol use* components; (2) *thought disorder*, composed of the *OCD* and *psychosis* components; and (3) *internalizing*, composed of the *disinhibited negative affect* and *fear* components. The lower-order *disinhibited negative affect* component is composed of *disinhibition (anger and attentional dysregulation)* and *distress (suicidality/hopelessness, with cross-loadings from social anxiety symptom clusters)*. The *fear* component was composed of *anxiety*, *PTSD*, and *social anxiety* components. Notably, there were two sets of secondary correlations that helped to clarify the hierarchical structure: The strongest correlation for the *social anxiety* component was with the higher-order *internalizing* component, reflecting its secondary correlations with both the lower-order *distress* and *fear* components. Similarly, *distress* was most strongly associated with *internalizing*, but had a secondary correlation with *disinhibited negative affect* that clarified the nature of the latter component.

The extended bass-ackward approach highlighted several important relationships that were not evident taking the traditional bass-ackward approach, as mentioned above. For example, the three-component structures of both *general psychopathology* and *fear* could not emerge when exclusively focusing on associations between sequential levels. There were also substantive differences in inferences based on the two structures: Taking the traditional bass-ackward approach, *PTSD* (the last component to emerge) appeared to be most closely related to *social anxiety* (Figure

6A), but when examining associations across all levels of the hierarchy, it became evident that the strongest relationship was between *PTSD* and the higher-order *fear* component (Figure 6B). Similarly, *social anxiety* was not most strongly associated with *distress*, as appeared in the traditional bass-ackward approach (Figure 6A), but with *internalizing*, reflecting its secondary correlation with *fear* (Figure 6B). The associations of *thought disorder* and *internalizing* with *general psychopathology* were also obscured due to apparent artefacts in the three-component structure. More broadly, with fewer components to interpret (the final model had 20 correlations among 18 components, versus 58 correlations among 55 components in the traditional model), the hierarchy based on the extended approach is simpler to interpret.

When a two- or three-tier hierarchy emerges that can be parameterised as a first- or second-order confirmatory factor analysis, the fit of the hypothesised hierarchical model to the data can be tested directly (cf. Greene et al., 2021). In this example, the four tiers of hypothesised latent variables do not align well with that approach, so we relied on convergence with the cluster analytic hierarchy to check for robustness of the structure (Figure S1). Notably, the structures were highly similar, including substance use, thought disorder, disinhibition, fear, and distress clusters (social anxiety was part of the latter). The one substantive difference in interpretation was that fear and distress formed a traditional internalizing cluster before merging with disinhibition (i.e., a disinhibited negative affect cluster did not emerge). The extended bass-ackward approach was also able to capture cross-loadings that were not evident in the hierarchical agglomerative clustering solution, due to the mutually exclusive clusters (e.g., *social anxiety* with both *fear* and *distress*; *distress* with both *internalizing* and *disinhibited negative affect*).

Limitations

While the proposed approach has several advantages over other methods for elucidating hierarchies in individual differences data—such as capturing complexity in the associations between items and components, and capturing a more complete (and simplified) hierarchical model by looking across all levels—it also has several limitations. For example, by starting with a single factor that is included in every model by default, we are in danger of reifying *general factors* that can be found in many forms of individual differences data (e.g., intelligence, Spearman, 1904; personality, Muek, 2007; psychopathology, Lahey et al., 2012; and stress, Mann et al., 2021). Care should be taken to examine the reliability and interpretability of all elements that emerge in these hierarchical models. It is beyond the scope of this paper to prescribe specific circumstances under which factors or components are (un)interpretable, but there is a substantial and growing literature on this topic (e.g., Brunner et al., 2012; Rodriguez et al., 2016a, 2016b). External validity of the constructs that emerge can also be tested (e.g., predictive validity of external correlates) to explicate their nomological net (Cronbach & Meehl, 1955).

In addition to reification, overextraction is also a risk of this approach. In the applied example above, I chose a maximum number of components to extract based on parallel analysis and Velicer's minimum average partial, but stopped including new levels in the hierarchy when components were not uniquely identified by a set of items (i.e., had less than three items with primary loadings $> .4$ on the component and without substantial cross-loadings). Goldberg (2006) recommended stopping when there is a component or factor on which no variable has its highest loading.

As discussed above, swapping the automation of the traditional bass-ackward approach for subjective interpretation of whether an emergent component or factor is likely to be an artefact brings with it a risk of confirmation bias—eliminating constructs that do not fit with our expectations, or are inconvenient for our narrative. This can be mitigated by setting *a priori* scenarios in which we might deem a component or factor an artefact. For example, when amalgamations emerge in the bass-ackward framework, but not a cluster analytic framework; when multiple components on a given level of the hierarchy are uninterpretable and/or reflect patterns of co-occurrence among observed variables that are not evident in any other levels; or when a component or factor falls just short of being redundant across multiple levels in the model (i.e., the constructs are conceptually, if not statistically, identical, as was the case for the *alcohol use* [C3] component in the applied example). In each of these cases, removing the likely artefacts can simplify and clarify the resulting hierarchy. However, as discussed above, these decisions should be made with caution and transparency, and compared to the interpretation of a traditional bass-ackward solution.

Conclusion

I have proposed an extended bass-ackward approach based on three simple modifications to Goldberg's (2006) traditional bass-ackward approach for delineating hierarchical structural models: 1) identify and remove redundant components that perpetuate through multiple levels of the hierarchy; 2) (optionally) identify and remove artefactual components; and 3) plot the strongest component correlations among the remaining components to identify their hierarchical structure. I believe that this approach can offer us new and valuable perspectives when exploring the hierarchical structures that emerge among constructs of varying breadth in many fields of individual differences research.

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