A Basic Introduction to Messy Structures and Generative Models

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Abstract

While networks have been the standard mathematical representation for detailed arrangements of relations in society, the simplicity of the representation limits what information can be preserved in the abstraction, and subsequently, what can be mathematically analyzed. In this paper, we introduce a more rigorous and flexible abstraction for representing arrangements of relations in society, the Messy Structure. After introducing the representational form and theoretically justifying it, we introduce some basic generative procedures for creating Messy Structures with “hierarchical” properties.

Author Contributions

Karl Vachuska conceived the research; Karl Vachuska designed the methodology; Karl Vachuska wrote the computational programs; Karl Vachuska performed the research; Karl Vachuska analyzed the data; Karl Vachuska and Stephen Rodriguez-Elliott worked on the theory. Karl Vachuska and Stephen Rodriguez-Elliott wrote the paper.

Introduction

Social Networks have generally been the central mathematical abstraction representing societal patterns of relation. Specific empirical social networks are mathematically analyzed to understand the properties of the specific network. Additionally, generative models for types of random graphs (networks) are studied to examine the generative mechanisms behind real-world networks' general properties. In this paper, we introduce a more flexible mathematical abstraction for representing relations between objects, which we call Messy Structures.

Messy Structures are capable of mathematically representing more detailed and complicated sets of relations compared to networks. We draw upon the sociological theory of Institutions and Capital in justifying our representation. We further outline an approach for generating random "Hierarchical" Messy Structures and examine the properties of such Messy Structures.

Introduction to Messy Structures

A Messy Structure is at its core, similar to what is mathematically called a graph. A graph can broadly be defined as a set of objects such that specific pairs of objects are related to one another. A Messy Structure can be more broadly defined as a set of objects such that objects relate to one another via belonging to common subsets, and that the nature of objects belonging to subsets can be qualified in one or more discrete forms.

In this paper, we focus on institutions as networks consisting of interrelating roles occupied by individuals/objects to represent the structural dynamics of institutions. We assume a teleological account of institutions, meaning that there is a collective end(s) that determines individuals' collective actions. Furthermore, we assume that the collective end(s) transcend the institutions in that individuals, through their relationship status to institutions, pursue the collective end(s) through their means. Not to suggest that institutions are a means to an end for individuals; instead, due to the messy structure of society, there are many overlapping institutions of various scales coexisting with

individuals being members of those same institutions. So, this messy structure can be represented meaningfully.

To redefine Messy Structures in the terms we will use for the rest of the paper, a Messy Structure is a set of discrete objects, such that the objects (*individuals*) are related to one another not directly but indirectly through common groupings (*institutions*) and that individuals are part of indirect groupings not through a singular relation but one or more discrete types of relations (*roles*). While all networks/graphs can be represented as Messy Structures, the inverse is not true.

Consider this friendship social network in adjacency matrix form:

Table One

	Karl	Stephen	Shanaya	Haley
Karl		1	1	0
Stephen	1		0	1
Shanaya	1	0		0
Haley	0	1	0	

This adjacency matrix is notably symmetric. This diagram indicates Karl and Stephen are both friends with one another. Karl is also friends with Shanaya and Stephen is also friends with Haley.

We can represent this same social network as a Messy Structure in Messy adjacency matrix form:

Table Two

Institution 1	Friend
Karl	1
Stephen	1
Shanaya	0
Haley	0

Table Three

Institution 2	Friend
Karl	1
Stephen	0
Shanaya	1
Haley	0

Table Four

Institution 3	Friend
Karl	0
Stephen	1
Shanaya	0
Haley	1

The social network had three different groupings of individuals, which our Messy representation writes as three different matrices. The entire collection of matrices thus depicts the Messy Structure. Consistent across the matrices, the rows display a unique individual, and the columns depict a unique role. The binary presence of 0's and 1's thus indicate whether or not a unique individual occupies a unique role in the specific institution (grouping). For this specific network data, writing things in Messy Structure form is relatively useless, but for more complex data, it can be quite useful in representing complex data.

Consider how we represent the main characters of the show *Friends* in Messy Structure matrices:

Table Five

Institution 1	Friend	Roommate	Partner	Sibling
Rachel	1	0	0	0
Monica	1	0	0	0
Phoebe	1	0	0	0
Joey	1	0	0	0
Chandler	1	0	0	0
Ross	1	0	0	0

Table Six

Institution 2	Friend	Roommate	Partner	Sibling
Rachel	0	0	0	0
Monica	0	0	0	0
Phoebe	0	0	0	0
Joey	0	1	0	0
Chandler	0	1	0	0
Ross	0	0	0	0

Table Seven

Institution 3	Friend	Roommate	Partner	Sibling
Rachel	0	1	0	0
Monica	0	1	0	0
Phoebe	0	0	0	0
Joey	0	0	0	0
Chandler	0	0	0	0
Ross	0	0	0	0

Table Eight

Institution 4	Friend	Roommate	Partner	Sibling
Rachel	0	0	1	0
Monica	0	0	0	0
Phoebe	0	0	0	0
Joey	0	0	0	0
Chandler	0	0	0	0
Ross	0	0	1	0

Table Nine

Institution 6	Friend	Roommate	Partner	Sibling
Rachel	0	0	0	0
Monica	0	0	0	1
Phoebe	0	0	0	0
Joey	0	1	0	0
Chandler	0	1	0	0
Ross	0	0	0	1

If we want to refine the level of analysis we can refine the categorization of the roles:

Table Ten

Institution 1	Friend	Roommate	Boyfriend	Girlfriend	Brother	Sister
Rachel	1	0	0	0	0	0
Monica	1	0	0	0	0	0
Phoebe	1	0	0	0	0	0
Joey	1	0	0	0	0	0
Chandler	1	0	0	0	0	0
Ross	1	0	0	0	0	0

Table Eleven

Institution 2	Friend	Roommate	Boyfriend	Girlfriend	Brother	Sister
Rachel	0	0	0	0	0	0
Monica	0	0	0	0	0	0
Phoebe	0	0	0	0	0	0
Joey	0	1	0	0	0	0
Chandler	0	1	0	0	0	0
Ross	0	0	0	0	0	0

Table Twelve

Institution 3	Friend	Roommate	Boyfriend	Girlfriend	Brother	Sister
Rachel	0	1	0	0	0	0
Monica	0	1	0	0	0	0
Phoebe	0	0	0	0	0	0
Joey	0	0	0	0	0	0
Chandler	0	0	0	0	0	0
Ross	0	0	0	0	0	0

Table Thirteen

Institution 4	Friend	Roommate	Boyfriend	Girlfriend	Brother	Sister
Rachel	0	0	0	1	0	0
Monica	0	0	0	0	0	0
Phoebe	0	0	0	0	0	0
Joey	0	0	0	0	0	0
Chandler	0	0	0	0	0	0
Ross	0	0	1	0	0	0

Table Fourteen

Institution 5	Friend	Roommate	Boyfriend	Girlfriend	Brother	Sister
Rachel	0	0	0	0	0	0
Monica	0	0	0	1	0	0
Phoebe	0	0	0	0	0	0
Joey	0	0	0	0	0	0
Chandler	0	0	1	0	0	0
Ross	0	0	0	0	0	0

Table Fifteen

Institution 6	Friend	Roommate	Boyfriend	Girlfriend	Brother	Sister
Rachel	0	0	0	0	0	0
Monica	0	0	0	0	0	1
Phoebe	0	0	0	0	0	0
Joey	0	1	0	0	0	0
Chandler	0	1	0	0	0	0
Ross	0	0	0	0	1	0

Note that our representation of *Friends* includes a level of detail that a network could not preserve. We have six individuals in our Messy Structure, the main characters of *Friends*: Rachel, Monica, Phoebe, Joey, Chandler, Ross. We can depict the Messy Structure with either 4 or 6 different roles and have six institutions in total: including friendship groups, roommate arrangements, romantic unions, and families.

Messy Structures ultimately can be used for analyzing relations with a much higher level of detail. Standard mathematical analysis of networks assumes all types of relations within a network are the same, when they likely may not be. Messy Structures allow for the discrete categorization of relations to be involved in a network. As our example with *Friends* data highlights, noting the types of relations in data can be distinguished as specific effects can only manifest through certain relations. Ross can have a different effect on Rachel because they are dating then he may have on Monica, his sister. Referring to the point of view of Capital, Messy Structures can represent effects specific to Capital, such as differing effects in different institutions, like on an episode of *Friends*, when Monica lost her job and financial Capital, and how that had different implications across the different institutions she is part of. However, effects that have little to do with Capital, such as implications for sickness and disease spread, can still be analyzed through Messy

Structures. Thus while the Messy Structure is set up to represent relations that pertain to Capital, the effects it can analyze may have nothing to do with Capital.

Messy Structures additionally can be used for analyzing relations that connect *more than two* objects simultaneously. Capital itself does not transfer linearly, but may only be transferred or recognized in specific arrangements, for instance, consider an assembly line and how a product is produced, while everyone is contributing Capital, only when everyone is performing their role is the final product and created Capital fully realized. A customer is a critical part of the institution as well because, without a customer for the product, no one receives the benefit of the engineered Capital. Again though, simply because Capital creates the motivation for the arrangement of Messy Structures, effects having little to do with Capital can still be analyzed with them. In pertinence to the fact that institutions can contain more than two objects, it is essential to recognize that specific social contagions are *viral*, and others are more *resistant*. Diseases, for example, tend to be viral. If a school has a single infected student, it is possible that soon most of the people affiliated with the school could be infected. Oppositely, certain phenomena, such as collective identity, are unlikely to be affected by a single individual, but instead, they require a certain critical mass or threshold of individuals within the institution in order to affect the phenomena at all.

To summarize, a Messy Structure has the following terminology:

An ***individual*** describes a singular object

An ***institution*** describes a subset of objects in a single grouping

A ***role*** describes one way in which an individual relates to an institution

A ***Messy structure*** is a collection of institutions, similar to a graph

Messy Structures possess the following representational attributes in distinction to a graph/network:

1. Relations do not exist between pairs but between any number of objects via indirect linkage through common membership in a grouping.
2. There is not a singular type of relation but a number of discrete types of relations.

Motivation for Messy Structure

The relationships between institutions, between individuals, and the interrelations between institutions, individuals, and the society they compose can be networked and represented through Capital and its many manipulations. The manipulation of Capital is meant to be broadly inclusive of how Capital is exchanged, used, transformed, and created. These relationships can be mapped graphically. Capital can explain institutions' arrangements and the interrelationships they have to individuals, so mapping it graphically can adequately represent the relationships through which Capital is manipulated. Capital is manifested through the presence of a relationship between individuals and institutions.

Our theoretical approach considers society as a complex of institutions. Institutions then consist of interrelating roles occupied by individuals. With this in mind, we add to our analysis and representations the dynamics of Capital to generate representations of Messy Structures. Our consideration of Capital stems, theoretically, from Pierre Bourdieu's definition of Capital as accumulated labor which "when appropriated on a private, i.e., exclusive, basis by agents or groups of agents, enables them to appropriate social energy in the form of reified or living labor" (Bourdieu, 1986). More so, Capital is thought of as consistent and constant force present in and perpetuating objective or subjective structures. In short, Capital is ever-present and ever-consequential in social life. So, Capital is inherent to institutions and how they operate allowing a clear and consistent means to network relationships on all scales.

Marxist theory provides an original conception of the possible dynamics of roles and Capital within an institution. The proletariat produces Capital, which is accumulated and used by the Bourgeoisie (Marx, 1848). Individuals occupy the broad roles of the Proletariat and Bourgeoisie. These roles interrelate within the economic institution of Capital which

operates within society. Capital is manipulated between both roles and individuals, which then has relevance for the individuals outside of their roles and into other aspects of their lives as individuals. The consequence of Capital is then outside the roles of the one economic institution and impacts other institutions to which the individuals are members of. Standard definitions of Capital have evolved substantially since Marx wrote on it, and we conceptualize Capital in a more modern sense, as "a stock or reserve of anything of social or economic significance". (Hodgson 2014)

Capital is manipulated, reproduced, and modified into various forms and is transferred through relationships between individuals within common institutions and between institutions within individuals. For example, Capital can be invested in schools in various forms, economic, human, and cultural. Money, financial Capital, is given to schools while teachers need to be embodied with human and cultural Capital providing the appropriate and sufficient amount of skills, human Capital, necessary to teach within that school. Depending on this initial investment, Capital is manipulated in how it is transferred to students. Different types of teachers may be systemically obligated to deliver different types of capital in different ways to students, effectively distinguishing their roles from one another. We conceptually define a role as a pattern of behavior that is unique in what type(s) of Capital it is directly involved with and how the pattern of behavior manipulates, reproduces, modifies, transfers, or receives such Capital.

Individuals' roles do not operate independently, however, but instead in relation to specific institutions. We conceptually define institutions as being a specific and unique practice of a social institution. Turner (1997) defines social institutions as "a complex of positions, roles, norms and values lodged in particular types of social structures and organizing relatively stable patterns of human activity concerning fundamental problems in producing life-sustaining resources, in reproducing individuals, and in sustaining viable societal structures within a given environment."

Ultimately by examining specific practices of social institutions, the relationships through which Capital is affected or affects individuals can be mapped. This mapping can then represent a network-like structure consisting of relationships between individuals occupying specifically defined roles within institutions. However, the benefit is that the representation exists outside of any one institution, and instead, creates a network of

interrelating institutions through individuals and the various roles they occupy, preserving information such as the nature of through what roles individuals relate to one another, and within what common grouping individuals relate to one another.

Hierarchical Messy Structure

Albert and Barbási (2002) articulated the Scale-Free network and outlined the sensible generative model in the Barabási-Albert model. The generative model, which produces scale-free networks (networks that's degree distribution is scale-free), rests in the concept of Preferential Attachment. Within the context of network growth, Preferential Attachment refers to the notion that the probability of connecting to a node depends on the node's degree. Albert and Barabasi discuss an example, "a web page will more likely include hyperlinks to popular documents with already high degrees because such highly connected documents are easy to find and thus well known, or a new manuscript is more likely to cite well-known and thus much-cited publications than less-cited and consequently less-known papers."

Albert and Barbási's thoughts on network growth and structure tendencies serve as the motivation for this simple model on how Messy Structures may grow and develop. Preferential Attachment is highly relevant in our conceptualization of Messy Structures, in similar and different ways. An additional relevant set of mechanisms pertains to matching theory and related constraints.

Generative Mechanisms

Preferential Attachment

Rational individuals tend to join/attach themselves to institutions where there is more *Capital* present, or they are better connected to more *Capital*. In the job market, this generally manifests in individuals desiring to be employed at businesses that provide more financial Capital in compensation or possibly demonstrate a higher likelihood of prosperity (connected to a successful investment firm or a promising industry or promising consumer demographic). Potential employees also find appeal in working with pleasant individuals

that possess higher degrees of Human Capital and lead lives with otherwise low degrees of stress and are well-supported, and in working in locations that have more middle-class/affluent families residing and have better-funded schools or even simply just offer better scenery (aesthetic Capital).

Preferential Initiation

Institutions tend to be created and controlled/operated by individuals with high degrees of *Capital*. In terms of businesses, financial Capital tends to be the primary factor. However, social Capital and human Capital broadly are highly relevant. Outside of business, in attempting to make friends or start a social organization, social Capital and cultural Capital increase the likelihood of success in initial organization and attraction.

Preferential Selection

Individuals with higher degrees of Capital generally have the first choice in joining an institution. This can partially be attributed to the institution's side within a two-sided matching model. In the case of businesses hiring employees, for example, the business will generally first offer a job to the applicant they believe most fit for the job, which often comes back to Human or Social Capital. This tendency can also be attributed to individuals with higher Capital temporally having access to institutions earlier. In terms of social trends, individuals with higher social Capital may be able to access information earlier and jump on a trend earlier. In another way, individuals lacking Capital may not have the financial Capital or time to immediately pursue (or ever pursue) certain opportunities.

Institutional Limitations

Institutions may have a limited capacity for how many individuals may actively participate in them. These limits may be the product of either artificial or natural constraints. A company will only employ a certain number of individuals because they may only have a certain amount of financial Capital to pay employees, a natural constraint based on the amount of Capital available. Additionally, profit-oriented businesses may intentionally limit the number of employees they employ in order to maximize profit. In

another scenario, a person may be able to seemingly join an institution, but if the institution is beyond a specific capacity for the amount of capital they can manipulate, their membership may be meaningless if they cannot access any capital within the institution. Popular professors may offer to have students join their research lab or team but may end up having trouble directly including the student in the research work available, especially if they are already working with many other students, and thus do not have the time to share their intellectual Capital with students at the bottom of the symbolic queue.

Hierarchical Generative Procedure

We want to preface this generative procedure by articulating that this generative procedure is distinct from the otherwise highly-complex role-based analysis one can perform on Messy Structures. For the sake of simplicity and focusing on a small number of simple generative parameters, we only include two broad types of roles in this generative procedure, "operators" and "members". Institution operators are meant to generally represent leaders or individuals that have a disproportionate degree of influence in a particular institution. "Members" are meant to represent the general/subordinate role within an institution, that is subject to the influence of the "operator". If the institution examined was a class at a school, the "operator" would most fittingly be the teacher, and the members would most fittingly be the students. In order to keep things simple additionally, each institution has only one operator.

Setup: Take n *individuals*, an institutional size of m , x institutions where $x < n$, and a number of steps s .

Randomly select x unique individuals within the set of n . These x individuals will "operate" x unique institutions.

In random order, assign each of the x individuals to another individual's (not their own) institution. If an institution already has m individuals assigned to it, do not attach that individual to the institution and move on to the next individual in the sequence.

Once, this has been done for the x individuals that “operate” institutions, follow the same procedure for the remaining $n - x$ individuals.

Once this has been done for Step 1, repeat the following procedure for all remaining steps:

Choose x individuals with probability based on the total number of institutions each individual is a member of (not including the number of institutions they “operate”, but the number they are members of, assigned to)

Next, sort all individuals in descending order based on the total number of institutions each individual is a member of (not including the number of institutions they control, but the number they are members of, assigned to). (The individual that is a member of the most institutions is first)

Going through the ranking of individuals in order, allow each individual to choose one of the x institutions with a probability of choice based on the total number of institutions the individual that “operates” the institution is a member of. Always, if an institution already has m individuals assigned to it, do not attach/assign that individual to that institution, and move on to the next individual in the ranking.

Resulting Messy Structure:

The resulting Messy Structure has s types of institutions and sx total institutions.

More importantly, the resulting Messy Structure has unique properties and is virtually hierarchical, with the number of institutions individuals are members of almost appearing to follow a power law.

Based on 50,000 simulations with the following parameters: $n = 100$, $m = 3$, $x = 10$, $s = 100$
The following depicts the frequency of the number of institutions individuals are members of:

Figure One

An elite group of nodes exists, such that they are members of the maximal number of institutions as well as are operators of a high number of institutions. The top 10 nodes operate an average of 34.8 institutions, while the bottom 10 nodes operate no institutions at all.

In order to highlight the collective importance of the generative mechanisms, we also examine three additional generative procedures that each lack one of three generative mechanisms:

Random Operating

All steps in this generative model are the same as the original, except that in this model, the individuals that are chosen to “operate” institutions in each step are always completely random.

Random Selection

All steps in this generative model are the same as the original, except that in this model, the institutions from the selection that individuals choose to join are always completely random, not based at all on the number of institutional memberships of the institution operator.

Random Order

All steps in this generative model are the same as the original, except that in this model, the order by which individuals get to choose their institution is always completely random (randomized every step), not based at all on the number of institutional memberships each individual has.

Measures

Institutional Memberships pertain to the total number of institutions an individual is a member of, not including those they operate.

Institutions Operated pertain to the total number of institutions an individual operates.

Peer Quality pertains to the average total number of institutions an individual’s peer (not the individual operating the institution) are members of.

Positive Influence refers to a stochastic measure for how positive influence may spread between individuals within a Messy Structure. Individuals who operate an institution in Step 1 are assigned a positive influence value of 1 if they are a member of another

institution in the step, otherwise, they are assigned a positive influence value of 0. All other individuals are then assigned the positive influence value of the operator of the institution they belong to. If they do not belong to any institution they are assigned a value of 0. Step by step, positive influence values are then calculated for each individual by adding the positive influence value from the prior step to the product of the positive influence value of the controller of the institution the individual is part of in that step and a constant β . If an individual is not part of any institution in a given step (excluding operation), their positive influence value remains the same as in the prior step.

Positive Influence is meant to represent the flow of behavior through the Messy Structure, such that the underlying behavior is characterized by being increased by participation in institutions and imprinted on from others. An example of a social contagion that may follow such a pattern might be social skills, since absence of interaction/participation in an institution does little to improve social skills, while interaction/participation in more institutions, especially institutions filled with individuals with strong social skills themselves, substantially helps to improve social skills.

Negative Influence refers to a stochastic measure for how negative influence may spread between individuals within a Messy Structure. Individuals who operate an institution in Step 1 are assigned a negative influence value of 0 if they are members of another institution in that step, otherwise if not are assigned a negative influence value of 1. All other individuals are then assigned the negative influence value of the operator of the institution they belong to. Step by step, negative influence values are then calculated for each individual by adding the negative influence value from the prior step to the product of the negative influence value of the controller of the institution the individual is part of in that step and a constant β . If an individual is not part of any institution in a given step (excluding operation), their negative influence value has 1 added to it.

Negative Influence is meant to represent the flow of behavior through the Messy Structure, such that the underlying behavior is characterized by being increased by lack of participation in institutions as well as imprinted on from others. An example of a social contagion that may follow such a pattern might be stress, since not being included in institutions that provide capital and satisfy human needs, can amplify stress, while

participating in institutions with individuals that have a high degree of stress can also amplify stress.

Positive Influence with Peer Effects refers to a stochastic measure for how positive influence may spread between individual operators and individual members as well as in between individual members, within a Messy Structure. Individuals who operate an institution in Step 1 are assigned a positive influence value of 1 if they are a member of another institution in the step, otherwise, they are assigned a positive influence value of 0. All other individuals are then assigned the positive influence value of the operator of the institution they belong to. If they do not belong to any institution they are assigned a value of 0. Step by step, positive influence values are then calculated for each individual by adding the positive influence value from the prior step to the product of the positive influence value of the operator of the institution the individual is part of in that step and a constant β and a second constant $1 - \beta$ as well as the product of the average positive influence value of the other members in the institution and a constant β and a second constant β . If an individual is not part of any institution in a given step (excluding operation), their positive influence value remains the same as in the prior step.

Negative Influence with Peer Effects refers to a stochastic measure for how negative influence may spread between individual operators and individual members as well as in between individual members, within a Messy Structure. Individuals who operate an institution in Step 1 are assigned a negative influence value of 0 if they are members of another institution in that step, otherwise if not are assigned a negative influence value of 1. All other individuals are then assigned the negative influence value of the operator of the institution they belong to. Step by step, negative influence values are then calculated for each individual by adding the negative influence value from the prior step to the product of the negative influence value of the operator of the institution the individual is part of in that step and a constant β and a second constant $1 - \beta$ as well as the product of the average negative influence value of the other members in the institution and a constant β and a second constant β . If an individual is not part of any institution in a given step (excluding operation), their negative influence value has 1 added to it.

Analysis

The following tables depict a variety of measures on the four generative types. For each generative approach, 1000 simulations were done, with:

$$n = 100, m = 3, x = 10, s = 100, \beta = .01, \beta_2 = .5$$

Table Sixteen

Generative Approach	Member Inequality	Operation Inequality	Positive Influence Inequality	Negative Influence Inequality	Positive Influence (w Peer) Inequality	Negative Influence (w Peer) Inequality	Peer Quality Inequality
Hierarchical	0.5973936	0.6371557	0.6871792	0.1858821	0.6696185	0.2054242	0.2334171
Random Operating	0.5769384	0.3049122	0.7296258	0.07477705	0.6985297	0.101605	0.2652802
Random Selection	0.626873	0.6366147	0.6633068	0.203538	0.6578222	0.2322391	0.1797563
Random Order	0.08958005	0.174859	0.2967021	0.01964419	0.3018301	0.02230698	0.07893164

Table Seventeen

Generative Approach	Total Positive Influence	Total Negative Influence	Total Positive Influence w Peer	Total Negative Influence w Peer	Total Peer Quality
Hierarchical	59.99242	7722.205	54.30496	7531.352	5100.257
Random Operating	39.06335	8970.242	40.75048	8640.966	3672.629
Random Selection	60.44599	7549.351	55.99604	7322.006	6313.132
Random Order	2.952807	8313.711	2.795663	8100.735	1947.422

Table Eighteen

Generative Approach	Positive Influence Bottom 10 individuals	Negative Influence Bottom 10 individuals	Positive Influence (w Peer) Bottom 10	Negative Influence (w Peer) Bottom 10
Hierarchical	0.0008628926	99.52934	0.002400117	99.4242
Random Operating	5.64E-05	99.53674	0.002029307	99.35935
Random Selection	0	100	0	100
Random Order	0.00137119	87.97895	0.001512511	86.37366

Table Nineteen

Generative Approach	Positive Influence Top 10 individuals	Negative Influence Top 10 individuals	Positive Influence (w Peer) Top 10	Negative Influence (w Peer) Top 10
Hierarchical	2.229938	15.45837	1.877148	8.426011
Random Operating	1.638164	48.15282	1.563034	33.21968
Random Selection	2.026332	19.39267	1.819769	9.819537
Random Order	0.2263659	77.98711	0.223311	75.312

The following tables depict a variety of measures on the four generative types. For each generative approach, 1000 simulations were done, with:

$n = 100$, $m = 3$, $x = 10$, $s = 100$, $\beta = .5$, $\beta_2 = .5$

Table Twenty

Generative Approach	Member Inequality	Operation Inequality	Positive Influence Inequality	Negative Influence Inequality	Positive Influence (w Peer) Inequality	Negative Influence (w Peer) Inequality	Peer Quality Inequality
Hierarchical	0.5971291	0.6363916	0.724346	0.724346	0.6698895	0.6698895	0.2332455
Random Offering	0.5774326	0.3056891	0.8901514	0.887702	0.6786233	0.6786233	0.2664627
Random Selection	0.6267836	0.636178	0.6605944	0.6605944	0.6324426	0.6324426	0.1789381
Random Order	0.06729001	0.1304453	0.07011195	0.1933164	0.06363566	0.1755775	0.05930792

Table Twenty-One

Generative Approach	Total Positive Influence	Total Negative Influence	Total Positive Influence w Peer	Total Negative Influence w Peer	Total Peer Quality
Hierarchical	2.11E+15	1.12E+15	7.15E+12	3.33E+12	5100.781
Random Operating	32311249	197627965	1429722371	3660623194	3669.369
Random Selection	4.46E+14	3.41E+14	4.84E+12	3.23E+12	6314.202
Random Order	1553460	350888370	144159.2	48167602	1462.106

Table Twenty-Two

Generative Approach	Positive Influence Bottom 10 individuals	Negative Influence Bottom 10 individuals	Positive Influence (w Peer) Bottom 10	Negative Influence (w Peer) Bottom 10
Hierarchical	21669412	4.96E+13	1017090	127295434712
Random Operating	7.00E+00	16243946	18683.38	165483264
Random Selection	0	1.23E+13	0	107199547465
Random Order	5014.727	6593579	534.5227	860322.3

Table Twenty-Three

Generative Approach	Positive Influence Top 10 individuals	Negative Influence Top 10 individuals	Positive Influence (w Peer) Top 10	Negative Influence (w Peer) Top 10
Hierarchical	9.37E+13	12002208	2.74E+11	473623.7
Random Operating	2664306	332.382	64656620	48512.87
Random Selection	1.61E+13	99.99021	160433932955	99.99014
Random Order	29157.94	1119307	2579.082	177385

Ultimately, we generally see the basic Hierarchical and Random Selection with somewhat similar measure values, posting high levels of inequality but higher levels of Positive Influence (higher is better) and lower levels of Negative Influence (lower is better). Both the best-off and worst-off individuals in the Hierarchical process score better on Positive and Negative Influence compared to their Random Selection counterparts. Generally, these results are as we would expect. Since the probability of individuals operating an institution is already dependent on the number of memberships they have, having individuals choose an institution to join once again on that probability basis is

somewhat redundant. The central difference it makes is increasing the number of individuals that can choose before all institutions are filled up, thus increasing the chance that a lower-ranked individual gets a higher chance of successfully joining an institution in the basic Hierarchical model. Ultimately, while the differences are very minute, Random Selection has slightly better Negative Influence scores, and slightly better Positive Influence scores when β is low, the Hierarchical has better Positive Influence scores when β is higher, however.

The Random Operating model scores next best generally. Individuals with high numbers of Institutional Memberships possess high degrees of Positive Influence and low degrees of Negative Influence. When these individuals are not disproportionately represented in operating Institutions, even when individuals make choices based on Operators' number of memberships, the whole Messy Structure cannot benefit as significantly from these individuals. Additionally, these Messy Structures tend to have higher degrees of inequality, likely attributable to the fact that individuals choose their institution in each step based on a more unequal set of probabilities.

The Random Order model scores worst generally, but best in terms of inequality. When individuals all have equal opportunity in joining institutions, only the inequality we would expect to arise randomly in terms of the number of institutional memberships arises. This means there is little variation in the level of Positive or Negative Influence individuals possess as well, so the Messy Structure as a whole can not benefit from the most elite nodes abundance of Positive Influence nor can be insulated from the worse-off nodes abundance of Negative Influence.

Albert, Jeong, and Barbási (2000) highlight one of the critical properties of scale-free networks as the networks' resistance to failure. They cite common examples in real-world scale-free-like networks that are subject to high failure rates but easily withstand the failures with virtually no impediment to the functioning of the network. They attribute the resilience of the network's functionality to the fact that if a node were to fail at random, it is likely to have a low degree and, thus, have little implication in connecting other nodes. Even if a node that fails is a "hub" (a node with high-degree), it is unlikely to disconnect the network, due to the presence of multiple hubs.

A similar idea can be discussed as a core attribute of hierarchical Messy Structures. Here, the probability of failure for a given individual can be correlated with the level of Positive and Negative Influence an individual possesses. This understanding stems directly from Capital's definition since positive Influence increases Capital, and Negative Influence decreases Capital. In either case, more Capital means one has more considerable resources with which to solve one's functionality-threatening problems and thus less chance of being unable to perform one's duties within an institution. Since individuals with the lowest likelihood of "failure" have the highest degree of Influence and individuals with the highest likelihood of failure, have the lowest degree of Influence, for a certain number of failures, this type of Messy Structure is likely to be minimally affected. Further, if "failures" are seen as being a social contagion, where the number of institutional memberships an individual has is inversely related to an individual's likelihood of being subject to "failure" when being in contact with "failure", hierarchical Messy Structures best contain "failures". To some extent, Negative Influence captures this phenomenon, but further research utilizing Poisson processes would better analyze it.

An example sheds some light on this idea. In the early 1970s, a hotel was under design and construction in Singapore. (Lim 2011) While a structural engineer usually leads the design of a building, an unqualified draftsman was in charge. The unqualified draftsman made substantial errors in designing the building, including calculating the building's weight incorrectly by an enormous margin. Additionally, to save money, the owner of the building directed the construction of the building himself, utilizing low-quality building materials. With few government regulations in the construction of buildings in Singapore, there were no mechanisms to catch these mistakes. Unfortunately, these mistakes had consequences, and on March 15th, 1986, the six-story Hotel New World in Singapore collapsed completely. Following the collapse, first responders realized they were utterly unprepared to respond to such an incident, both in training and equipment. Tunneling experts from Europe who happened to be in the county were eventually utilized, and specialized equipment for dealing with equipment was brought in by the Civil Defense Force and private organizations. Ultimately, as a result of a substantial building collapse unaided by the prolonged, haphazard response, 33 people perished, and a total of 35 children lost a parent in the collapse.

Ultimately, the Messy Structure surrounding the incident proved problematic in containing "failure," but instead allowed it to spread in the absence of institutional checks. When the building owner decided to direct construction himself and have a highly unqualified individual design the building, he fails as an individual. Perhaps if he had better institutional access to financial Capital, to hire a better lead designer and buy better materials, or institutions had afforded him human Capital more relevant to the task of constructing a building, he would not have made such poor decisions. Similarly, if institutions had afforded the building's designer better human Capital, perhaps he would not have made such a grave error. Perhaps if he had better resources with which to perform the calculations, or had access to an institution that would have made the calculations for him, he would have avoided the mistake. Similarly, if the government had an agency to regulate the construction of buildings better, the manifestation of these mistakes could have been avoided. After the building had collapsed, if there was an institutional rescue team prepared to respond to such a disaster, some of the deaths could have been avoided. Ultimately, 33 died, and 35 children lost a parent. Presumably, many many lives were indirectly affected, all by an individual with a substantial level of institutional Influence failing in the absence of immediate and subsequent institutional checks.

Conclusion

In this paper, we have outlined an approach for creating mathematically-useful abstract representations of society, which we term Messy Structures. We have grounded our theoretical conceptualization of Messy Structures in theories related to Capital and Institutions. We further introduced a basic generative model for creating random "Hierarchical" Messy Structures, which we believe accurately encapsulate and represent some of society's qualities. Our analysis of these Hierarchical Messy Structures identifies a vital property in maximizing the spread of Positive Influence, minimizing the spread of Negative Influence, and hypothetically durable insulation from "failure" and general negative perturbation.

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