Calculation of the Hubble Constant, the Minimum Mass, and the Proton Charge Radius Using the Dirac's Hypothesis on the Ratio of the Electrostatic Force to the Gravitational Force (Preprint)

Paul Talbot

St-Hubert, Qc, Canada

ORCID: [0000-0002-9919-9034](https://orcid.org/0000-0002-9919-9034)

DOI : <https://osf.io/j5gz4/>

Abstract

This publication suggests that some physical values could be calculated using the Dirac's hypothesis on the observed ratio of the electrostatic force to the gravitational force. The calculated value of the Hubble constant is $H \approx 72.013 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and that of the minimum mass, $M_{\min} \approx 1.7206 \times 10^{-68} \text{ kg}$. Recent observations suggest that the proton charge radius could also be calculated using an additional but related assumption: $r_p \approx 0.8264 \text{ fm}$.

Keywords

Cosmology, Hubble constant, minimum mass, graviton, proton charge radius, dimensionless constants.

Introduction

Currently, the local value of the Hubble constant is determined by observation. The observed values are more or less precise, and oscillate between $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $74 \text{ km s}^{-1} \text{ Mpc}^{-1}$, depending on the techniques used [1] [2] [3] [4] [5] [6].

We know that some physical quantities are limited by minimum values. For example, the quantum of action h is the minimum value of action. The elementary charge e can also be regarded as the minimum charge. Quarks are associated with a fractional charge, but they are confined inside hadrons, which do have an integer charge [7].

Hypothesis About Minimum Values

The working hypothesis is that other physical quantities would also be limited by minimum values (subscript $\min$). Using algebraic equations and the observed ratio of the electrostatic force to the gravitational force, some minimum values could be calculated precisely.

The Hubble constant H (of dimension T^{-1}) would correspond to the minimum frequency:

$$f_{\min} = H. \quad (1)$$

Since $E = hf$, the minimum energy would be

$$E_{\min} = hf_{\min} = hH. \quad (2)$$

Since $M = E / c^2$, the minimum mass would be

$$M_{\min} = E_{\min} / c^2 = hH / c^2. \quad (3)$$

To calculate H , another, but more precise value must be involved. Since May 2019, the value of the elementary charge is fixed by definition at $e = 1.602176634 \times 10^{-19} \text{ C}$.

The Ratio of the Electrostatic Force to the Gravitational Force

The ratio, say β , of the electrostatic force to the gravitational force between an electron and a proton is

$$\beta = e^2 / 4\pi \varepsilon_0 G m_e m_p \approx 2.268\,66 \times 10^{39} \quad (4)$$

ε_0 is the vacuum permittivity.

G is the gravitational constant .

m_e is the mass of the electron.

m_p is the mass of the proton.

In the last century, Paul Dirac popularized this β ratio in a cosmological context [8] [9] [10]. The value of the β ratio depends on the masses of the interacting particles. Here, I stick to the natural choice made by Dirac (the electron and the proton). In this context, these two particles are regarded as two sides of the same coin: the electron-proton couple.

The gravitational force is tiny compared to other forces, so this β ratio is sometimes called a dimensionless “Large Number”. It still gets attention nowadays [11] [12] [13] [14].

Calculation of H Using the Minimum Energy

Gravitational and electrostatic forces are conservative, which implies that the β ratio applies to both forces and energies. An energy E_{ep} can thus be associated with the electron-proton couple:

$$E_{ep} = \beta E_{\min} = \beta h H, \quad (5)$$

that could be calculated precisely using the assumed relation

$$E_{ep}^2 = m_e m_p c^4, \quad (6)$$

where c is the speed of light in vacuum. Using the 2018 CODATA recommended values, we get

$$E_{ep} = \sqrt{m_e m_p} c^2 \approx 3.508\,2 \times 10^{-12} \text{ J, or } 22.346 \text{ MeV.} \quad (7)$$

Since $E_{ep} = \beta h H$, we deduce that

$$H = E_{ep} / \beta h \approx 2.333\,8 \times 10^{-18} \text{ s}^{-1}, \text{ or } 72.013 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (8)$$

halfway between low and high observed values.

This means that a frequency f_{ep} can be associated with the electron-proton couple:

$$f_{ep} = \beta H = E_{ep} / h \approx 5.294\,5 \times 10^{21} \text{ s}^{-1}. \quad (9)$$

Expanding equation (8), we note that physical constants would be interlinked:

$$H = 4\pi \varepsilon_0 G (m_e m_p)^{3/2} c^2 / h e^2. \quad (10)$$

The value of the minimum energy would be

$$E_{\min} = h H \approx 1.546\,4 \times 10^{-51} \text{ J,} \quad (11)$$

and that of the minimum mass

$$M_{\min} = h H / c^2 \approx 1.720\,6 \times 10^{-68} \text{ kg.} \quad (12)$$

The very definition of the β ratio suggests that this tiny mass might be that of the graviton.

Previous Calculations of Minimum Values

Remarkably, the value of $M_{\min}$ coincides with the Wesson's mass ($m_E \approx 2 \times 10^{-68}$ kg), i.e., the quantum of mass calculated by Pr. Wesson [15] in 2003 using the constants c , h and Λ (the cosmological constant). According to Wesson, $m_E = (h/c) (\Lambda/3)^{1/2}$, compared to $M_{\min} = H h / c^2$ under the present hypothesis. We deduce that

$$\Lambda = 3H^2 / c^2 \approx 1.818 \times 10^{-52} \text{ m}^{-2}, \quad (13)$$

according to Wesson's Λ hypothesis. In the conformal cosmological model (Λ CDM), this value is multiplied by a factor $\Omega_\Lambda \approx 0.7$ [16] [17].

Using dimensional analysis, Pr. Dimitar Valev [18] [19] [20] [21] formulated an equation for a minute mass that could be that of the graviton: $m_H \approx \hbar H / c^2$. This tiny mass is of the same order of magnitude as the minimum mass $M_{\min}$.

Likewise, using dimensional analysis, Pr. Ali Muhammad Rushdi, and Dr. Mostafa Ali Rushdi [22] deduced a dimensionless product ($\pi_{a4} = hH/m_2 c^2$) where m_2 would equal to $M_{\min}$ when $\pi_{a4} = 1$.

In 2021, Mr. Ramón Garza Wilmot posted a research on ResearchGate: «*Exact Value of the Hubble Constant - The Most Precise Value of the Hubble Constant Deduced from Other Constants of Physics*» [23]. Although the reasoning differs and implies a fictitious particle called «*mason*», the calculations and the resulting values of H and $M_{\min}$ agree. Mr. Wilmot's work was posted on September 8, 2021, five days before mine on Amazon [24] [25]. The anteriority of the calculation of H and $M_{\min}$ thus belongs to Mr. Wilmot.

Among related research, some calculated results come close to the suggested value of H :

This work	(72.013 km s ⁻¹ Mpc ⁻¹),
Dr. Kritov's [26]	(71.995 km s ⁻¹ Mpc ⁻¹),
Mr. Mercier's [27]	(72.095 km s ⁻¹ Mpc ⁻¹),
Mr. Wolf's [28]	(71.994 km s ⁻¹ Mpc ⁻¹).

The Charge Radius of the Proton

In addition to the agreement of the calculated values with observation and previous works, there is something else peculiar about this value of E_{ep} . Indeed, we calculate that

$$e^2 / \varepsilon_0 E_{ep} \approx 0.8264 \text{ fm} \approx r_p, \quad (14)$$

where r_p is the proton charge radius. When measured by an electron-proton interaction, the calculated value of r_p matches (within 1σ) many recent observed values [29] [30] [31] [32]. It is unlikely that an arbitrary value of E_{ep} could lead to such consistent results, so this fact makes up an additional but related assumption.

Replacing E_{ep} by $\beta \hbar H$ (5), this new assumption can be written

$$r_p = (e^2 / \varepsilon_0 \hbar) / \beta H. \quad (15)$$

Using the fine-structure constant ($\alpha = e^2 / 2 \varepsilon_0 \hbar c$), equation (15) then becomes

$$r_p = 2 \alpha c / \beta H = 2 \alpha c / f_{ep}, \quad (16)$$

where αc is the speed of the electron in the ground state of the hydrogen atom. Inversely,

$$H = 2 \alpha c / \beta r_p. \quad (17)$$

This last equation might be a clue to the enigmatic values of the dimensionless constants α and β . It is worth highlighting that in agreement with (14) and (16), we do observe

$$r_p \approx e^2 / \varepsilon_0 E_{ep} \approx 2 \alpha c / \beta H \approx 0.83 \text{ fm}. \quad (18)$$

According to known physics, r_p should neither be related to E_{ep} nor to H . It is like winning the lottery twice in a row with the same combination.

Chart of the Assumed Relations Between Physical Values

Figure 1: Assumed relations between physical values

Physical values are represented by circles, and the relationships between them by arrows, each with a boxed formula. Each arrow indicates the direction of the relationship expressed by the formula linking two quantities. This direction is arbitrary and can be reversed by changing the formula. For example, the relation ($M_{\min} = Hh / c^2$) can be written ($H = M_{\min} c^2 / h$). The dotted lines depict the additional assumption (16) about the calculated value of r_p .

Conclusion

Using the observed ratio of the electrostatic force to the gravitational force, the assumed relation (6) allows precisely calculating the Hubble constant and the minimum mass. These values agree with observations and previous works. Recent observations suggest that the proton charge radius could also be calculated using an additional but related assumption (16). It is unlikely that an arbitrary value of E_{ep} could lead to such consistent results. More precise observed values of H and r_p could confirm or refute some of these hypotheses.

References

- [1] A. Riess, Y. Wenlong, M. Lucas, M. Macri, D. Scolnic, D. Brout, S. Casertano, D. O. Jones, Y. Murakami, L. Breuval, T. G. Brink, A. V. Filippenko, S. Hoffmann, S. W. Jha, W. D'arcy Kenworthy, J. Mackenty, B. E. Stahl et W. Zheng, «A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s⁻¹ Mpc⁻¹ Uncertainty from the Hubble Space Telescope and the SH0ES Team,» *arXiv:2112.04510v2 [astroph.CO]* 10 Jan 2022, p. 67, 2022.
- [2] W. Freedman, «Measurements of the Hubble Constant: Tensions in Perspective,» *The Astrophysical Journal*, vol. 919:16, p. 22, 2021.
- [3] N. Khetan, L. Izzo, M. Branchesi, R. Wojtak, M. Cantiello, C. Murugesan, A. Agnello, E. Cappellaro, M. Della Valle, C. Gall, J. Hjorth, S. Benetti, E. Brocato, J. Burke, D. Hiramatsu, D. A. Howell, L. Tomasella et S. Valenti, «A new measurement of the Hubble constant using Type Ia supernovae calibrated with surface brightness fluctuations,» *Astronomy & Astrophysics*, vol. 647, n° %1A72, p. 20, 2021.
- [4] K. Liao, A. Shafieloo, R. E. Keeley et E. V. Linder, «A Model-independent Determination of the Hubble Constant from Lensed Quasars and Supernovae Using Gaussian Process Regression,» *The Astrophysical Journal Letters*, vol. 886, n° %1L23, p. 7, 2020.
- [5] D. W. Pesce, J. A. Braatz, M. J. Reid, A. G. Riess, D. Scolnic, J. Condon, F. Gao, C. Henkel, C. M. V. Impellizzeri, C. Y. Kuo et K. Y. Lo, «The Megamaser Cosmology Project. XIII. Combined Hubble Constant Constraints,» *The Astrophysical Journal Letters*, vol. 891, n° %1L1, p. 9, 2020.
- [6] T. Yang, S. Birrer et B. Hu, «The first simultaneous measurement of Hubble constant and post-Newtonian parameter from Time-Delay Strong Lensing,» *astro-ph.CO*, vol. MNRASS Preprint, p. 6, 2020.
- [7] M. L. Perl, E. R. Lee et D. Loomba, «A Brief Review of the Search for Isolatable Fractional Charge Elementary Particles,» *Modern Physics Letters*, vol. 19:35, p. 16, 2004.
- [8] P. A. Dirac, «A new basis for cosmology,» *Proceedings of the Royal Society of London*, vol. 165, pp. 199-208, 1938.
- [9] P. A. Dirac, «Cosmological Models and the Large Numbers Hypothesis,» *Proceedings of the Royal Society of London*, vol. 338, pp. 439-446, 1974.
- [10] P. Dirac, «The Large Numbers hypothesis and the Einstein theory of gravitation,» *Proceedings of the Royal Society*, n° %1365, pp. 19-30, 1979.
- [11] Saibal Ray, Utpal Mukhopadhyay, Soham Ray et Arjak Bhattacharjee, «Dirac's large number hypothesis: A journey from concept to implication,» *International Journal of Modern Physics D*, vol. 28, n° %108, 2019.
- [12] A. Cetto, L. de la Peña et E. Santos, «Dirac's large-number hypothesis revised,» *Astronomy and Astrophysics*, vol. 164, pp. 1-5, 1986.
- [13] M. S. Berman, «Large number hypothesis,» *International Journal of Theoretical Physics*, vol. 31, pp. 1447-1450, 1992.

- [14] Y. K. Lau; S. J. Prokhorovnik, «The Large Numbers Hypothesis and a Relativistic Theory of Gravitation,» *Australian Journal of Physics*, vol. 39, pp. 339-346, 1986.
- [15] P. S. Wesson, «Is mass quantized,» University of Waterloo, Waterloo, CA, 2003.
- [16] E. Lusso, E. Piedipalumbo, G. Risaliti, M. Paolillo, S. Bisogni, E. Nardini et L. Amati, «Tension with the flat Λ CDM model from a high-redshift Hubble diagram of supernovae, quasars, and gamma-ray bursts,» *Astronomy & Astrophysics*, vol. 628, n° 14, p. 5, 2019.
- [17] M. Shuntov, «Unveiling the Concordance Model of Cosmology,» Aix Marseille Université, Marseille, 2018.
- [18] D. Valev, «Three Fundamental Masses Derived by Dimensional Analysis,» *American Journal of Space Science*, pp. 145-149, 2013.
- [19] D. Valev, «Evidence of Dirac Large Numbers Hypothesis,» *PROCEEDINGS OF THE ROMANIAN ACADEMY*, vol. 20, n° 14, pp. 361-368, 2019.
- [20] D. Valev, «Neutrino and Graviton Mass Estimations by a Phenomenological Approach,» *Aerospace Research in Bulgaria*, vol. 22, 2008.
- [21] D. Valev, «Estimations of Neutrino and Graviton Masses by a Phenomenological Mass Relation for Stable Particles,» *Physics International*, pp. 82-88, 2015.
- [22] M. A. Rushdi et A. M. Rushdi, «On the Fundamental Masses Derivable by Dimensional Analysis,» *Journal of King Abdulaziz University : Engineering Sciences*, vol. 27, pp. 35-42, 2016.
- [23] R. G. Wilmot, «Exact Value of the Hubble Constant - The Most Precise Value of the Hubble Constant Deduced from Other Constants of Physics,» *ResearchGate*, p. 28, 2021.
- [24] P. Talbot, *The Cosmopheric Principle*, St-Hubert, Qc, Canada: Paul Talbot (Amazon), 2021.
- [25] P. Talbot, *Le principe cosmosphérique*, St-Hubert, Qc, Canada: Paul Talbot (Amazon), 2021.
- [26] A. Kritov, «Explicit Values for Gravitational and Hubble Constants from Cosmological Entropy Bound and Alpha-Quantization of Particle Masses,» *Progress in Physics*, vol. 17, pp. 158-163, 2021.
- [27] C. Mercier, «Calculation of the Universal Gravitational Constant, of the Hubble Constant, and of the Average CMB Temperature,» *Journal of Modern Physics*, vol. 10, pp. 641-662, 2019.
- [28] C. G. Wolf, «Hubble constant H_0 is derived from Newtonian gravitational constant G , speed of light in vacuum c , electron mass m_e , classical electron radius r_e squared and fine structure constant α ,» *ResearchGate*, p. 7, 2022.
- [29] W. Xiong, A. Gasparian, H. Gao, D. Dutta, M. Khandaker, N. Liyanage, E. Pasyuk, C. Peng, X. Bai, L. Ye, K. Gnanvo, C. Gu, M. Levillain, X. Yan, D. W. Higinbotham, M. Mezziane, Z. Ye, K. Adhikari, B. Aljawrneh, H. Bhatt, D. Bhetuwal, J. Brock, V. Burkert, C. Carlin, A. Deur, D. Di, P. Ekanay, P. Ekanayaka, L. El-Fassi, B. Emmich, L. Gan, O. Glamazdin, M. L. Kabir, A. Karki, C. Keith, S. Kowalski, V. Lagerquist, I. Larin, T. Liu, A. Liyanage, J. Maxwell, D. Meekins, S. J. Nazeer, V. Nelyubin, H. Nguyen, R. Pedroni, C. Perdrisat, J. Pierce, V. Punjabi, M. Shabestari, A. Shahinyan, R. Silwal, S. Stepanyan, A. Subedi, V. V. Tarasov, N. Ton,

- Y. Zhang et Z. W. Zhao, «A Small Proton Charge Radius From an Electron–Proton Scattering Experiment,» *Nature*, vol. 575, p. 147–150, 2019.
- [30] N. Bezginov, T. Valdez, M. Horbatsch, A. Marsmsn, A. Vuta et E. Hessels, «A Measurement of the Atomic Hydrogen Lamb Shift and the Proton Charge Radius,» *Science*, vol. 365, pp. 1007-1012, 2019.
- [31] A. Beyer, L. Maisenbacher, A. Matveev, R. Pohl, K. Khabarova, A. Grinin, T. Lamour, D. Yost, T. Hänsch, N. Kolachevsky et T. Udem, «The Rydberg constant and proton size from atomic hydrogen,» *Science*, vol. 358, pp. 79-85, 2017.
- [32] D. Djukanovic, T. Harris, G. von Hippel, P. M. Junnarkar, H. B. Meyer, D. Mohler, K. Ottnad, J. W. Schulz, H. Wittig et J. Wilhelm, «Isovector electromagnetic form factors of the nucleon from lattice QCD and the proton radius puzzle,» *Physical Review D*, vol. 103, n° %1094522, p. 25, 2021.
- [33] Z.-F. Cui, D. Binosi, C. D. Roberts et S. M. Schmidt, «Fresh extraction of the proton charge radius from electron scattering,» *High Energy Physics - Phenomenology*, vol. Lett 127, n° %1092001, p. 7, 2021.
- [34] H. Gao et M. Vanderhaeghen, «The proton charge radius,» *Rev. Mod. Phys.*, vol. 94, n° %11ss. 1, p. 43, 2022.