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Materiality and Numerical Cognition

A Material Engagement Theory Perspective

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Introduction

Material engagement theory (MET) envisions materiality as an integral part of cognition (Malafouris, 2013). This is similar to the idea that perception involves not only processing in auditory and association cortex, but also sound waves striking the eardrum and something in the world producing noise; MET, however, assumes greater interactivity and bidirectional change between brains and materiality both over short- and long-term time spans than is characteristic of the causal linkage of audition. Viewing materiality in this way necessarily redraws the boundaries of cognition: It is no longer something performed by the brain alone, but a system that includes the body and the environment, including material culture. This is not the view of mainstream neuroscience, which intently focuses on the brain, has just begun to recognize the contribution of the body, and has seemingly not yet realized that materiality has a constitutive role. This focus is reasonable, given the investigative methodologies currently available and the need to formulate testable hypotheses and limit potential confounds. Nor is it certain that materiality is constitutive of cognition, thus calling its inclusion in cognition into question, as Malafouris (2013) readily admits. Nonetheless, as Malafouris also suggests, seeing cognition through MET's paradigmatic lens—considering it in terms of nontraditional boundaries—has the potential to yield new insights into how humans think, as the present review will argue.

Taking the perspective that cognition is a complex system that includes not only brain but body and materiality provides an apt framework for examining numerical cognition because all three contribute to it, albeit each in different ways. This functionality will be viewed through Malafouris's three central tenets: first, cognition is *embodied and extended*. That is, in addition to the brain,

the body and materiality not only contribute to but are constitutive of cognition. This positions the body and materiality as meaningful parts of numerical cognition while preserving the brain's unambiguous and important role. It also provides a basis for comparing the numerical cognition of ancient and existing peoples, particularly through the material and behavioral components. Second, in shaping behavior and psychological responses, materiality has *agency*: it influences the other components of cognition through its properties (or affordances, described further below), thereby shaping numerical system outcomes. This suggests that the materials used to realize and represent numbers have a particular influence on the way we conceptualize them. Finally, materiality has a semiotic function (or a meaning potential), making it an *enactive sign*. This is important in numerical cognition, where material meaning precedes language for numbers, and change in material form changes how numbers are conceptualized.

These central tenets are then applied to representational artifacts used as counting technologies: the fingers, tallies, and strings, suggested by the hand stencils, notched bones, and beads of the Upper Paleolithic and Middle Stone Age. Considering these artifacts as the material component of numerical cognition allows us to interpret them in terms of potential functions, properties, and signification, thus yielding new insights into how ancient peoples may have thought about numbers.

The Mind Is Extended

The idea that the brain is not all there is to cognition is a philosophical perspective known as the *extended mind hypothesis*. It is important to clarify that seeing cognition as extended beyond the body does not entail that the brain is uninvolved or unimportant; rather, the brain is a vital part of a complex, dynamic system that also includes the body and materiality. As an illustration, writing a note to assist memory performs the same function as remembering without material assistance. Because they are functionally indistinct, the standard by which the second is deemed cognitive whereas the first is excluded can be questioned (Clark, 2008; Clark & Chalmers, 1998). And, once paper, pen, or the iPhone Notes application and acts of writing and reading have been admitted into the human memory system, the definition of cognition has necessarily been expanded from something performed by the brain alone to something encompassing material devices and the activities of interacting with them (Estany & Martínez, 2014; Malafouris, 2013). This is not to say that brain, body, and materiality play the same role within the cognitive system; rather, they are seen as contributing in different but complementary and mutually interacting ways.

The new boundaries position material culture as an integral part of cognition, not merely its byproduct, assistant, external repository, or source of

information over which its computations might range. Once material culture is viewed as part of a larger cognitive system, insights about the functionality of material culture within the system follow, as do insights about how our brains have evolved to exploit material structures. For example, the material environment, broadly speaking, consists of social institutions, cultural practices, and material artifacts. These help us decompose problems into smaller tasks that are more easily solved because they are smaller (Beer, 2003). This effect is seen in numbers when a complex task like multiplying 8,912 by 7,252 can be reduced to “a series of simpler problems beginning with 2×2 ” (Hodder, 2012, p. 35). Decomposing problems also makes it easier for multiple individuals to collaborate in solutions, thus opening up new possibilities for outcomes in addition to the variation gained through “difference, localism, and choice” (Robson, 2008, p. xxii). Solutions can take the form of artifacts, making them available to other individuals and future generations; this opens up further opportunities to refine or apply artifacts to new uses, thus affording additional possibilities for change (Damerow, 2010).

When material culture enables future individuals to perform tasks and solve new problems, cognitive effort becomes distributed over space and time, an extensionality that is one of the major differences in the use of materiality by the human species (Hutchins, 1995). For example, the calculator represents inventions of numbers, notations, algorithms, mathematical pedagogy, metallurgy, plastics, electronics, software, data storage, marketing, transportation, and so on, an amount of cognitive effort that no one individual or society could easily formulate and synthesize. Someone using a calculator to perform a mathematical task today is unlikely to be any of the people who invented the things that went into the device, which instantiates all sorts of knowledge and makes it available to other individuals as a resource for performing tasks and discovering new applications. That is, someone with a slide rule need not reinvent calculus; she need merely apply the device’s ability to manipulate numbers to the problem at hand.

Cumulatively, processes of invention, learning, use, refinement, and extension afforded opportunities for both artifacts and brains to change, especially when long spans of time are considered. Artifactual designs are refined and extended to become more efficient, capable, and tailored to support new or increasingly specialized cultural purposes, while brain functions become repurposed for cultural inventions (e.g., object recognition in literacy; see Perfetti & Tan, 2013). Over even longer spans of time, new brain functions or regions may come into being (e.g., regions of the human intraparietal sulcus specialized for detailed vision; see Orban et al., 2006). Modern calculation devices instantiate, in a very meaningful sense, much of this ancestral cognitive activity: electronic calculators are not invented *ex nihilo* but rather from the number concepts, numerical algorithms, and notations; supporting technologies; and repurposed

brain functions realized through the past and present interactions, interdependencies, and multidirectional changes of brains, bodies, and materiality.

Understanding how the incorporation of materiality into the human cognitive system and its interaction with brains and bodies might have led to the realization of number concepts requires insight into the unique expansion of the parietal lobe in *Homo sapiens* and its consequences, including globularization, reorganization of neural connectivity between and within various brain regions, and enhanced parietal functioning (Bruner, 2004, 2010, 2014; Bruner et al., 2014; Bruner, Manzi, & Arsuaga, 2003; Coolidge & Overmann, 2012; Coolidge, Wynn, Overmann, & Hicks, 2015). Cognitive functions likely enhanced by the morphometric rescaling of the parietal lobe include analytical abilities for relationships of space, time, and number; multimodal association of visuospatial and other sensory information; motor sequence planning and execution; the ability to “know” and control the fingers; internal representations (to whatever degree such exist) of external space; and abilities for understanding concepts through associations with embodied experience (i.e., metaphor).

As previously noted, regions in the human intraparietal sulcus (a portion of the parietal lobe located on its lateral surface that is not present in the brains of macaques) represent additional aspects of dimensionality in shape, and this detailed vision, in conjunction with fine motor control of the fingers, might have informed our ability to make and use tools, especially those with compound or moving parts (Orban & Caruana, 2014; Orban et al., 2006). However, it is also possible that the new intraparietal regions were the result of long-term interaction with tools, an activity-dependent change in neural function known as metaplasticity, because the use of tools by the hominid lineage extends more than 2.5 million years. Even more likely is the possibility that it was—and still is, because human becoming is hardly finished—the interaction between tools and brains that changed the form and function of both, alterations detectable especially when viewed over long spans of time.

The interaction between parietal functions for quantity perception (numerosity, which is shared by many species) and the fingers (finger gnosis, the ability to know the fingers; haptic perception, the sense of active touch; and neural reactions to tools and graspable objects) may inform another unique human capability: engaging materiality with the fingers may have allowed us to express our innate, alinguistic quantity perception in material form, making numerical properties tangible, visible, and manipulable and thereby bridging the mental and physical senses of quantity and facilitating the formation of explicit concepts of number (Coolidge & Overmann, 2012; Malafouris, 2010). Few species use their paws, hooves, or pads to explore and gain information about the world (Gallagher, 2013). The unique capabilities of the hand that enable us to use it for these purposes are consistent with the amount of our sensorimotor cortex dedicated to its sensation and control (Penfield & Jasper, 1954). Without the

ability to engage materiality with our hands and fingers, our quantity appreciation might otherwise have remained locked within the limits of our perceptual experience of quantity.¹ More severely, a lack of material engagement might also have precluded us from forming explicit concepts of even small quantities salient within the subitizing range because the tactile manipulation of objects and reference sets appears integral to the process of realizing higher order relations of shared cardinality for both subitizable and larger numbers (Overmann, 2015a). The engagement of materiality by brains and bodies is thus vital, the very mechanism by which our numerosity becomes human numerical cognition.

Materiality Has Agency

The second tenet of MET is the idea that materiality has agency, the capacity of an agent to act in the world (Malafouris, 2013). Agency is often conceptualized as something humans have related to their abilities to form and pursue courses of action (sometimes called *free will* and often connoting intentionality). However, agency can also be understood as a relational property, something that causes change within a system; thus, material artifacts, in changing human behavior, display agency. Thus, an agent need not be alive in order to act. For example, a speed bump insists that drivers slow down or risk car damage and the attendant need for repairs (Malafouris, 2013). But material agency is no mere passive repository of human intentionality, like that of the fellows who design and install speed bumps to moderate or penalize driving behavior. A stone has agency in producing (or not) a sharp, usable edge, thus making it suitable (or not) for producing a handaxe and affecting outcomes of activities like hunting and informing decisions about whether a group returns to its source area; a totem has agency in enforcing taboos, altering beliefs and behavior in those who acknowledge its power (Gell, 1998); the sextant has agency in enabling sailors to estimate latitude and longitude, thus improving their chances of reaching their desired destination (Hutchins, 1995). Although neither equivalent to nor interchangeable with human agency, material agency is nonetheless complementary and relationally intertwined with human agency to the extent that it can be difficult to separate the two in any meaningful sense (Knappett, 2005).

Whereas materiality has agency in being able to change numerical cognition, the specific change it is able to effect is a function of its *affordances*. As theorized by psychologist James Gibson, affordances are what the environment offers to an agent, and they are found in the interaction between an agent's abilities and its environment, rather than being properties of either. Affordances are inherently related to an agent's abilities, both physical and mental; if a species lacks the ability to exploit a particular affordance, that affordance is not one for that species (Gibson, 1977, 1979). A monkey, for example, has little use for a

calculator beyond mashing its buttons or using it as a projectile. Such employments undoubtedly fall within the human behavioral repertoire, but humans can also apply calculators to mathematical tasks (and, indeed, might not be able to perform them without the affordances the calculator provides).

The example of the calculator illustrates that affordances are not just features of the natural environment: they can be encoded in material artifacts, making them available for other individuals and generations and distributing cognitive effort geographically and temporally (Hutchins, 1995; Smith, 2007). When solutions to problems and enablers of task performance take the form of artifacts, their capabilities influence behaviors and outcomes in subsequent performances and problem-solving. An example is the sextant, a navigation instrument that instantiates the “kinds of [wayfinding] knowledge that would be exceedingly difficult to represent mentally” (Hutchins, 1995, p. 96). The sextant (as a calculator does for mathematical knowledge) represents navigational knowledge that has been accumulated, changed, and improved by generations of individuals, as well as algorithms implicit in and automated through the artifact’s design. Artifactual structures reflect the requirements of the tasks for which they were created, just as their designs complement and compensate for human psychological and anatomical abilities and limitations. Their structures, designs, and capabilities shape and constrain task performance: the sextant entails that navigation involves measuring celestial altitudes; the calculator solves square roots when buttons that activate preformed algorithms are pressed in certain combinations. Such devices can also offer different and perhaps new possibilities for action, thus expanding the range of potential outcomes.

What do material agency and affordances look like in numerical cognition? Answering this requires us first to consider what numbers and numerical cognition are: numbers are concepts of discrete quantity like “one,” “two,” and “three” and the relations between them. Numerical relations are manipulated by arithmetical operations like addition and subtraction. Numerical cognition is thinking with numbers, a characterization that is not intended to limit its scope to numeracy or reasoning with numbers, but rather to include the process by which number concepts are realized through the interaction of brains, bodies, and materiality, a model of which is proposed in Figure 5.1. Simply, numerical concepts and words like “one,” “two,” and “three” result from the interaction of the perceptual system for quantity, the fingers, and material objects. As materiality—both the body and artifacts—is incorporated into the cognitive system for numbers, the innate sense of quantity that is common to many species is given opportunities for explication and elaboration in a way unique to humans. Tangibility yields structure and stability and (depending on the materiality that comprises the artifact) a certain amount of manipulability for the quantity instantiated. Once explicit, quantity also becomes discrete, and its material form gives it persistence. These qualities increase the likelihood that

Figure 5.1 Conceptual model of counting in numerical cognition, based on the neuroscience of number, ethnographic observations about the use of the body and material devices for counting, linguistic data about characteristics that number words share across languages, and ideas about how the components contribute and interact when considered through Material Engagement Theory. The model is not reflective of any particular society or culture and should not be understood as implying either progress or linearity, but rather the typical changes that occur as materiality is incorporated into numerical cognition. Images from OpenClipArt.com.

Components include brain (left), body (middle), and materiality (right). The brain supports functions such as numerosity and categorization, while the body provides fingers that interact with the brain and materiality through processes like finger gnosis, haptic perception, and neural reactions to tools and graspable objects. Materiality includes both naturally occurring objects used as reference sets and artifacts made for counting—respectively, things like fingers and notched sticks. Brain, body, and materiality overlap to characterize their interactivity and because brains do not occur in a vat, nor bodies in a vacuum.

The model differentiates things used to count (top) from enumerated objects (bottom), with relations between the two achieved through strategies such as pairing (a behavior within the capabilities of non-human primates; see Conway & Christiansen, 2001; Frigaszy, Galloway, Johnson-Pynn, & Brakke, 2002) and one-to-one correspondence (behavior unique to humans). Highest number counted is related to the things used to count; it increases as materiality is incorporated into the system through the changes that result from the interaction of material affordances and agent abilities.

Number system characteristics suggest their origins. Initial quantity expressions are generally ‘one’, ‘two’, ‘three’, ‘big many’ and ‘small many’ (Menninger, 1992), which are consistent with the neurological constraints on quantity perception and attention. Involvement of the body leaves traces in the form of anatomic bases (e.g., decimal) and embodied vocabulary like the word ‘digit’ (Comrie, 2013). Material devices used as counting technologies are not necessarily limited to ‘three’ or anatomically patterned; their characteristic signature is the extension of highest number counted.

someone interacting with the artifact will notice similarities and patterns in the quantity it instantiates, like each notch being one more than the notch it follows. Patterns and similarities, in turn, can be codified and thus endure in the form of artifacts and language.

Different counting technologies offer different affordances, each of which shapes or constrains the resultant quantification behavior and numerical system outcomes in different ways. The use of the fingers for counting, for example, appears to be cross-culturally universal, independent of how elaborate a culture's number system might be. This behavior reflects the neurological interaction between the perceptual system for quantity and control of the fingers (reviewed in Overmann, 2014). Because the fingers interact with materiality and are literally ready to hand, they bridge psychological capabilities like numerosity with material objects that have quantity. They are also typically the first things used as a reference in counting: they instantiate the concept of quantity, making it accessible and communicable. In addition to being found in emerging number systems, finger-counting persists in highly elaborated ones as well, where it acts as a convenient materiality for enumeration and an aid to memory, likely because of the neurological interaction between numerosity and control of the fingers (i.e., between the intraparietal sulcus and angular gyrus). In comparison, beads can also be used as a counting technology, have a comparable cross-cultural prevalence, and similarly span the full spectrum of numerical elaboration. For example, the rosary requires neither concepts of nor words for numbers, whereas beads have been used to count into the tens of thousands by the North American people known as the Pomo (Barrett, 1952). However, many societies make and wear beads, but few count with them, an uneven ratio that perhaps reflects the need to repurpose beads from ornamentation to quantification and the lack of a specific neurological function integrating the material form with numerosity. Fingers and beads also have different capacities for quantity: Fingers are generally (but not invariably) limited to low numbers, whereas beads (as attested by Pomo counting) have a much greater potential.

A similar differential is found in material capacities for instantiating relations between numbers. Accumulation is something that all early counting technologies appear to have in common: it is easy to extend another finger or add another notch to a tally, knot a string, add a stone to a pile, or mark the ground—at least up to the capacity of the material form: the number of fingers, the surface area for adding notches, the length of string to be knotted, and so on. Fewer of these technologies have a ready potential for subtraction, however. Notches are difficult to fill in, although knots can be untied, stones removed, and marks on the ground erased. Fingers, despite their universality in counting, are seemingly not a factor in developing subtraction (or it might be developed sooner), perhaps because the embodied experience of quantity does not disappear when an extended finger is flexed in the same way that it does when, for example, a stone

is removed from the pile. Given the potential of material counting technologies for accumulation (all) and subtraction (some), it cannot be coincidental that addition is generally the first arithmetical operation to emerge, with subtraction developing later.

Regularity, manipulability, and integrity are other characteristics in which material affordances differ. Notches on sticks and knots in strings instantiate the kind of stable-order linearity that fingers afford, a similarity that is perhaps a factor in why such technologies tend to follow finger-counting. By comparison, stones and other noncontiguous objects lack implicit order, but their individual elements can be manipulated in ways that notches, knots, and fingers cannot. Greater manipulability is a likely factor in extending the recognition of numerical relations beyond plus-one. Four pebbles, for example, can be decomposed and rearranged as “three” and “one,” “one” and “three,” or “two” and “two,” groupings with the potential to illuminate not only the quantity relations involved but their relative equality (i.e., “two” and “two” balance in a way that “three” and “one” do not). A notched tally, however, maintains its integrity as an instantiated quantity in a way that loose, manipulable counters do not unless they are somehow contained. As these examples serve to illustrate, potential outcomes for numerical elaboration may be enabled or constrained by the affordances of particular material forms (Hodder, 2012; Knappett, 2005), just as affordance differences may be potential factors in the incorporation of new material forms. Material agency and the variability admitted through affordance differences may be as significant in influencing numerical system outcomes as the various decisions, applications, and values of the social dimension.

The psychological functioning of the brain changes as the cognitive system for numbers incorporates materiality. Although parietal functions such as numerosity and finger control are significant to the realization of initial discrete quantities, arithmetical and mathematical tasks additionally engage the frontal lobe and executive functions such as intentionality and working memory (Zamarian, Ischebeck, & Delazer, 2009), thus increasing goal-directed behavior. Demands on various forms of memory may decrease: spatial memory (memory for object location used as a heuristic in societies with few numbers; e.g., Butterworth & Reeve, 2008), working memory, and visuospatial working memory (a component of working memory that supports mental imagery). Reduced demands on memory, along with the visualization and problem-solving facilitated by material engagement, allow freed attentional capacity to be repurposed to things like pattern recognition, categorization, and abstraction, thus enabling the formation of new concepts. At the same time, interaction with material linearity increases the linearity of the mental number line, an internal resource for estimating numerical magnitudes (Brannon, 2006; Fischer, 2008; Previtali, Rinaldi, & Girelli, 2011), and this may assist the realization of higher quantities in regularized and productive ways.

The numerical system also changes: highest number counted goes up as materiality is incorporated; this seems only commonsensical, given that material devices for counting seem like just the very thing to help us count. An increase in highest number counted, in turn, affords an increased likelihood that the relations between numbers will be discovered (this is simply a function of exemplar availability; see Beller & Bender, 2011) and codified in the form of devices, algorithms, and language. As numbers become more elaborated, they become more useful, a utility that makes them familiar and habitual. This positions them as a cognitive technology, a body of knowledge that structures understanding and behavior (De Cruz, 2008, 2012). The more familiar and habitual a cognitive technology becomes, the more likely it is that it will be extended to new domains (Overmann, 2013) and become entangled in cultural practices and behaviors (Hodder, 2012). The more ingrained in culture numbers become, the more likely they are to be elaborated; their usefulness may intensify the need to develop and extend them.

Material Signs Are Enactive

The third tenet of MET is enactive signification, the idea that things acquire meaning in virtue of what they are and what we do with them. Integral to enactive signification is the material sign, which functions semiotically in a manner quite different from that of the linguistic sign. Where a word is ephemeral—speak it, and it is just as quickly gone—the material sign persists; where a string of words occur in a linear sequence, a function of sound production in language, the material sign has no obvious start or stop (Malafouris, 2013). Linguistic signs are communicative and symbolic, denoting quantity through arbitrary conventions, while material signs are enactive and instantiate quantity by simply being what they are (Malafouris, 2013), objects of a certain quantity in interaction with numerosity and perhaps the fingers. Language and the linguistic signs that comprise it have been said to form a “longer, slower, and more complex” bridge between communication and cognition, implying that “things, concepts of things, and signs of concepts of things” may constitute a shorter, faster, simpler bridge, one that is more directly connected to thinking (Brandt, 2010, p. 253), including numerical thinking. Essentially, both are critical to accessing numerical cognition and elaborating numerical intuitions into number systems; however, they differ in more than the speed of the bridge they provide (i.e., in things like their semiotic function, as discussed later), and any account of numerical cognition that does not consider both is necessarily incomplete.

The semiotic differentiae illuminate some differences in the roles of material and linguistic signs in the development of number concepts. By instantiating

discrete quantities, the material sign (a term encompassing material counting technologies, including the body) anchors and stabilizes them, facilitating the realization of explicit number concepts and their interrelations (Coolidge & Overmann, 2012; Hutchins, 2005; Malafouris, 2010). Materiality (and forms of numerical notation as well, as will be discussed) enable the manipulation of numerical relations and algorithms in ways that language does not (Sfard & Linchevski, 1994). Simply, language, counting devices, and numerical notations represent numbers differently (as attested by the different universal properties of language for numbers and numerical notations [Chrisomalis, 2010; Greenberg, 1978] and the fact that neither considers the properties of non-notational material forms). Algebraic problems in the form of spoken narratives are difficult to solve, a challenge not resolved by writing the language phonographically. Material structures and numerical notations, by comparison, enable a more direct manipulation of the logical and spatial relationships that are the essence of numerical cognition and mathematics. However, if language is a “slow and hesitant” route to numerical cognition (Sfard & Linchevski, 1994, p. 198), linguistic signs nonetheless enable numerical concepts to be communicated, learned, and applied in social contexts in ways not possible through visuospatial means alone (Ferrari, 2003). Furthermore, lexicalization may help number concepts become conceptualized as entities in their own right, allowing them to acquire new properties (as stabilized and represented by material realizers) and become available to be acted upon by new operations and processes.

Numerosity (the perceptual system for quantity shared with other species) and language (which we do not share) are both essential to numeracy (earlier defined as the ability to reason with numbers). Language per se, however, is insufficient for developing number concepts, or there would be no languages that lack them, as the Pirahã language spoken in Amazonian Brazil famously does (e.g., Everett, 2005). As cognitive functions, numeracy (which depends on numerosity) and language are independent in that they are doubly dissociable (Brannon, 2005). That is, a person can lose the ability to calculate but retain the ability to converse, and vice versa (Ardila & Rosselli, 2002; Varley, Klessinger, Romanowski, & Siegal, 2005). Independence is also suggested by the multiplicity of species with numerosity but lacking language and the fact that human infants demonstrate the ability to appreciate quantity before they develop language (e.g., Cantlon, Brannon, Carter, & Pelphrey, 2006; Van Marle & Wynn, 2011). Furthermore, material signs precede linguistic signs in ethnographic accounts of initial quantification (e.g., Chagnon, 2013; da Silva, 1962; Karsten, 1935; Murphy & Quain, 1955; Staden, 1928; Turrado Moreno, 1945), and material and linguistic signs differ semiotically. These differences suggest that materiality and language be analyzed both separately and in interaction with each other for their contributions to numerical cognition.

Material and Linguistic Signs

An initial analysis of the semiotic differences of materiality and language is shown in Table 5.1. The basic systems (material objects and non-numerical speech) appear in the outermost columns and are related as signified and signifier (i.e., the concept *jar* and the word “jar,” or, in Peircean terms, *interpretant* and *representatem*; Gottdiener, 1995). From the left toward the middle, the order reflects increasing elaboration in material signs for numbers: material objects (jars), counting technologies (e.g., tallies or tokens), and cumulative and ciphered numerical notations² (respectively, the Roman numeral III and the Hindu–Arabic numeral 3). From the right toward the middle, the order reflects increasing elaboration in linguistic signs for numbers: non-numerical language (“jar”), emergent adjectival number terms (“threeness”), full-fledged lexical terms for numbers (“three”), and phonographically written numbers (three).³

The semiotic qualities considered in the analysis included type (metrological, related through properties of capacity and weight; numerical, denoting quantity; and non-numerical), sense mode (tactile, visual, or auditory), persistence (durable or ephemeral), sequentiality (no start/stop or linear), logic (enactive or communicative), and representation (instantiate or symbolize). Composition (metrological, quantitative, or grammatical) examined syntactic structure in larger combinations; decomposition (metrological, quantitative, phonemic, or meaningless) explored the meaning of individual elements when separated from a larger group. Finally, conventionality and language (low, medium, or high) examined the amount of cultural or linguistic knowledge required to understand meaning (Houston, 1994).

The basic systems (material objects and non-numerical speech) had no qualities in common despite their signifier–signified relation. Systems adjacent to the basic ones were similar to them in terms of the semiotic qualities considered; as systems became elaborated (with ciphered and phonographic notations containing the greatest mix of semiotic qualities from the two basic systems), they adopted more of the qualities belonging to the other basic system. Such hybridization is presumably incremental and nonlinear, and it depends on the extensive interaction and interdependence of materiality and language (i.e., their interaction and interdependence comprise the mechanism that enables their elaboration). It also seems reasonable to conjecture that not all qualities would be equally changeable (thus, not all transitions would represent change of the same magnitude) and that few transitions likely represent deliberate, consciously thought-through change (though these would not be excluded) but rather combinations of extended usage, discovery, happenstance, and mistakes, and the contributions of many individuals.

Table 5.1 Comparison of Materiality, Notational Numbers, and Speech

Quality	Material			Numerical Notation			Speech		
	Object	Sign		Cumulative	Ciphered	Phonographic	Lexical Nr.	Adjective	Noun
Example		• • •		III	3	three	'three'	'-ness'	'jar'
Type	Metrological	Numerical		Numerical		Numerical	Numerical	Numerical	Non-Numeric
Sense Mode	Tactile/visual	Tactile/visual		Visual	Visual	Visual	Auditory	Auditory	Auditory
Persistence	Durable	Durable		Durable	Durable	Durable	Ephemeral	Ephemeral	Ephemeral
Sequentiality	No start/stop	No start/stop		No start/stop	Linear	Linear	Linear	Linear	Linear
Logic	Enactive	Enactive		Enactive	Communicative	Communicative	Communicative	Communicative	Communicative
Representation	Instantiate	Instantiate		Instantiate	Symbolize	Symbolize	Symbolize	Symbolize	Symbolize
Composition	Metrological	Quantitative		Quantitative	Quantitative	Qty. or gram.	Qty. or gram.	Grammatical	Grammatical
Decomposition	Metrological	Quantitative		Quantitative	Meaningless	Phonemic	Phonemic	Phonemic	Phonemic
Conventionality	Low	Low		Low	Medium	Medium	High	High	High
Language	Low	Low		Low	Medium	Medium	High	High	High

Note: Comparison of type, sense mode, semiotic function (persistence, sequentiality, logic, and representation), composition–decomposition, and amount of cultural–linguistic knowledge needed to understand meaning (conventionality and language) in material objects and signs, numerical notations (cumulative, ciphered, and phonographic), and speech (lexical numbers, emerging adjectival quantity terms, and non-numerical language). Developed from concepts in Chrisomalis, 2010 (numerical notations and lexical numbers); Houston, 1994 (conventionality and language); and Malafouris, 2013 (semiotic functions of material and linguistic signs). Material objects and non-numerical spoken language appear in the outer columns with no qualities in common, though they are related through reference (as the word ‘jar’ refers to the object jar). The other systems generally share qualities with one or more of the basic systems.

Objects, counting technologies, and cumulative notations share most characteristics; the only difference is that objects can be related metrologically (through characteristics of capacity, weight, size, measure, etc.), and objects and counting technologies are perhaps more tactile than written notations. Ciphered notations share several characteristics with material objects and counting technologies. These similarities perhaps explain why material instantiations of quantity and numerical notations are recognized translinguistically, making them highly likely to diffuse across linguistic and cultural boundaries and generally the first thing to be recognized in unknown scripts.

Overall, language and materiality interact in ways that contribute to the development of numerical cognition. Language provides an access that is critical to explicating and elaborating numerical concepts in material form, and it absorbs material characteristics like the productive cycles that structure numerical systems (e.g., decimal, quinary/vigesimal, etc.). Materiality provides the stability, anchoring, and representation that are critical to realizing and expressing numerical concepts in language, and it structures numerical thinking (e.g., linearity, stable order, etc.).

Material and Linguistic Affordances

Additional differences are found in the affordances that material and linguistic signs offer to numerical cognition. Linguistic signs for quantity are arranged in ordinal sequences (e.g., “one, two, three. . .”), and the enumeration enabled by ordinal sequencing can facilitate the bundling of material signs for quantity (e.g., in which six or ten units of a smaller quantity equal and are exchanged for one unit of a larger quantity⁴). Material signs for quantity can also be rearranged and decomposed (to the degree permitted by the material form, as previously noted), thus enabling access to the individual elements and their various permutations in a way that linguistic signs do not permit. There is no way to get inside the spoken word “four” to understand “two plus two,” let alone “ten minus six,” “twenty divided by five,” “square root of sixteen,” or indeed any of the infinity of possible relations. The modern reader may understand these relations as implicit in the definitions of the ordinal numbers, but, in actuality, such relations and algorithms are unknown until they have been explicated and codified, including their general extensibility (“–1,365,788 divided by –341,447,” for example, also equals “four”). In such explication and extension, materiality is critical to visualizing and stabilizing numerical concepts in symbolic form, and the example illustrates the observation of archaeologist Colin Renfrew regarding its role: “the symbol cannot exist without the substance, and the material reality of the substance precedes the symbolic role” (Renfrew, 2004, p. 23).

Where the affordances of materiality and language coincide, there may be an increased potential for long-term stability of the numerical system. For example, peoples in Papua New Guinea count up to 27 using their bodies (Lean, 1992; Saxe, 2012). The material structure used for counting is an ordinal counting sequence, similar to the ordinal counting sequence provided by language. The affordances are mutually reinforcing, so the system is geared toward stability, and, indeed, body-counting systems have a long history in Papua New Guinea (Lean, 1992; Saxe, 2012). Alternatively, where the affordances of materiality and language contrast, there may be an opportunity to notice and analyze the differences to gain insight into the nature of each, thus enabling their exploitation, extension, and elaboration (Beller & Bender, 2011). This is simply the principle that contrast can illuminate differences better than similarities might. However, a differential opportunity for elaboration does not entail that the potential will be realized: any potential for elaboration is still subject to cultural needs and priorities. No matter how dissimilar the affordances of material and language might be for analytic comparison, a society that does not need or value numbers will not elaborate them to the same extent as a society that finds them vital to its survival.

Such affordance similarities and differences are neither advantageous nor disadvantageous, merely circumstances that inform numerical system outcomes.

Furthermore, the potential for contrastive difference to afford opportunities for analysis and change is not the only consideration. For example, Chinese lexical and notational numbers share common structuring (i.e., affordance similarities), a circumstance that at least partially explains the mathematical task performance advantages of Chinese speakers relative to English speakers, whose lexical and notational numbers are structured differently (Cantlon & Brannon, 2007; Chrisomalis, 2010). Thus, it is not simply affordance similarities or differences per se, but their holistic interactions with psychological and physical abilities that influence potential outcomes in numerical systems.

Using Material Engagement Theory to Interpret Artifacts

Fingers

In ethnographic descriptions of how modern peoples count, finger-counting is cross-culturally prevalent. Whereas there are neurological reasons that likely explain why finger-counting is ubiquitous (i.e., the interaction between the perceptual system for quantity and the sensory and motor systems controlling the fingers; reviewed in Coolidge & Overmann, 2012), there is an interesting point that should be highlighted: not only is finger-counting seemingly independent of how elaborate the numerical system might be, it spans the full spectrum of numerical system elaboration because it is found both in systems with few numbers and in those with computers and calculus. Moreover, widespread phenomena of embodied vocabulary (words like the English word “digit,” which means both “finger” and “number,” or numbers such as “seven” composed as “five plus two”) and anatomical cycles (decimal, quinary, vigesimal, etc., which characterize most of the world’s number systems; see Comrie, 2013) suggest that finger-counting has been fundamental to forming initial number concepts in most—if not all—number systems.

What makes the fingers such an apt material technology for early counting? As was previously mentioned, counting with the fingers may be cognitively prepotent because of the interaction between the parts of the parietal lobe involved in quantity perception and the sensorimotor control of the fingers. The sense of touch is also unique in its bipolarity, having both an external aspect in what the objects we touch feel like, as well as an internal aspect in what we feel when an object touches us (Gallagher, 2013), that may help broker relationships between our psychological processes and our material culture. Counting involves constructing relations between enumerated objects and a reference set, and fingers are literally ready to hand when a reference set is needed. However, when fingers are used as a reference set to form initial concepts of quantity, a more fundamental principle than their mere availability and convenience is in action: their

use indicates the judgment of relations between world and body, in which a quality such as size in each is assessed in relation to the other (Gallagher, 2013). Whereas many number systems transition to the use of reference sets that are not embodied, enabling world-to-world comparisons (e.g., a tally as a reference), such need not be the case, as attested by numerical systems based on body-counting (Lean, 1992; Saxe, 2012). Arguably, the distinction between fingers and tallies is not large because the former are as much a material technology as the latter; the difference is the sensory bipolarity that finger-counting entails.

Upper Paleolithic Hand Stencils

Finger-counting may have a fairly deep history if Rouillon (2006) is correct that the 27,000-year-old hand stencils in Cosquer Cave represent integers (also see Overmann, 2014, for a comparison of the hand stencils at Cosquer and Gargas to finger-counting behavior across cultures). When the fingers are used as a material counting technology, their affordances shape the resultant number system: five fingers on two hands impose anatomically based cycles of five and ten onto our number systems. The affordances of the hand mean that we generally start with an outside finger (the thumb or the little finger) and work our way across the fingers sequentially, imposing linearity and stable order (Gelman & Gallistel, 1978). Their limited capacity necessitates repeated use of the same hand for numbers up to five; numbers higher than five generally require the recruitment of other hands or the inclusion of nonembodied materiality with similar affordances (e.g., tallies, which share with fingers the potential for conceptualizations of stable order and the successor function).

Thus, a number system based on finger-counting, were it represented in paint, would use only the finger-patterns involved in sequentially flexing or extending the fingers to form integers, as the ones at Cosquer Cave appear to do (Rouillon, 2006). One hand would be used for the stencils if it counted no higher than five (though this could also be an effect of the production process); it might also underrepresent the number “four,” given that “five” tends to emerge first (as scaffolded by the hand entire) after “one,” “two,” and “three,” suggesting “four” later fills in the gap (as scaffolded by the finger sequence). The occurrence of tallies in roughly the same region and time period suggests a numerical system incorporating additional material forms, with the effect of increasing highest number counted.

Beyond Fingers

Aside from the possible occasional recording in paint, finger-counting does not persist, making it a particularly perishable medium; even marks on the ground tend to persist longer than someone is willing to sustain a finger-pattern. Given

a social requirement for remembering or communicating quantity over time or distance, some more durable, transportable form may be sought. In body-counting systems, this might take the form of body-paint (something that will not preserve archaeologically, unless it is represented in parietal art or petroglyphs, or indirectly and rather speculatively construed from the presence of ochre); in other systems, material devices might be used. In either case, the capacity for enumeration increases, which has an effect on the number system itself because, as previously mentioned, increased highest number counted increases the discoverability of numerical relations (Beller & Bender, 2011). Discovery of numerical relations or the limits of particular material forms, in turn, may inspire the inclusion of other forms of materiality with different affordances (e.g., manipulability). As was the case when affordances of language and materiality differed, contrastive differences in material forms affordances can serve to scaffold numerical concepts. For example, whereas both fingers and tallies instantiate plus-one stable order, the latter's greater capacity may also suggest a more indefinite accumulation. This is not to say that all such possibilities will be realized (as the Romans apparently failed to recognize the concept of "zero" implicit in their abacus design).

Upper Paleolithic Notched Tallies

Tallies in the form of notched bones are fairly common in the Upper Paleolithic, as for example, the artifacts from Abri Cellier (dated to 28,000 years ago) and Grotte du Taï (14,000 years ago). The older bones are marked by deep cutmarks in a single row, regular in their size (large) and disposition (even) but fairly simple in their representation of quantity (if indeed that was their purpose); by comparison, the younger artifact contains a complex, multirow sequence of small cutmarks, still regular and even but presenting the appearance of a fairly elaborate quantification (Marshack, 1991). If the use for quantification remains ambiguous for the Abri Cellier bones, the potential quantificational purpose of the later Taï plaque seems less so, likely because its similarity to more recent quantificational artifacts and the higher number of its marks lends it a plausible recognizability. However, archaeologists have approached the potential purposes of such artifacts cautiously, justly pointing out that cutmarks can represent, in addition to counting and decoration, things like butchery, record-keeping, divination, music, fiber- or leather-working, or other tool use (Reese, 2002).

Disambiguating such purposes has been attempted through various methods. Cutmark surface disposition has been assessed for randomness; patterns deemed more random are associated with butchery, those less haphazard thought to be more intentional and indicative of associated tool use. Microscopic analysis has also been used: wear patterns and trace elements can indicate tool use; cutmark distribution across artifactual surfaces and by time (the latter is

Figure 5.2 Bridging argument for material artifacts possibly used for quantification (modeled after Botha, 2008).

- **Intentional behavior inference:** From data about Upper Paleolithic and Middle Stone Age artifacts, inferences can be made regarding the behaviors that produced them (e.g., punctures in shells were made by humans rather than caused by animals or natural processes; bone marks were intentionally made rather than occurred inadvertently during butchery; etc.).
- **Material engagement inference:** From the artifacts and the behaviors that produced them, it can be inferred that either archaic peoples engaged specific material forms in ways that had the potential to generate number concepts, like notching bones (the minimal inference), or they produced material forms that reflected the availability of number concepts, like the hand stencils (the maximal inference).
- **Social context inference:** From material engagements with the potential to generate number concepts situated in certain social contexts, it can be inferred that social needs increased the likelihood of realizing and elaborating number concepts.
- **Numerical concepts inference:** From the increased likelihood of realizing numbers in certain social contexts, it can be inferred that Upper Palaeolithic peoples had number concepts and Middle Stone Age peoples may have had numbers (to the degree consistent with the material structures used for possible quantification and social needs for numbers).

assessed by characteristics indicating use of the same or different tools; see d'Errico, 1991) may distinguish decoration from quantification. The benefit of using such techniques is undeniable and an essential first step in inferring whether artifacts were used for quantification (see Figure 5.2), but material engagement must also be considered. That is, notches, even if made for decoration, represent a motor activity that provides an opportunity for the development of skill, and they instantiate the kind of tactile and visual regularities that can scaffold numerical cognition by providing affordances (as previously discussed) that engage psychological processes such as categorization, abstraction, working memory, and language. Behaviors are situated in social contexts that act to increase (or not) the need for numbers as a cognitive technology:

the elaboration of overall complexity in numeral systems—i.e., the development from highly restricted to generative systems that can name precise quantities into the tens, hundreds, or thousands—is motivated (at least in part) by social and cultural practices that

encourage counting and/or keeping track of precise quantities. (Epps, Bowerin, Hansen, Hill, & Zentz, 2012, p. 42)

Artifacts such as the Cellier and Tāi bones instantiate quantities that can be compared, communicated, and used across time and distance; furthermore, these purposes can be accomplished regardless of the availability of words for numbers or even concepts of the specific quantities represented by the artifacts (though it seems likely that, especially for the Tāi plaque, both concepts and words for numbers would have been available to some extent; see Overmann, 2013).

Middle Stone Age Stringed Beads

Beads of marine shells, ostrich eggshell, and stone, many with wear patterns of stringing and some with traces of ochre, have been found at several dozen Middle Stone Age sites throughout Africa and the Near East (covered in detail by Wynn, Overmann, Coolidge, & Janulis, Chapter 9, in this volume), most famously at Blombos Cave in South Africa, where they have been dated to approximately 77,000 years ago (d'Errico, Henshilwood, Vanhaeren, & Van Niekerk, 2005; Henshilwood, d'Errico, Vanhaeren, Van Niekerk, & Jacobs, 2004; Vanhaeren, d'Errico, Van Niekerk, Henshilwood, & Erasmus, 2013). This ubiquity is consistent with the prevalence of beads in existing cultures, as most cultures make beads. Stringed beads, although generally made and used for ornamentation, have affordances that may (in comparison to other artifacts) increase their likelihood of being repurposed from ornamentation to counting. That is, many objects do not have the same potential as beads for being used as an initial counting technology: knives and can openers, for example, have sharp edges and bulky shapes that tend to make them less easily manipulated—less handleable—as the possible elements of a reference set (as differentiated from being enumerated as objects; see Figure 5.1), no matter what quantity relations they might also happen to instantiate when placed in a row.

In addition to their handleability, beads have other affordances that suggest that their use in counting will take similar form across cultures. For example, as beads are slid along a string, they instantiate stable-order, plus-one, accumulative relations and are analogous in this regard to other initial counting technologies like fingers and tallies. Beads can be applied to counting methods that range from extremely complex to fairly simple. Exemplifying the former, the Pomo, who were mentioned previously, used beads to count to impressively high numbers (Barrett, 1952), although such elaborated counting with beads is rarely described in the ethnographic literature, suggesting it is unusual. A fairly simple counting method, one-to-one correspondence, is much more typical; moreover, it requires neither concepts of nor vocabulary for numbers (an example is the rosary, an artifact found in fairly similar form in many cultures around

the world). These affordances may explain why beads, when repurposed, are more likely to be used as an early counting technology: they are similar to but can extend the capabilities of other early technologies like fingers and tallies, and their repurposing may occur when numerical concepts and language are minimal.

Prehistoric Perishable Materials

Knotted strings, if such artifacts were used by Upper Paleolithic or Middle Stone Age societies, would not likely preserve. Neither would tallies made of wood or leaves, tokens made of seeds or grain, or marks made on the ground or body; we might also have difficulty recognizing tokens in the form of manuports, like a collection of pebbles. All of these counting technologies are attested in the ethnographic literature, and their prevalence in early counting suggests that the archaeological record may significantly underrepresent the time depth for numerical cognition, which likely included fingers and perishable materials before incorporating nonperishable materials. This circumstance suggests that archaeologists approach the material record with a caveat in mind: in addition to the need for caution in construing quantificational intent, there is an equal and somewhat opposed requirement to recognize that by the time nonperishable materials are used in counting, people have generally been counting for some time.

Conclusion

Materiality is so much more than tools that merely serve and externalize our mental computations, even if it is currently underappreciated for its constitutive functionality in human cognition (Malafouris, 2013). The lack of appreciation, however, need not remain the case. The way forward includes questioning the assumptions that sequester mental activity to the brain (Renfrew & Malafouris, 2008; Tallis, 2011), expanding and improving our investigative techniques so they include the larger cognitive system (i.e., not just brains but bodies and materiality; not just after the fact but during their interactivity, etc.), and finding ways to reconcile the short time spans considered by psychology with the longer temporalities involved in material, cultural, and genetic change (Overmann 2016*a*, 2016*b*). Such an approach is necessarily interdisciplinary, which also means adopting theoretical frameworks capable of reconciling data from evidentiary sources whose foundational assumptions will often be in conflict with one another.

Once we adopt the perspective that materiality is constitutive of our cognition, we can “notice phenomena like artifactual accumulation and the emergence of addition, affordance similarities of body-counting and language, or the

structural properties of Chinese numerical notations and mathematical task performance, and investigate the ways that materiality co-creates and influences our numerical thinking” (Overmann, 2015*b*, p. 9). That is, we will start to see how materiality shapes the human mind through its semiotic qualities and agency and become able to question why its role in this regard is generally so transparent. We can start to investigate how materiality interacts with language to yield abstract conceptual domains and get a better grasp of what abstract conceptual thought is as a phenomenon distributed over psychological, behavioral, and material components. If there is much to be done, and discomfort in the challenge of it, the end result may be that we are better able to understand the mysteries of our past and present human becoming.

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Notes

1. The perceptual experience of quantity is governed by attentional constraints on object individuation and the Weber-Fechner constant of just-noticeable differences; these functionally divide quantity appreciation into *subitization*, the ability to appreciate small quantities (up to 3 or 4) as individuals, and *magnitude appreciation*, the ability to appreciate larger quantities (above 3 or 4) as differentials (i.e., bigger and smaller) if their difference falls above the threshold of noticeability (Piazza, 2011).
2. Chrisomalis (2010) notes the tendency for cumulative numerical systems to precede ciphered ones.
3. Because phonographically represented numbers like “three” require a system of writing be in place, they develop after spoken lexical numbers (Chrisomalis, 2010), so they appear to the left of the spoken forms.
4. Bundling and debundling arguably require the ability to count if they are to be performed with reasonable convenience; an alternative would be the use of one-to-one correspondence for every bundling or debundling operation, which seems infeasible given its laboriousness.

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