An insulator becomes a conductor under the quantised, critical field: (The Insuductor) :

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A linear array of hydrogen or Na atoms, or any array of atoms have an odd number of electron per unit cell. The order of the lattice constant(a) is roughly ($1 \text{ \AA}$). The wavefunctions of the electron on each hydrogen atom according to band theory, are overlapping which means the electrons hop from site to site. The system is a conductor when the electrons delocalise to form a half-filled band where per each unit cell, there exists one electron. Physically, the band formation is a result of delocalisation of the electrons when gaining kinetic energy.

One could imagine that if the lattice constant (a) is increased, then what will be the consequences? According to band theory, as long as one starts with a half-filled band (i.e. one electron per unit cell), then, the system will not change but will remain metallic no matter how the atoms were pulled far apart, that will lead to an absurd. Where, at very large lattice separation, there exists a limit, where the conductor becomes just an array of atoms which implies the delocalisation of the electrons at each atom around their nucleus, Hence, the conductor in this limit tends to be an insulator. The big question now is (Why for large values of the lattice constant that array must be an insulator?).

At very small lattice separation, the quantum mechanical tunnelling occurs with perfect transmission coefficient, hence, with perfect delocalisation, which ensures the case of a conductor. At very large lattice separation, the quantum mechanical tunnelling is forbidden with zero transmission coefficient, hence, with zero delocalisation. Hence, the localisation coefficient is perfect, and indeed this is the case of an insulator. At very large lattice separation, the conductor becomes an insulator. Applying the negative, critical potential on the lattice separation region allows the delocalisation coefficient to be perfect due to the quantisation of the critical potential, then, an insulator becomes a conductor. Therefore, (The Insuductor) is an insulator which converted into a conductor under the quantised, critical field.

I. THE ASSUMPTION

"Quantum mechanical tunnelling^{1,3} only occurs at the limit of the very small lattice separation², which is an analogy to the perfect delocalisation. In contrast, the quantum tunnelling is forbidden at the limit of the very large lattice separation which is an analogy to the perfect localisation".

II. MY THESIS

- Consider 3 regions, two atoms and a separation (a) in between them, one can solve a quantum tunnelling³ equations for these 3 regions:

1. Region 1: atom 1, which has a negative potential ($-V$) since the attractive interaction between the negative electron and the positive nucleus. Hence, it will be considered a potential well. Then, the wavefunction of the electron on region 1, in the range: $x < 0$, will be:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi_1}{dx^2} - V = E \psi_1,$$

2. Region 2: separation (a) between atom (1) and atom (2), which has a positive potential ($V_0 > E$) since the strong

repulsive interaction between the electrons passing, and between the neighbouring positive nuclei in this separation. Hence it will be considered a potential barrier. Then, the wavefunction of the electron on region 2, in the range: $0 < x < a$, will be:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi_2}{dx^2} + V_0 = E \psi_2,$$

3. Region 3: atom 3, which has a negative potential ($-V$) since the attractive interaction between the negative electron and the positive nucleus. Hence, it will be considered a potential well. Then, the wavefunction of the electron on region 1, in the range: $x < a$, will be:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi_3}{dx^2} - V = E \psi_3.$$

III. THE MATHEMATICAL PROOF

Solutions to the the independent Schrodinger equation in each region are most generally³,

Region 1: $x < 0$

$$\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x},$$

Region 2: $0 < x < a$

$$\psi_2 = C e^{k_2 x} + D e^{-k_2 x},$$

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Region 3: $a < x$

$$\psi_3 = Fe^{ik_1x} + Ge^{-ik_1x},$$

Where,

$$K_1 = K_3 = \sqrt{\frac{2m(E+V)}{\hbar^2}},$$

Next,

$$K_2 = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}.$$

The derivatives of the functions are,

Region 1: $x < 0$

$$\frac{d\psi_1}{dx} = Aik_1e^{ik_1x} - Bik_1e^{-ik_1x},$$

Region 2: $0 < x < a$

$$\frac{d\psi_2}{dx} = Ck_2e^{k_2x} - Dk_2e^{-k_2x},$$

Region 3: $a < x$

$$\frac{d\psi_3}{dx} = Fik_1e^{ik_1x} - Gik_1e^{-ik_1x}.$$

Since (ψ) and its derivative are finite, single-valued and continuous, we can apply the following continuity relationships,

$$[\psi_1]_{x=0} = [\psi_2]_{x=0}, \quad (1)$$

$$[\psi_2]_{x=a} = [\psi_3]_{x=a}, \quad (2)$$

$$\left[\frac{d\psi_1}{dx}\right]_{x=0} = \left[\frac{d\psi_2}{dx}\right]_{x=0}, \quad (3)$$

$$\left[\frac{d\psi_2}{dx}\right]_{x=a} = \left[\frac{d\psi_3}{dx}\right]_{x=a}. \quad (4)$$

Since there should be no particles coming in from the far right moving in the negative x-direction, the constant (G) must be zero. When, applying the continuity relationships on the equations (1), (2), (3) and (4),

$$A + B = C + D, \quad (5)$$

$$iK_1A - iK_1B = K_2C - K_2D, \quad (6)$$

$$Ce^{k_2a} + Dke^{-k_2a} = Fe^{ik_1a}, \quad (7)$$

$$Ck_2e^{k_2a} - Dk_2e^{-k_2a} = iFk_1e^{ik_1a}. \quad (8)$$

Hence, equations (5), (6), (7) and (8) are the four necessary relationships between the five arbitrary constants A , B , C , D and F . When rearranging equation (6),

$$A - B = \frac{k_2}{ik_1}(C - D). \quad (9)$$

Then, adding equation (9) to (5),

$$2A = C\left(1 + \frac{k_2}{ik_1}\right) + D\left(1 - \frac{k_2}{ik_1}\right). \quad (10)$$

Then, rearranging equation (8),

$$Ce^{k_2a} - De^{-k_2a} = \left(\frac{ik_1}{k_2}\right)Fe^{ik_1a}. \quad (11)$$

Then, adding equation (7) to equation (11) and subtract them after,

$$C = \left(1 + \frac{ik_1}{k_2}\right)\left(\frac{e^{ik_1a}}{2e^{k_2a}}\right)F, \quad (12)$$

$$D = \left(1 - \frac{ik_1}{k_2}\right)\left(\frac{e^{ik_1a}}{2e^{-k_2a}}\right)F. \quad (13)$$

Then, substituting the value of C and D into equation (10),

$$2A = \left(1 + \frac{k_2}{ik_1}\right)\left(1 + \frac{ik_1}{k_2}\right)\left(\frac{e^{ik_1a}}{2e^{k_2a}}\right)F + \left(1 - \frac{k_2}{ik_1}\right)\left(1 - \frac{k_2}{ik_1}\right)\left(\frac{e^{ik_1a}}{2e^{-k_2a}}\right)F.$$

Then, factorising $[F(\frac{e^{ik_1a}}{2})]$,

$$2A = F\left(\frac{e^{ik_1a}}{2}\right)\left[\frac{\left(1 + \frac{k_2}{ik_1}\right)\left(1 + \frac{ik_1}{k_2}\right)}{e^{k_2a}} + \frac{\left(1 - \frac{ik_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right)}{e^{-k_2a}}\right].$$

Then, rearranging to find the transmission coefficient ($\frac{F}{A}$),

$$\frac{F}{A} = 4e^{-ik_1a}\left[\frac{1}{\left(1 + \frac{k_2}{ik_1}\right)\left(1 + \frac{ik_1}{k_2}\right)e^{-k_2a} + \left(1 - \frac{ik_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right)e^{k_2a}}\right].$$

Then, expanding the brackets,

$$\frac{F}{A} = 4e^{-ik_1a}\left[\frac{1}{\left(2 + \frac{k_2}{ik_1} + \frac{ik_1}{k_2}\right)e^{-k_2a} + \left(2 - \frac{ik_1}{k_2} - \frac{k_2}{ik_1}\right)e^{k_2a}}\right].$$

Then, taking (ik_1k_2) as a common factor,

$$\frac{F}{A} = 4ik_1k_2e^{-ik_1a} \times \left[\frac{1}{(2ik_1k_2 - k_1^2 + k_2^2)e^{-k_2a} + (2ik_1k_2 + k_1^2 - k_2^2)e^{k_2a}} \right].$$

Then, expanding,

$$\frac{F}{A} = 4ik_1k_2e^{-ik_1a} \times \left[\frac{1}{(2ik_1k_2e^{k_2a} - k_1^2e^{-k_2a} + k_2^2e^{-k_2a}) + (2ik_1k_2e^{k_2a} + k_1^2e^{k_2a} - k_2^2e^{k_2a})} \right].$$

Then, rearranging,

$$\frac{F}{A} = 4ik_1k_2e^{-ik_1a} \times \left[\frac{1}{(2ik_1k_2e^{-k_2a} + 2ik_1k_2e^{k_2a}) + (k_1^2e^{-k_2a} - k_1^2e^{k_2a}) + (k_2^2e^{-k_2a} - k_2^2e^{k_2a})} \right].$$

$$\frac{F}{A} = 4ik_1k_2e^{-ik_1a} \times \left[\frac{1}{2ik_1k_2(e^{-k_2a} + e^{k_2a}) + k_1^2(e^{-k_2a} - e^{k_2a}) + k_2^2(e^{-k_2a} - e^{k_2a})} \right].$$

$$\frac{F}{A} = 4ik_1k_2e^{-ik_1a} \times \left[\frac{1}{2ik_1k_2(e^{-k_2a} + e^{k_2a}) + (k_1^2 + k_2^2)(e^{-k_2a} - e^{k_2a})} \right].$$

Using the complex trigonometric identities,

$$\frac{F}{A} = 4ik_1k_2e^{-ik_1a} \left[\frac{1}{4ik_1k_2 \cosh(k_2a) - 2(k_1^2 + k_2^2) \sinh(k_2a)} \right].$$

Simplifying the equation,

$$\frac{F}{A} = \frac{e^{-ik_1a}}{\cosh(k_2a) + \frac{i}{2} \frac{(k_1^2 + k_2^2)}{k_1k_2} \sinh(k_2a)}.$$

$$\frac{F}{A} = \frac{e^{-ik_1a}}{\cosh(k_2a) + \frac{i}{2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh(k_2a)}.$$

Then, multiplying by the complex conjugate,

$$\left| \frac{F}{A} \right|^2 = \frac{e^{-ik_1a} e^{ik_1a}}{[\cosh(k_2a) - \frac{i}{2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh(k_2a)][\cosh(k_2a) + \frac{i}{2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh(k_2a)]}. \quad (14)$$

Finally, simplifying,

$$\left| \frac{F}{A} \right|^2 = \frac{1}{\cosh^2(k_2a) + \frac{1}{4} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh^2(k_2a)}. \quad (15)$$

Considering equation (15), $\left| \frac{F}{A} \right|^2$ represents the transmission coefficient, and in our case, I argue that $\left| \frac{F}{A} \right|^2$ will be the delocalisation coefficient. Indeed, I argue that this coefficient depends mainly on the lattice separation a . Where, a very small lattice separation (a) ensures the delocalisation coefficient is very large, i.e the case of conductor. In contrast, a very large a lattice separation (a) ensures the delocalisation coefficient is very low, hence, very high localisation, i.e the case of conductor. In order to prove that, one can take the limit for $\left| \frac{F}{A} \right|^2$ in equation (57) twice. Once, in the limit (a) tends to zero and the second, in the limit (a) tends to infinity.

$$\begin{aligned} \lim_{a \rightarrow 0} \left| \frac{F}{A} \right|^2 &= \lim_{a \rightarrow 0} \frac{1}{\cosh^2(k_2a) + \frac{1}{4} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh^2(k_2a)} \\ &= \frac{1}{\cosh^2(0) + \frac{1}{4} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh^2(0)} = \frac{1}{1+0} = 1 \end{aligned} \quad (16)$$

Equation (16), ensures that at very low lattice separation, the delocalisation is perfect. Hence, this is the case of a conductor.

$$\begin{aligned} \lim_{a \rightarrow \infty} \left| \frac{F}{A} \right|^2 &= \lim_{a \rightarrow \infty} \frac{1}{\cosh^2(k_2a) + \frac{1}{4} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh^2(k_2a)} \\ &= \frac{1}{\cosh^2(\infty) + \frac{1}{4} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \sinh^2(\infty)} = \frac{1}{\infty + \infty} = 0 \end{aligned} \quad (17)$$

Equation (17), ensures that at very large lattice separation, the delocalisation is zero. Hence, the localisation coefficient is perfect, and indeed this is the case of an insulator.

IV. THE PROPOSED SOLUTION

Since, at very large lattice separation, the conductor becomes an insulator. Hence, at very small lattice separation, the insulator becomes a conductor.

Therefore, I proposed that if we apply a critical, negative potential, ($-V_c$), such that the overall potential in the separation (a) between the two atoms becomes negative,

$$-V_c + V_0 = -V_i$$

In fact, the lattice separation is induced to be converted from being a potential barrier with potential energy (V_0), into a potential well with potential energy ($-V_i$)

V. MY THESIS

When applying the critical, negative potential (V_c), that will induce a change of the overall potential in region 2. Therefore,

the separation (a) between atom (1) and atom (2), will have an induced, negative potential, ($-V_i$) such that ($E > -V_i$). Hence it will be considered a potential well. Then, the wavefunction of the electron on region 2, in the range: $0 < x < a$, will be:

$$\frac{-\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} - V_i = E\psi_2,$$

VI. THE MATHEMATICAL PROOF

Solutions to the the independent Schrodinger equation in each region are most generally,

Region 1: $x < 0$

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x},$$

Region 2: $0 < x < a$

$$\psi_2 = Ce^{ik_2x} + De^{-ik_2x},$$

Region 3: $a < x$

$$\psi_3 = Fe^{ik_1x} + Ge^{-ik_1x}.$$

Where,

$$K_1 = K_3 = \sqrt{\frac{2m(E + V)}{\hbar^2}},$$

And,

$$K_2 = \sqrt{\frac{2m(E + V_i)}{\hbar^2}}.$$

The derivatives of the functions are,

Region 1: $x < 0$

$$\frac{d\psi_1}{dx} = Aik_1e^{ik_1x} - Bik_1e^{-ik_1x},$$

Region 2: $0 < x < a$

$$\frac{d\psi_2}{dx} = Cik_2e^{ik_2x} - Dik_2e^{-ik_2x},$$

Region 3: $a < x$

$$\frac{d\psi_3}{dx} = Fik_1e^{ik_1x} - Gik_1e^{-ik_1x}.$$

Since (ψ) and its derivative are finite, single-valued and continuous, we can apply the following continuity relationships,

$$[\psi_1]_{x=0} = [\psi_2]_{x=0}, \quad (18)$$

$$[\psi_2]_{x=a} = [\psi_3]_{x=a}, \quad (19)$$

$$\left[\frac{d\psi_1}{dx}\right]_{x=0} = \left[\frac{d\psi_2}{dx}\right]_{x=0}, \quad (20)$$

$$\left[\frac{d\psi_2}{dx}\right]_{x=a} = \left[\frac{d\psi_3}{dx}\right]_{x=a}. \quad (21)$$

Since there should be no particles coming in from the far right moving in the negative x-direction, the constant (G) must be zero. When, applying the continuity relationships on the equations (18), (19), (20) and (21), then, dividing all by (i),

$$A + B = C + D, \quad (22)$$

$$K_1A - K_1B = K_2C - K_2D, \quad (23)$$

$$Ce^{ik_2a} + Dke^{-ik_2a} = Fe^{ik_1a}, \quad (24)$$

$$Ck_2e^{ik_2a} - Dk_2e^{-ik_2a} = Fk_1e^{ik_1a}. \quad (25)$$

Hence, equations (22), (23), (24), and (25) are the four necessary relationships between the five arbitrary constants A, B, C, D and F . When rearranging equation (23),

$$A - B = \frac{k_2}{k_1}(C - D). \quad (26)$$

Then, adding equation (26) to (22),

$$2A = C\left(1 + \frac{k_2}{k_1}\right) + D\left(1 - \frac{k_2}{k_1}\right). \quad (27)$$

Then, rearranging equation (25),

$$Ce^{ik_2a} - De^{-ik_2a} = \left(\frac{k_1}{k_2}\right)Fe^{ik_1a}. \quad (28)$$

Then, adding equation (24) to equation (28), and subtract them after,

$$C = \left(1 + \frac{k_1}{k_2}\right)\left(\frac{e^{ik_1a}}{2e^{ik_2a}}\right)F.$$

$$D = \left(1 - \frac{k_1}{k_2}\right)\left(\frac{e^{ik_1a}}{2e^{-ik_2a}}\right)F.$$

Then, substituting the value of (C) and (D) into equation (27),

$$2A = \left(1 + \frac{k_2}{k_1}\right)\left(1 + \frac{k_1}{k_2}\right)\left(\frac{e^{ik_1a}}{2e^{ik_2a}}\right)F + \left(1 - \frac{k_1}{k_2}\right)\left(1 - \frac{k_2}{k_1}\right)\left(\frac{e^{ik_1a}}{2e^{-ik_2a}}\right)F$$

Then, factorising $[F(\frac{e^{ik_1a}}{2})]$,

$$2A = F(\frac{e^{ik_1a}}{2})[\frac{(1+\frac{k_2}{k_1})(1+\frac{k_1}{k_2})}{e^{ik_2a}} + \frac{(1-\frac{k_1}{k_2})(1-\frac{k_2}{k_1})}{e^{-ik_2a}}].$$

Then, rearranging to find the transmission coefficient ($\frac{F}{A}$),

$$\frac{F}{A} = 4e^{-ik_1a}[\frac{1}{(1+\frac{k_2}{k_1})(1+\frac{k_1}{k_2})e^{-ik_2a} + (1-\frac{k_1}{k_2})(1-\frac{k_2}{k_1})e^{ik_2a}}].$$

Then, expanding the brackets,

$$\frac{F}{A} = 4e^{-ik_1a}[\frac{1}{(2+\frac{k_2}{k_1}+\frac{k_1}{k_2})e^{-ik_2a} + (2-\frac{k_1}{k_2}-\frac{k_2}{k_1})e^{ik_2a}}].$$

Then, taking (k_1k_2) as a common factor,

$$\frac{F}{A} = 4k_1k_2e^{-ik_1a}[\frac{1}{(2k_1k_2+k_1^2+k_2^2)e^{-ik_2a} + (2k_1k_2-k_1^2-k_2^2)e^{ik_2a}}].$$

Then, expanding,

$$\frac{F}{A} = 4k_1k_2e^{-ik_1a} \times [\frac{1}{(2k_1k_2+k_1^2+k_2^2)e^{-ik_2a} + (2k_1k_2-k_1^2-k_2^2)e^{ik_2a}}],$$

$$\frac{F}{A} = 4k_1k_2e^{-ik_1a} \times [\frac{1}{(2k_1k_2e^{-ik_2a}+k_1^2e^{-ik_2a}+k_2^2e^{-ik_2a})+(2k_1k_2e^{ik_2a}-k_1^2e^{ik_2a}-k_2^2e^{ik_2a})}].$$

Then, rearranging,

$$\frac{F}{A} = 4k_1k_2e^{-ik_1a} \times [\frac{1}{(2k_1k_2e^{-ik_2a}+2k_1k_2e^{ik_2a})+(k_1^2e^{-ik_2a}-k_1^2e^{ik_2a})+(k_2^2e^{-ik_2a}-k_2^2e^{ik_2a})}],$$

$$\frac{F}{A} = 4k_1k_2e^{-ik_1a} \times [\frac{1}{2k_1k_2(e^{-ik_2a}+e^{ik_2a})+k_1^2(e^{-ik_2a}-e^{ik_2a})+k_2^2(e^{-ik_2a}-e^{ik_2a})}],$$

$$\frac{F}{A} = 4k_1k_2e^{ik_1a} \times [\frac{1}{2k_1k_2(e^{-ik_2a}+e^{ik_2a})+(k_1^2+k_2^2)(e^{-ik_2a}-e^{ik_2a})}].$$

Using the complex trigonometric identities,

$$\frac{F}{A} = 4k_1k_2e^{-ik_1a}[\frac{1}{4k_1k_2\cos(k_2a) - 2i(k_1^2+k_2^2)\sin(k_2a)}]$$

Simplifying the equation,

$$\frac{F}{A} = \frac{e^{-ik_1a}}{\cos(k_2a) - \frac{i}{2}(\frac{k_1^2+k_2^2}{k_1k_2})\sin(k_2a)},$$

$$\frac{F}{A} = \frac{e^{-ik_1a}}{\cos(k_2a) - \frac{i}{2}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin(k_2a)}. \quad (29)$$

Then, multiplying equation (29) by the complex conjugate,

$$|\frac{F}{A}|^2 = \frac{e^{-ik_1a}e^{ik_1a}}{[\cos(k_2a) - \frac{i}{2}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin(k_2a)][\cos(k_2a) + \frac{i}{2}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin(k_2a)]}. \quad (30)$$

Finally, simplifying the equation (30),

$$|\frac{F}{A}|^2 = \frac{1}{\cos^2(k_2a) + \frac{1}{4}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin^2(k_2a)}. \quad (31)$$

Then, one can take the limit for $|\frac{F}{A}|^2$ in equation (31) twice. Once, in the limit (a) tends to zero and the second, in the limit (a) tends to infinity.

$$\begin{aligned} \lim_{a \rightarrow 0} |\frac{F}{A}|^2 &= \lim_{a \rightarrow 0} \frac{1}{\cos^2(k_2a) + \frac{1}{4}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin^2(k_2a)} \\ &= \frac{1}{\cos^2(0) + \frac{1}{4}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin^2(0)} = \frac{1}{1+0} = 1 \end{aligned} \quad (32)$$

Equation (32), confirms that at very low lattice separation, the delocalisation is perfect. Hence, this is the case of a conductor.

But, the surprise now, ever at very large lattice separation, one can still get a perfect delocalisation coefficient which is equal to unity, such that:

$$\lim_{a \rightarrow \infty} |\frac{F}{A}|^2 = \lim_{a \rightarrow \infty} \frac{1}{\cos^2(k_2a) + \frac{1}{4}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin^2(k_2a)} = 1 \quad (33)$$

Equation (33), can be achieved if and only if the argument of the $\cos^2$ function is equal to $(n\pi)$. where this critical case implies whatever large the lattice separation is, as long as,

$$k_2a = n\pi$$

Therefore,

$$\cos^2(k_2a) = \cos^2(n\pi) = 1$$

The result is, a perfect delocalisation coefficient such that:

$$|\frac{F}{A}|^2 = \lim_{a \rightarrow \infty} \frac{1}{\cos^2(k_2a) + \frac{1}{4}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin^2(k_2a)}$$

$$= \frac{1}{\cos^2(n\pi) + \frac{1}{4}(\frac{k_1}{k_2} + \frac{k_2}{k_1})\sin^2(n\pi)} = \frac{1}{1+0} = 1$$

Therefore one can find the critical potential (V_c), such that:

$$k_2 a = n\pi$$

$$K_2 = \sqrt{\frac{2m(E + V_i)}{\hbar^2}}$$

Therefore,

$$\sqrt{\frac{2m(E + V_i)}{\hbar^2}} = \frac{n\pi}{a}$$

$$\frac{2m(E + V_i)}{\hbar^2} = \left(\frac{n\pi}{a}\right)^2$$

$$-V_i = E - \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Since,

$$-V_c + V_0 = -V_i$$

Therefore, one needs to apply a critical potential ($-V_c$):

$$-V_c = (E - V_0) - \frac{n^2\pi^2\hbar^2}{2ma^2}$$

One predicts the expression, $(E - V_0)$ is unique for each lattice, which in general is the kinetic energy of the electron in the lattice separation with positive potential (V_0), hence,

$$E - V_0 = K_E = \frac{\hbar^2 K^2}{2m} = \frac{s^2\pi^2\hbar^2}{2ma^2}$$

Where (s) is a specific integer and it is a unique integer for each lattice. Therefore, one can write the critical potential as:

$$-V_c = (s^2 - n^2) \frac{\pi^2\hbar^2}{2ma^2} \quad (34)$$

Equation (35), introduces an explicit form of the quantised critical potential. Therefore, the quantisation of the critical potential ensures that the delocalisation coefficient to be perfect, then, an insulator becomes a conductor.

VII. THE PROTOTYPE: (THE INSUDUCTOR)

1. Consider a type-X insulator.
2. Determine the unique kinetic energy of the electron in the lattice separation

$$E - V_0 = \frac{s^2\pi^2\hbar^2}{2ma^2}$$

, in particular, determine (s), the unique integer for each lattice.

3. A cylindrical interface will contain the insulator inside it to allow to the lattice separations to be parallel to Nano-components which generate a quantised, critical field:

$$-V_c = (s^2 - n^2) \frac{\pi^2\hbar^2}{2ma^2},$$

such that, ($n \neq s$).

4. Then, the delocalisation coefficient becomes one.
5. Therefore, the insulator becomes a conductor, in fact, becomes an Insuductor.

VIII. THE CONCLUSION

1. At very small lattice separation, the quantum mechanical tunnelling occurs with perfect transmission coefficient, hence, with perfect delocalisation, which ensures the case of a conductor.
2. At very large lattice separation, the quantum mechanical tunnelling is forbidden with zero transmission coefficient, hence, with zero delocalisation. Hence, the localisation coefficient is perfect, and indeed this is the case of an insulator.
3. At very large lattice separation, the conductor becomes an insulator.
4. Applying the negative, critical potential on the lattice separation region allows the delocalisation coefficient to be perfect due to the quantisation of the critical potential, then, an insulator becomes a conductor.

ACKNOWLEDGMENTS

I dedicate this research and these results to my beloved mother/ Magda Zaky Kamel and my beloved father/ Magdy Samy Ibrahim; you are the most precious treasure I own.

IX. DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

X. REFERENCES

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