Convertible Bond Valuation Model

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ABSTRACT

This paper presents a new model for pricing convertible bonds. We model both equities and bonds as defaultable in a consistent way. The default proceedings provide a justification for our modeling assumptions: Different classes of securities issued by the same company have the same default probability but different recovery rates. Given this model, we are able to back out the market prices. We find that convertibles have relatively large positive gammas. As a typical convertible arbitrage strategy employs delta-neutral hedging, a large positive gamma can make the portfolio highly profitable, especially for a large movement in the underlying stock price.

Key Words: hybrid financial instrument, convertible bond, convertible underpricing, convertible arbitrage, default time approach, default probability approach, jump diffusion.

1. Introduction

This article makes a theoretical and empirical contribution to the study of convertible bonds. In contrast to the above mentioned literature, we present a model that is based on the probability distribution (or intensity) of a default jump (or a default time) rather than the default jump itself, as the default jump is usually inaccessible (see Duffie and Huang (1996), Jarrow and Protter (2004), etc).

We model both equities and bonds as defaultable in a consistent way. When a firm goes bankrupt, the investors who take the least risk are paid first. Secured creditors have the best chances of seeing the value of their initial investments come back to them. Bondholders have a greater potential for recovering some their losses than stockholders who are last in line to be repaid and usually receive little, if anything. The default proceedings provide a justification for our modeling assumptions: Different classes of securities issued by the same company have the same default probability but different recovery rates. Given this model, we are able to back out the market prices.

Valuation under our risky model can be solved by common numerical methods, such as, Monte Carlo simulation, tree/lattice approaches, or partial differential equation (PDE) solutions. The PDE algorithm is elaborated in this paper, but of course the methodology can be easily extended to tree/lattice or Monte Carlo.

Using the model proposed, we conduct an empirical study of convertible bonds. We obtain a data set from FinPricing (2013). The data set contains 164 convertible bonds and 2 years of daily market prices as well as associated interest rate curves, credit curves, stock prices, implied Black-Scholes volatilities and recovery rates.

The most important parameter to be determined is the volatility input for valuation. A common approach in the market is to use the at-the-money (ATM) implied Black-Scholes volatility to price convertible bonds. However, most liquid stock options have relatively short maturates (rarely more than 8 years). As a result, some authors, such as Ammann *et al* (2003), Loncarski *et al* (2009), Zabolotnyuk *et al* (2010), have to make do with historical volatilities. Therefore, we segment the sample into two sets

according to maturity: a short-maturity class (0 ~ 8 years) and a long-maturity class (> 8 years). For the short-maturity class, we use the ATM implied Black-Scholes volatility for valuation, whereas for the long-maturity class, we calculate the historical volatility as the annualized standard deviation of the daily log returns of the last 2 years and then price the convertible bond based on this real-world volatility.

The empirical results show that the model prices fluctuate randomly around the market prices, indicating the model is quite accurate. Our empirical evidence does not support a systematic underpricing hypothesis. A similar conclusion is reached by Ammann *et al* (2008) who use a Monte-Carlo simulation approach. Moreover, market participants almost always calibrate their models to the observed market prices using implied convertible volatilities. Therefore, underpricing may not be the main driver of profitability in convertible arbitrage.

It is useful to examine the basics of the convertible arbitrage strategy. A typical convertible bond arbitrage employs delta-neutral hedging, in which an arbitrageur buys a convertible bond and sells the underlying equity at the current delta (see Choi *et al* (2009), Loncarski *et al* (2009), etc.). With delta neutral positions, the sign of Gamma is important. If Gamma is negative, the portfolio profits so long as the underlying equity remains stable. If Gamma is positive, the portfolio will profit from large movements in the stock price in either direction (Somanath, 2011).

We study the sensitivities of convertible bonds and find that convertible bonds have relatively large positive gammas, implying that convertible arbitrage can make a profit on a large upside or downside movement in the underlying stock price. Since convertible bonds are issued mainly by start-up or small companies (while more established firms rely on other means of financing), the chance of a large movement in either direction is very likely. Even for very small movements in the underlying stock price, profits can still be generated from the yield of the convertible bond and the interest rebate for the short position.

2 Model

Three sources of randomness exist in a convertible bond: the stock price, the interest rate, and the credit spread. Grimwood and Hodges (2002) conduct a sensitivity study and find that accurately modeling the equity process appears crucial. This is why all convertible bond models in the market capture, at a minimum, the dynamics of the underlying equity price. Since convertible bonds are issued mainly by start-up or small companies (while more established firms rely on other means of financing), credit risk plays an important role in the valuation. Grimwood and Hodges (2002) further note that the interest rate process is of second order importance. Similarly, Brennan and Schwartz (1980) conclude that the effect of a stochastic interest rate on convertible bond prices is so small that it can be neglected. Furthermore, Ammann *et al* (2008) notice that the overall pricing benefit of incorporating stochastic interest rates would be very limited and would not justify the additional computational costs. For these reasons, most practical convertible models in the market do not take stochastic interest rate into account.

The risk-free stock price process can be described as

$$dS(t) = r(t)S(t)dt + \sigma S(t)dW(t) \quad (1)$$

where $S(t)$ denotes the stock price, $r(t)$ denotes the risk-free interest rate, σ denotes the volatility, $W(t)$ denotes a Wiener process.

The expectation of equation (1) is

$$E(dS(t)|\mathcal{F}_t) = r(t)S(t)dt \quad (2)$$

where $E\{\bullet|\mathcal{F}_t\}$ is the expectation conditional on the $\mathcal{F}_t$.

Equation (2) tells us that in a risk-neutral world, the expected return on a risk-free stock is the risk-free interest rate $r(t)$, i.e., the discounted stock price under the risk neutral measure is a martingale process.

Next, we turn to a defaultable stock. The defaultable stock process proposed by Davis and Lischka (1999), Andersen and Buffum (2004), and Bloomberg (2009), etc., is given by

$$dS(t) = (r(t) + h(t))S(t_-)dt + \hat{\sigma}S(t_-)dW(t) - S(t_-)dU(t) \quad (3)$$

where $U(t)$ is an independent Poisson process with $dU(t)=1$ with probability $h(t)dt$ and 0 otherwise, $h(t)$ is the hazard rate or the default intensity, $S(t_-)$ is the stock price immediately before any jump at time t . The expectation of $dU(t)$ is $E(dU(t)|\mathcal{F}_t) = h(t)dt$.

The expectation of equation (3) is given by

$$E(dS(t)|\mathcal{F}_t) = (r(t) + h(t))S(t)dt - S(t)h(t)dt = r(t)S(t)dt \quad (4)$$

It is shown in equation (4) that the expected return of a defaultable stock under a jump-diffusion model also grows at the risk-free interest rate. Equation (3) is a simpler version of the Merton's Jump-diffusion model where the number of jumps is 1.

The jump-diffusion model was first proposed in the context of market risk, which naturally exhibits high skewness and leptokurtosis levels and captures the so-called implied volatility smile or skew effects. Ederington and Lee (1993) find that the markets tend to have overreaction and underreaction to the outside news. The jump part of the model can be interpreted as the market response to outside news. If there is not any outside news, the asset price changes according to a geometric Brownian motion. Since the market price of risk and risk preferences are irrelevant to asset pricing within the market risk context, the expected rate of return to the stockholders is equal to the risk-free rate under a risk-neutral measure.

However, we wonder whether it is appropriate to propagate the jump-diffusion model directly from the market risk domain to the credit risk domain, as credit risk actually impacts the valuation of assets. This is why financial institutions are required by regulators to report CVA. In fact, we will show in the following derivation that the expected return of a defaultable asset under a risk-neutral measure is actually equal to a risky rate instead of the risk-free rate. This conclusion is very important for risky valuation.

The world of credit modeling is divided into two main approaches: structural models and reduced-form (or intensity) models. The structural models regard default as an endogenous event, focusing on the capital structure of a firm. The reduced-form models do not explain the event of default endogenously, but instead characterize it exogenously as a jump process. In general, structural models are based on the information set available to the firm's management, such as the firm's assets and liabilities; while reduced-

form models are based on the information set available to the market, such as the firm's bond prices or credit default swap (CDS) premia. Many practitioners in the credit trading arena have tended to gravitate toward the reduced-form models given their mathematical tractability. The reduced-form models can be made consistent with the risk-neutral probabilities of default backed out from corporate bond prices or CDS spreads/premia.

In the reduced-form models, the stopping (or default) time τ of a firm is modeled as a Cox arrival process (also known as a doubly stochastic Poisson process) whose first jump occurs at default and is defined as,

$$\tau = \inf \left\{ t : \int_0^t h(s, \Phi_s) ds \geq \Delta \right\} \quad (5)$$

where $h(t)$ or $h(t, \Phi_t)$ denotes the stochastic hazard rate or arrival intensity dependent on an exogenous common state Φ_t , and Δ is a unit exponential random variable independent of Φ_t .

It is well-known that the survival probability from time t to s in this framework is defined by

$$p(t, s) := P(\tau > s \mid \tau > t, Z) = \exp\left(-\int_t^s h(u) du\right) \quad (6)$$

The default probability for the period (t, s) in this framework is given by

$$q(t, s) := P(\tau \leq s \mid \tau > t, Z) = 1 - p(t, s) = 1 - \exp\left(-\int_t^s h(u) du\right) \quad (7)$$

We consider a defaultable asset that pays nothing between dates t and T . Let $V(t)$ and $V(T)$ denote its values at t and T , respectively. Risky valuation can be generally classified into two categories: the *default time approach* (DTA) and the *default probability (intensity) approach* (DPA).

The DTA involves the default time explicitly. If there has been no default before time T (i.e., $\tau > T$), the value of the asset at T is $V(T)$. If a default happens before T (i.e., $t < \tau \leq T$), a recovery payoff is made at the default time τ as a fraction of the market value¹ given by $\phi V(\tau)$ where ϕ is the default recovery

¹ Here we use the recovery of market value (RMV) assumption.

rate and $V(\tau)$ is the market value at default. Under a risk-neutral measure, the value of this defaultable asset is the discounted expectation of all the payoffs and is given by

$$V(t) = E\left[\left(D(t, T)V(T)1_{\tau > T} + D(t, \tau)\phi V(\tau)1_{\tau \leq T}\right) | \mathcal{F}_t\right] \quad (8)$$

where 1_Y is an indicator function that is equal to one if Y is true and zero otherwise, and $D(t, \tau)$ denotes the stochastic risk-free discount factor at t for the maturity τ given by

$$D(t, \tau) = \exp\left[-\int_t^\tau r(u)du\right] \quad (9)$$

Although the DTA is very intuitive, it has the disadvantage that it explicitly involves the default time/jump. We are very unlikely to have complete information about a firm's default point, which is often inaccessible. Moreover, in a derivative transaction, the market-value-at-default $V(\tau)$ usually reflects the replacement cost of the transaction, where the replacement is also defaultable². Therefore $V(\tau)$ should be determined via another risky valuation and so forth. Usually, valuation under the DTA is performed via Monte Carlo simulation.

The DPA relies on the probability distribution of the default time rather than the default time itself. We divide the time period (t, T) into n very small time intervals (Δt) and assume that a default may occur only at the end of each very small period. In our derivation, we use the approximation $\exp(y) \approx 1 + y$ for very small y . The survival and default probabilities for the period $(t, t + \Delta t)$ are given by

$$\hat{p}(t) := p(t, t + \Delta t) = \exp(-h(t)\Delta t) \approx 1 - h(t)\Delta t \quad (10)$$

$$\hat{q}(t) := q(t, t + \Delta t) = 1 - \exp(-h(t)\Delta t) \approx h(t)\Delta t \quad (11)$$

The binomial default rule considers only two possible states: default or survival. For the one-period $(t, t + \Delta t)$ economy, at time $t + \Delta t$ the asset either defaults with the default probability $q(t, t + \Delta t)$ or

² Many people in the market use the risk-free value as the market-value-at-default, which is inappropriate as any contract in the OTC market is risky when taking counterparty risk into account.

survives with the survival probability $p(t, t + \Delta t)$. The survival payoff is equal to the market value $V(t + \Delta t)$ and the default payoff is a fraction of the market value: $\phi(t + \Delta t)V(t + \Delta t)$. Under a risk-neutral measure, the value of the asset at t is the expectation of all the payoffs discounted at the risk-free rate and is given by

$$V(t) = E\left\{\exp(-r(t)\Delta t) [\hat{p}(t) + \phi(t)\hat{q}(t)]V(t + \Delta t) \middle| \mathcal{F}_t\right\} \approx E\left\{\exp(-y(t)\Delta t)V(t + \Delta t) \middle| \mathcal{F}_t\right\} \quad (12)$$

where $y(t) = r(t) + h(t)(1 - \phi(t)) = r(t) + c(t)$ denotes the risky rate and $c(t) = h(t)(1 - \phi(t))$ is called the (short) credit spread.

Similarly, we have

$$V(t + \Delta t) = E\left\{\exp(-y(t + \Delta t)\Delta t)V(t + 2\Delta t) \middle| \mathcal{F}_{t+\Delta t}\right\} \quad (13)$$

Note that $\exp(-y(t)\Delta t)$ is $\mathcal{F}_{t+\Delta t}$ -measurable. By definition, an $\mathcal{F}_{t+\Delta t}$ -measurable random variable is a random variable whose value is known at time $t + \Delta t$. Based on the *taking out what is known* and *tower* properties of conditional expectation, we have

$$\begin{aligned} V(t) &= E\left\{\exp(-y(t)\Delta t)V(t + \Delta t) \middle| \mathcal{F}_t\right\} \\ &= E\left\{\exp(-y(t)\Delta t)E\left[\exp(-y(t + \Delta t)\Delta t)V(t + 2\Delta t) \middle| \mathcal{F}_{t+\Delta t}\right] \middle| \mathcal{F}_t\right\} \\ &= E\left\{\exp\left(-\sum_{i=0}^1 y(t + i\Delta t)\Delta t\right)V(t + 2\Delta t) \middle| \mathcal{F}_t\right\} \end{aligned} \quad (14)$$

By recursively deriving from t forward over T and taking the limit as Δt approaches zero, the risky value of the asset can be expressed as

$$V(t) = E\left\{\exp\left[-\int_t^T y(u)du\right]V(T) \middle| \mathcal{F}_t\right\} \quad (15)$$

Using the DPA, we obtain a closed-form solution for pricing an asset subject to credit risk. Another good example of the DPA is the CDS model proposed by J.P. Morgan (1999).

The derivation of equation (15) takes into account all credit characteristics: possibility of a jump to default and recovery rate. It tells us that a defaultable asset under the risk-neutral measure grows at a *risky rate*. The risky rate is equal to a risk-free interest rate plus a credit spread. If the asset is a bond, the equation is the same as Equation (10) in Duffie and Singleton (1999), which is the market model for pricing risky

bonds. The market bond model says that the value of a risky bond is obtained by discounting the promised payoff using the risk-free interest rate plus the credit spread³.

Under a risk-neutral measure the market price of risk and risk preferences are irrelevant to asset pricing (Hull, 2003) and thereby the expectation of a risk-free asset grows at the risk-free interest rate. However, credit risk actually has a significant impact on asset prices. This is the reason that regulators, such as IASB and BCBS, require financial institutions to report a CVA in addition to the risk-free MTM value to reflect credit risk.

In asset pricing theory, the fundamental no-arbitrage theorems do not require expected returns to be equal to the risk free rate, but only that prices are martingales after discounting under the numeraire. For risk-free valuation, people commonly use a risk-free bond as the numeraire, whereas for risky valuation, they should choose an associated risky numeraire to reflect credit risk. The expected return is that of the numeraire.

If a company files bankruptcy, both bonds and stocks go into a default status. In other words, the default probabilities for both of them are the same (i.e., equal to the firm's probability of default). But the recovery rates are different because the stockholders are the lowest priority in the list of the stakeholders in the company, whereas the bondholders have a higher priority to receive a higher percentage of invested funds. The default proceedings provide a justification for our modeling assumptions: Different classes of securities issued by the same company have the same default probability but different recovery rates.

According to equation (15), we propose a risky model that embeds the probability of the default jump rather than the default jump itself into the price dynamics of an asset. The stochastic differential equation (SDE) of a defaultable stock is defined as

$$dS(t) = (r(t) + h(t)(1 - \varphi_s(t)))S(t)dt + \sigma S(t)dW(t) = y(t)S(t)dt + \sigma S(t)dW(t) \quad (16)$$

where φ_s is the recovery rate of the stock and $y(t) = r(t) + h(t)(1 - \varphi_s(t))$ is the risky rate.

³ There is a liquidity component in the bond spread. This paper, however, focuses on credit risk only.

For most practical problems, zero recovery at default (or jump to zero) is unrealistic. For example, the stock of Lehman Brothers fell 94.3% on September 15, 2008 after the company filed for Chapter 11 bankruptcy. Similarly, the shares of General Motors (GM) plunged 32% on June 1, 2009 after the firm initiated Chapter 11 bankruptcy. A good framework should flexibly allow people to incorporate different recovery assumptions into risky valuation.

Equation (16) is the direct derivation of equation (15). The formula allows different assumptions concerning recovery on default. In particular, $\varphi_s = 0$ represents the situation where the stock price jumps to 0, and $\varphi_s = 1$ corresponds to the risk-free case. The expectation of equations (16) is

$$E(dS(t)|\mathcal{F}_t) = (r(t) + h(t)(1 - \varphi_s(t)))S(t)dt \quad (17)$$

Equation (17) says that the expected return of a stock subject to credit risk is equal to a risky rate rather than the risk-free rate. The risky rate reflects the compensation investors receive for bearing credit risk.

3. PDE Algorithm

The defaultable stock price process is given by

$$dS(t) = (r(t) - q(t) + h(t)(1 - \varphi_s(t)))S(t)dt + \sigma S(t)dW(t) = \mu(t)S(t)dt + \sigma S(t)dW(t) \quad (18)$$

where $q(t)$ is the dividend and $\mu(t) = r(t) - q(t) + h(t)(1 - \varphi_s(t))$.

Suppose that $G(S, t)$ is some function of S and t . Applying Ito Lemma, we have

$$dG = \left(\mu S \frac{\partial G}{\partial S} + \frac{\partial G}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} \right) dt + \sigma S \frac{\partial G}{\partial S} dW \quad (19)$$

Since the Wiener process underlying S and G are the same, we can construct the following portfolio so that the Wiener process can be eliminated.

$$X = G - S \frac{\partial G}{\partial S} \quad (20)$$

Therefore, we have

$$dX = dG - \frac{\partial G}{\partial S} dS = \left(\frac{\partial G}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} \right) dt \quad (21)$$

In contrast to all previous studies, we believe that the defaultable equity should grow at the risky rate of the equity including dividends, whereas the equity part of the convertible bond should earn the risky rate of the equity excluding dividends, i.e.,

$$(r + h(1 - \varphi_s))Gdt - (r - q + h(1 - \varphi_s)) \frac{\partial G}{\partial S} Sdt = dX = \left(\frac{\partial G}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} \right) dt \quad (22)$$

So that the PDE of the equity component is given by

$$\frac{\partial G}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} + (r - q + h(1 - \varphi_s))S \frac{\partial G}{\partial S} - (r + h(1 - \varphi_s))G = 0 \quad (23)$$

Similarly applying Ito Lemma to the bond part of the convertible $B(S, t)$, we obtain

$$dB = \left(\mu S \frac{\partial B}{\partial S} + \frac{\partial B}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 B}{\partial S^2} \right) dt + \sigma S \frac{\partial B}{\partial S} dW \quad (24)$$

Let us construct a portfolio so that we can eliminate the Wiener process as follows

$$Y = B - S \frac{\partial B}{\partial S} \quad (25)$$

Thus, we have

$$dY = dB - \frac{\partial B}{\partial S} dS = \left(\frac{\partial B}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 B}{\partial S^2} \right) dt \quad (26)$$

The defaultable equity should grow at the risky rate of the equity including dividends, while the bond part of the convertible bond grows at the risky rate of the bond. Consequently, we have

$$(r + h(1 - \varphi_b))Bdt - (r - q + h(1 - \varphi_s)) \frac{\partial B}{\partial S} Sdt = dY = \left(\frac{\partial B}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 B}{\partial S^2} \right) dt \quad (27)$$

where φ_b is the recovery rate of the bond.

The PDE of the bond component is

$$\frac{\partial B}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 B}{\partial S^2} + (r - q + h(1 - \varphi_s))S \frac{\partial B}{\partial S} - (r + h(1 - \varphi_b))B = 0 \quad (28)$$

Equations (23) and (28) are coupled through appropriate final and boundary conditions reflecting the terms and conditions of each individual convertible and need to be solved simultaneously. Convertible bonds often incorporate various additional features, such as call and put provisions.

The final conditions at maturity T can be generalized as

$$G_T = \begin{cases} \eta S_T, & \text{if } \eta S_T > \min[P_c, \max(P_p, N + C)] \\ 0, & \text{otherwise} \end{cases} \quad (29)$$

$$B_T = \begin{cases} \min[P_c, \max(P_p, N + C)], & \text{if } \eta S_T \leq \min[P_c, \max(P_p, N + C)] \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

where N denotes the bond principal, C denotes the coupon, P_c denotes the call price, P_p denotes the put price and η denotes the conversion ratio. The final conditions tell us that the convertible bond at the maturity is either a debt or an equity.

The upside constraints at time $t \in [0, T]$ are

$$\begin{cases} G_t = \eta S_t, B_t = 0 & \text{if } \eta S_t > \min[P_c, \max(P_p, \tilde{L}_t)] \\ G_t = 0, B_t = P_p & \text{else if } \tilde{L}_t \leq P_p \\ G_t = 0, B_t = P_c & \text{else if } \tilde{L}_t \geq P_c \\ G_t = \tilde{G}_t, B_t = \tilde{B}_t & \text{else} \end{cases} \quad (31)$$

where $\tilde{L}_t = \tilde{B}_t + \tilde{G}_t$ is the continuation value of the convertible bond, $\tilde{B}_t$ is the continuation value of the bond component and $\tilde{G}_t$ is the continuation value of the equity component. Equation (31) says that the convertible is either in the continuation region or one of the three constraints (called, put or converted). One can use finite difference methods to solve the PDEs (23) and (28) for the price of a convertible bond.

4. Conclusion

Our study shows that risky asset pricing is quite different from risk-free asset pricing. In fact, the expectation of a defaultable asset actually grows at a risky rate rather than the risk-free rate. This conclusion is vital for risky valuation.

We propose a hybrid framework to value risky equities and debts in a unified way. The model relies on the probability distribution of the default jump rather than the default jump itself. As such, the model can achieve a high order of accuracy with a relatively easy implementation.

Empirically, we do not find evidence supporting a systematic underpricing hypothesis. We also find that convertible bonds have relatively large positive gammas, implying that convertible arbitrage can make a significant profit on a large upside or downside movement in the underlying stock price.

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