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[knowledge base]
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Vector space over a field

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Abstract

VECTOR SPACE OVER A FIELD and its underlying definitions are presented in this white paper (knowledge base).

keywords: vector space, field, abstract algebra, knowledge base

The most updated version of this white paper is available at
https://osf.io/g6ux5/download

Open Mathematics Knowledge Base
http://omkb.org

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Definition

1. **Vector Space over a Field (F-vector space)**

$$(V, F, +, \cdot)$$

   (a) $(V, +) :=$ commutative group

   (b) Closure under (left) scalar multiplication:
   $\forall k \in F,\ \ \forall v \in V,\ \ kv \in V$

   (c) Scalar (left) multiplication identity: $(1 \in F,\ v \in V) \to (1v = v)$

   (d) Associativity of (left) scalar multiplication:
   $\forall j, k \in F,\ \ v \in V,\ \ (jk)v = j(kv)$

   (e) (left) Distributivity of 1 scalar over 2 vectors:
   $\forall k \in F,\ \ v, w \in V,\ \ k(v + w) = kv + kw$

   (f) (left) Distributivity of 2 scalars over 1 vector:
   $\forall j, k \in F,\ \ v \in V,\ \ (j + k)v = jv + kv$

$V, F :=$ sets
$(F, +, \cdot) :=$ field
$+, \cdot :=$ binary operations on V, F
$\cdot : F \times V \to V$
(scalar multiplication between elements of F and V)

[1, 2]

Prerequisites

2. **Field**

$$(F, +, \cdot)$$

   (a) $(F, +) :=$ commutative group
   (b) $(F^*, \cdot) :=$ commutative group
   (c) Multiplication is distributive over addition in F
   (d) $0 \neq 1$

   $F :=$ set, $\quad F^* = F \backslash \{0\}$
   $+, \cdot :=$ binary operations on F (addition and multiplication)
   $0 :=$ additive identity, $\quad 1 :=$ multiplicative identity

   [1, 2]

3. **Group**

$$(G, \star)$$

   (a) Associativity: $\forall x, y, z \in G,\ (x \star y) \star z = x \star (y \star z)$
   (b) Identity: $\exists e \in G : \forall x \in G,\ e \star x = x \star e = x$
   (c) Inverse: $\forall x \in G\ \exists y \in G :\ x \star y = y \star x = e$

   $G :=$ set
   $\star :=$ binary operation

   [1]

4. **Commutative group (Abelian)**

$$G_b$$

$$\forall g_1, g_2 \in G_b,\ g_1 g_2 = g_2 g_1$$

   $G_b :=$ group

[2]

5. **Distributive**

$$\forall x, y, z \in R : x \cdot (y + z) = x \cdot y + x \cdot z \qquad \text{left distributive}$$

$$\forall x, y, z \in R : (y + z) \cdot x = y \cdot x + z \cdot x \qquad \text{right distributive}$$

[1,2,4]

6. **Binary operation**

$$\star : S \times S \to S$$

$S :=$ set
$S \times S :=$ Cartesian product

[1]

7. **Cartesian product**

$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

$A, B :=$ sets
$A \times B :=$ Cartesian product
$(a, b) :=$ ordered pair

[3]

8. **Ordered pair**

$$(a, b) = \{\{a\}, \{a, b\}\}$$

$a :=$ first coordinate
$b :=$ second coordinate

[1,3]

Open Invitation

Review, *add* content, and **co-author** this white *white paper* [5, 6].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [7].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [8], please cite it accordingly [9]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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+ **Center for Open Science**
  `https://cos.io`

+ **Open Science Framework**
  `https://osf.io`

References

[1] Warner, Steve. *Abstract Algebra for Beginners*. GET 800, 2018.
`https://books.google.com/books?id=UFleyAEACAAJ`

[2] Dummit, David Steven, and Richard M. Foote. *Abstract Algebra*. Vol. 3. Hoboken: Wiley, 2004.
https://books.google.com/books?id=znzJygAACAAJ

[3] Velleman, Daniel J. *How to prove it: A structured approach*. Cambridge University Press, 2019.
https://books.google.com/books?vid=ISBN0521861241

[4] Cain, Alan J. *Nine Chapters on the Semigroup Art*. AJC Porto & Lisbon, 2020. http://www-groups.mcs.st-and.ac.uk/alanc

[5] Lobo, Matheus P. "Microarticles." *OSF Preprints*, 28 Oct. 2019.
https://doi.org/10.31219/osf.io/ejrct

[6] Lobo, Matheus P. "Simple Guidelines for Authors: Open Journal of Mathematics and Physics." *OSF Preprints*, 15 Nov. 2019.
https://doi.org/10.31219/osf.io/fk836

[7] Lobo, Matheus P. "Open Journal of Mathematics and Physics (OJMP)." *OSF*, 21 Apr. 2020.
https://doi.org/10.17605/osf.io/6hzyp
https://osf.io/6hzyp/files

[8] COS. *Open Science Framework*. https://osf.io

[9] Lobo, Matheus P. "Vector Space over a Field." *OSF Preprints*, 14 Mar. 2021. https://doi.org/10.31219/osf.io/g6ux5

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