[microreview]

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A right zero semigroup is left-cancellative

Open Mathematics Collaboration^{*†}

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Abstract

We prove the proposition addressed in the title of this paper.

keywords: semigroup, right zero, left-cancellative, abstract algebra

The most updated version of this paper is available at

<https://osf.io/g6snd/download>

Notation & Definition

1. $[1, 2]$
2. $\mathcal{S}$ = semigroup
3. $\forall x, y, z \in \mathcal{S} : (zx = zy) \rightarrow (x = y)$ left-cancellative
4. $\forall x, y \in \mathcal{S} : xy = y$ right zero semigroup

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Proposition

5. *A right zero semigroup is left-cancellative.* [1]

Proof

6. Suppose $zx = zy$ for an arbitrary z .

7. From (4) and (6), $zx = x$ and $zy = y$.

8. From (6) and (7), $x = y$.

9. From (6) and (8), since z is arbitrary,

$$\forall x, y, z \in S : (zx = zy) \rightarrow (x = y).$$

10. Therefore, from (3) and (9), $\mathcal{S}$ is left-cancellative. □

Final Remarks

11. $(\forall x, y \in \mathcal{S} : xy = y) \rightarrow (\forall x, y, z \in \mathcal{S} : (zx = zy) \rightarrow (x = y))$

Open Invitation

*Review, add content, and **co-author** this paper [3, 4]. Join the **Open Mathematics Collaboration** (<https://bit.ly/ojmp-slack>). Send your contribution to mplobo@uft.edu.br.*

Open Science

The **latex file** for this paper together with other *supplementary files* are available [5].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

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