MAXIMUM DISJOINT SET

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1. DEFINITION AND CLAIM

Definition 1. (MAXIMUM-DISJOINT-SET (MDS) problem): We are given a collection S of sets. We would like to find the maximum number of disjoint sets in S . The decision problem asks for the existence of K sets that are disjoint from each other.

Claim 2. *We have that $\text{MIS} \leq_p \text{MDS}$*

In this claim, MIS denotes the well-known Maximum Independent Set problem. Garey and Johnson [1] prove its **NP**-hardness. Now, we prove our claim.

Proof. Given MIS is NP-complete, we can reduce MIS to MDS. Let $G = (V, E)$ the graph we have for MIS problem. For every vertex $v_i \in V$, create a set S_i with edges of v_i , however, be careful that write each edge element as $v_i v_j$ even if it is $v_j v_i$. Reduction is completed after creating the sets S_i . There is an independent set in G of size K if, and only if, there are K disjoint sets where sets are taken from independent set's vertices.

This reduction can be performed in polynomial time since we traverse each vertex v and its edges. It can be performed in $O(V + E)$. Additionally, given a collection of sets, we can verify if they are disjoint and there are at least K of them in polynomial time.

Consequently, MDS is NP-complete. □

2. CONCLUSION

As long as we study a mathematical conjecture, we should encourage ourselves of having enough labouring hours on Prasolov and Sharygin maths books. Then, reading some articles on theory of computing like [2] is a good practice. Only after that, could we think of the ultimate final for all mathematics sciences.

REFERENCES

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Key words and phrases. edge, graph, independent, maximum, set, vertex.
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