[microreview]

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A simple group of order 60 is isomorphic to a subgroup of S_6

Open Mathematics Collaboration^{*†}

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Abstract

We show that the permutation of six Sylow 5-subgroups by conjugation is a faithful action, so that G is isomorphic to a subgroup of S_6 .

keywords: permutation group, simple group, Sylow subgroup, symmetric group

The most updated version of this paper is available at

<https://osf.io/g43yj/download>

Introduction

1. This is an open science article that aims the reproducibility of a specific result in abstract algebra in order to foster and strengthen the research in pure mathematics.
2. We present **theorem** (3) and prove *lemma* (4).

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Theorem

3. *Any simple group G of order 60 is isomorphic to A_5 . [1]*

Lemma

4. G permutes the set Ω of six Sylow 5-subgroups by conjugation. This action is faithful, so that G is isomorphic to a subgroup of S_6 .

Notation & Definition

5. $\exists!_2 \equiv$ exists exactly two

6. $(H \trianglelefteq G) \equiv (H \text{ is a normal subgroup of } G)$

7. $(H \triangleleft G) \equiv (H \text{ is a proper normal subgroup of } G)$

8. $\rightsquigarrow$ bijection

9. $\cong$ isomorphism

10. $G \curvearrowright A$: G acts on A

11. $g \in G, \quad h \in H$

Proof of Lemma (4)

12. $G = \text{simple}$ if

(a) $G \neq \{1\}$

(b) $\exists!_2 S \trianglelefteq G : (S = \{1\}) \vee (S = G)$

13. G is simple.

14. $|G| = 60$

15. $(H \trianglelefteq G) \equiv (ghg^{-1} \in H)$
16. $G = (G, \circ) = \text{group}, \quad \Omega \neq \emptyset, \quad \text{points} = \text{elements of } \Omega$
 $\alpha \cdot g = \text{rule that determines a new element of } \Omega$
 $\alpha \cdot g = \text{defines an action of } G \text{ on } \Omega \text{ (} G \text{ acts on } \Omega \text{) if}$
 - (i) $\alpha \cdot 1 = \alpha, \forall \alpha \in \Omega;$
 - (ii) $(\alpha \cdot g) \cdot h = \alpha \cdot (gh), \forall \alpha \in \Omega, \forall g, h \in G.$
17. $\forall G, A \neq \emptyset \exists \alpha : (\text{actions } G \curvearrowright A) \twoheadrightarrow (\text{homomorphisms } f^h : G \rightarrow S_A)$
18. G permutes the set Ω of six Sylow 5-subgroups by conjugation;
 $\forall H \in \Omega \forall g \in G : \alpha_g(H) = gHg^{-1}.$
19. Let $N = N_G(P)$, then $n_p = |G : N|.$
20. Since $n_5 = 6$, then N has index 6 in G .
21. So the action α in (18) gives a permutation representation of G into S_6 , i.e., α is the homomorphism $g \mapsto \alpha_g$.
22. $\ker \alpha = \{g \in G : \alpha \cdot g = \alpha, \forall \alpha \in \Omega\} \triangleleft G$
23. From (12), (13) and (22), $\ker \alpha = \{1\}.$
24. So the action is faithful.
25. Since $\ker \alpha = \{1\}$, the map $G \rightarrow S_6$ is injective.
26. Therefore $G \cong \text{Im } \alpha \leq S_6.$ □

Open Invitation

*Review, add content, and **co-author** this paper [2, 3]. Join the **Open Mathematics Collaboration** (<https://bit.ly/ojmp-slack>). Send your contribution to mplobo@uft.edu.br.*

Open Science

The **latex file** for this paper together with other *supplementary files* are available [4].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [5], please cite it accordingly [6]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

Acknowledgement

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