Credit Valuation Adjustment Study

Tim Xiao

ABSTRACT

This paper presents a model for calculating credit value adjustment (CVA) by taking wrong way risk into account. Many people believe that the cash flows of a defaultable financial contract can be priced independently and then be summed up to give the final risky price of the contract. We emphasize here that this conclusion is only true of the financial contracts whose payoffs are always positive. In the cases where the promised payoffs could be positive or negative, the valuation requires not only a backward recursive induction procedure, but also a strategic selection of different discount factors according to the market value in time. This coupled valuation process allows us to capture correlation between counterparties and market factors.

Key Words: credit value adjustment (CVA), wrong way risk, credit risk modeling, collateralization, margin and netting.

For years, a widespread practice in the industry has been to mark derivative portfolios to market without taking counterparty risk into account. All cash flows are discounted using the LIBOR curve. But the real parties, in many cases, happen to be of lower credit quality than the hypothetical LIBOR party and have a chance of default.

CVA not only allows institutions to move beyond the traditional control mindset of credit risk limits and to quantify counterparty risk as a single measurable P&L number, but also offers an opportunity for banks to dynamically manage, price and hedge counterparty risk. The benefits of CVA are widely acknowledged. Many banks have set up internal credit risk trading desks to manage counterparty risk on derivatives.

CVA, by definition, is the difference between the risk-free portfolio value and the true (or risky or defaultable) portfolio value that takes into account the possibility of a counterparty's default. The risk-free portfolio value is what brokers quote or what trading systems or models normally report. The risky portfolio value, however, is a relatively less explored and less transparent area, which is the main challenge and core theme for CVA. In other words, central to CVA is risky valuation.

Although the DTA is very intuitive, it has the disadvantage that it explicitly involves the default time. We are very unlikely to have complete information about a firm's default point, which is often inaccessible (see Duffie and Huang (1996), Jarrow and Protter (2004), etc.). Usually, valuation under the DTA is performed via Monte Carlo simulation. On the other hand, however, the DPA relies on the probability distribution of the default time rather than the default time itself. Sometimes the DPA yields simple closed form solutions.

In this paper, we present a framework for risky valuation and CVA. In contrast to previous studies, the model relies on the DPA rather than the DTA. Our study shows that the pricing process of a defaultable contract normally has a backward recursive nature if its payoff could be positive or negative.

An intuitive way of understanding these backward recursive behaviours is that we can think of that any contingent claim embeds two default options. In other words, when entering an OTC derivatives transaction, one party grants the other party an option to default and, at the same time, also receives an option to default itself. In theory, default may occur at any time. Therefore, the default options are American style options that normally require a backward induction valuation.

The rest of this paper is organized as follows: Section 2 discusses unilateral risky valuation and unilateral CVA. Section 2 elaborates bilateral risky valuation and bilateral CVA. Section 3 presents numerical results. The conclusions are given in Section 4. . All proofs and a practical framework that embraces netting agreements, margining agreements and wrong/right way risk are contained in the appendices.

1. Unilateral Risky Valuation and Unilateral CVA

We consider a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ satisfying the usual conditions, where Ω denotes a sample space; $\mathcal{F}$ denotes a σ -algebra; $\mathcal{P}$ denotes a probability measure; $\{\mathcal{F}_t\}_{t \geq 0}$ denotes a filtration.

It is well-known that the survival probability from time t to s in this framework is defined by

$$p(t, s) := P(\tau > s \mid \tau > t, Z) = \exp\left(-\int_t^s h(u) du\right) \quad (2a)$$

The default probability for the period (t, s) in this framework is defined by

$$q(t, s) := P(\tau \leq s \mid \tau > t, Z) = 1 - p(t, s) = 1 - \exp\left(-\int_t^s h(u) du\right) \quad (2b)$$

Two counterparties are denoted as A and B . Let valuation date be t . Consider a financial contract that promises to pay a $X_T > 0$ from party B to party A at maturity date T , and nothing

before date T . All calculations in the paper are from the perspective of party A. The risk free value of the financial contract is given by

$$V^F(t) = E[D(t, T)X_T | \mathcal{F}_t] \quad (3a)$$

where

$$D(t, T) = \exp\left[-\int_t^T r(u)du\right] \quad (3b)$$

where $E\{\bullet | \mathcal{F}_t\}$ denotes the expectation conditional on the $\mathcal{F}_t$, $D(t, T)$ denotes the risk-free discount factor at time t for the maturity T and $r(u)$ denotes the risk-free short rate at time u ($t \leq u \leq T$).

Next, we turn to risky valuation. In a unilateral credit risk case, we assume that party A is default-free and party B is defaultable. Risky valuation can be generally classified into two categories: the *default time approach* (DTA) and the *default probability (intensity) approach* (DPA).

Under a risk-neutral measure, the value of this defaultable contract is the discounted expectation of all the payoffs and is given by

$$V(t) = E\left[\left(D(t, T)X_T 1_{\tau > T} + D(t, \tau)\phi V(\tau) 1_{\tau \leq T}\right) | \mathcal{F}_t\right] \quad (4)$$

where 1_Y is an indicator function that is equal to one if Y is true and zero otherwise.

Although the DTA is very intuitive, it has the disadvantage that it explicitly involves the default time/jump. We are very unlikely to have complete information about a firm's default point, which is often inaccessible. Usually, valuation under the DTA is performed via Monte Carlo simulation.

The survival and the default probabilities for the period $(t, t + \Delta t)$ are given by

$$\hat{p}(t) := p(t, t + \Delta t) = \exp(-h(t)\Delta t) \approx 1 - h(t)\Delta t \quad (5a)$$

$$\hat{q}(t) := q(t, t + \Delta t) = 1 - \exp(-h(t)\Delta t) \approx h(t)\Delta t \quad (5b)$$

The binomial default rule considers only two possible states: default or survival. For the one-period $(t, t + \Delta t)$ economy, at time $t + \Delta t$ the asset either defaults with the default probability $q(t, t + \Delta t)$ or survives with the survival probability $p(t, t + \Delta t)$. The survival payoff is equal to the market value $V(t + \Delta t)$ and the default payoff is a fraction of the market value: $\varphi(t + \Delta t)V(t + \Delta t)$. Under a risk-neutral measure, the value of the asset at t is the expectation of all the payoffs discounted at the risk-free rate and is given by

$$V(t) = E\left\{\exp(-r(t)\Delta t)[\hat{p}(t) + \varphi(t)\hat{q}(t)]V(t + \Delta t) \middle| \mathcal{F}_t\right\} \approx E\left\{\exp(-y(t)\Delta t)V(t + \Delta t) \middle| \mathcal{F}_t\right\} \quad (6)$$

where $y(t) = r(t) + h(t)(1 - \varphi(t)) = r(t) + c(t)$ denotes the risky rate and $c(t) = h(t)(1 - \varphi(t))$ is called the (short) credit spread.

Similarly, we have

$$V(t + \Delta t) = E\left\{\exp(-y(t + \Delta t)\Delta t)V(t + 2\Delta t) \middle| \mathcal{F}_{t+\Delta t}\right\} \quad (7)$$

Note that $\exp(-y(t)\Delta t)$ is $\mathcal{F}_{t+\Delta t}$ -measurable. By definition, an $\mathcal{F}_{t+\Delta t}$ -measurable random variable is a random variable whose value is known at time $t + \Delta t$. Based on the *taking out what is known* and *tower* properties of conditional expectation, we have

$$\begin{aligned} V(t) &= E\left\{\exp(-y(t)\Delta t)V(t + \Delta t) \middle| \mathcal{F}_t\right\} \\ &= E\left\{\exp(-y(t)\Delta t)E\left[\exp(-y(t + \Delta t)\Delta t)V(t + 2\Delta t) \middle| \mathcal{F}_{t+\Delta t}\right] \middle| \mathcal{F}_t\right\} \\ &= E\left\{\exp\left(-\sum_{i=0}^1 y(t + i\Delta t)\Delta t\right)V(t + 2\Delta t) \middle| \mathcal{F}_t\right\} \end{aligned} \quad (8)$$

By recursively deriving from t forward over T and taking the limit as Δt approaches zero, the risky value of the asset can be expressed as

$$V(t) = E\left\{\exp\left[-\int_t^T y(u)du\right]V(T) \middle| \mathcal{F}_t\right\} \quad (9)$$

We may think of $y(u)$ as the risk-adjusted short rate. Equation (9) is the same as Equation (10) in Duffie and Singleton [1999], which is the market model for pricing risky bonds. Using the DPA, we obtain a closed-form solution for pricing an asset subject to credit risk. Other good

examples of the DPA are the CDS model proposed by J.P. Morgan (1999) and a more generic risky model presented by Xiao (2013a).

For a derivative contract, usually its payoff may be either an asset or a liability to each party. Thus, we further relax the assumption and suppose that X_T may be positive or negative.

Proposition 1: *The unilateral risky value of the single-payment contract in a discrete-time setting is given by*

$$V(t) = E[F(t, T)X_T | \mathcal{F}_t] \quad (10a)$$

where

$$F(t, T) = D(t, T)[1 - 1_{X_T \geq 0} q(t, T)(1 - \varphi(T))] \quad (10b)$$

Proof: See the appendix.

Here $F(t, T)$ can be regarded as a risk-adjusted discount factor. Proposition 1 says that the unilateral risky valuation of the single payoff contract has a dependence on the sign of the payoff. If the payoff is positive, the risky value is equal to the risk-free value minus the discounted potential loss. Otherwise, the risky value is equal to the risk-free value.

Proposition 2: *The unilateral risky value of the multiple-payment contract is given by*

$$V(t) = \sum_{i=1}^m E\left[\left(\prod_{j=0}^{i-1} F(T_j, T_{j+1})\right)X_i | \mathcal{F}_t\right] \quad (11a)$$

where $t = T_0$ and

$$F(T_j, T_{j+1}) = D(T_j, T_{j+1})[1 - 1_{(X_{j+1} + V(T_{j+1})) \geq 0} q(T_j, T_{j+1})(1 - \varphi(T_{j+1}))] \quad (11b)$$

Proof: See the appendix.

The risky valuation in Proposition 2 has a backward nature. The intermediate values are vital to determine the final price. For a discrete time interval, the current risky value has a dependence on the future risky value. Only on the final payment date T_m , the value of the contract and the maximum amount of information needed to determine the risk-adjusted discount factor are revealed. The coupled valuation behavior allows us to capture wrong/right way risk properly where

counterparty credit quality and market prices may be correlated. This type of problem can be best solved by working backwards in time, with the later risky value feeding into the earlier ones, so that the process builds on itself in a recursive fashion, which is referred to as *backward induction*. The most popular backward induction valuation algorithms are lattice/tree and least square Monte Carlo.

The similarity between American style financial options and American style default options is that both require a backward recursive valuation procedure. The difference between them is in the optimal strategy. The American financial option seeks an optimal value by comparing the exercise value with the continuation value, whereas the American default option seeks an optimal discount factor based on the option value in time.

The unilateral CVA, by definition, can be expressed as

$$CVA(t) = V^F(t) - V(t) = \sum_{i=1}^m E \left[\left(D(t, T_i) - \prod_{j=0}^{i-1} F(T_j, T_{j+1}) \right) X_i \middle| \mathcal{F}_t \right] \quad (12)$$

Proposition 2 provides a general form for pricing a unilateral defaultable contract. Applying it to a particular situation in which we assume that all the payoffs are nonnegative, we derive the following corollary:

Corollary 1: *If all the payoffs are nonnegative, the risky value of the multiple-payments contract is given by*

$$V(t) = \sum_{i=1}^m E \left[\left(\prod_{j=0}^{i-1} \bar{F}(T_j, T_{j+1}) \right) X_i \middle| \mathcal{F}_t \right] \quad (13a)$$

where $t = T_0$ and

$$\bar{F}(T_j, T_{j+1}) = D(T_j, T_{j+1}) \left[1 - q(T_j, T_{j+1}) (1 - \varphi(T_{j+1})) \right] \quad (13b)$$

The proof of this corollary is easily obtained according to Proposition 2 by setting $(X_{j+1} + V(T_{j+1})) \geq 0$, since the value of the contract at any time is also nonnegative.

The CVA in this case is given by

$$CVA(t) = V^F(t) - V(t) = \sum_{i=1}^m E \left[D(t, T_i) \left(1 - \prod_{j=0}^{i-1} (1 - q(T_j, T_{j+1}) (1 - \varphi(T_{j+1}))) \right) X_i \middle| \mathcal{F}_t \right] \quad (14)$$

The current popular CVA model (e.g., equation (17) in Pykhtin and Zhu (2007) and equation (3) in Gregory (2009)) is quite different from above either equation (12) or equation (14). As a matter of fact, the current CVA model has never been rigorously proved. In order to reflect the economic value of counterparty credit risk, to measure the profit and loss of a bank and to provide proper incentives to traders, a good CVA model must be not only rigorous and accurate but also feasible to implement.

2. Bilateral Risky Valuation and Bilateral CVA

There is ample evidence that corporate defaults are correlated. The default of a firm's counterparty might affect its own default probability. Thus, default correlation and dependence arise due to the counterparty relations. Default correlation can be positive or negative. The effect of positive correlation is usually called contagion, whereas the latter is referred to as competition effect.

Consider a pair of random variables (Y_A, Y_B) that has a bivariate Bernoulli distribution. The joint probability representations are given by

$$p_{00} := P(Y_A = 0, Y_B = 0) = p_A p_B + \sigma_{AB} \quad (15a)$$

$$p_{01} := P(Y_A = 0, Y_B = 1) = p_A q_B - \sigma_{AB} \quad (15b)$$

$$p_{10} := P(Y_A = 1, Y_B = 0) = q_A p_B - \sigma_{AB} \quad (15c)$$

$$p_{11} := P(Y_A = 1, Y_B = 1) = q_A q_B + \sigma_{AB} \quad (15d)$$

where $E(Y_j) = q_j$, $\sigma_j^2 = p_j q_j$, $\sigma_{AB} := E[(Y_A - q_A)(Y_B - q_B)] = \rho_{AB} \sigma_A \sigma_B = \rho_{AB} \sqrt{q_A p_A q_B p_B}$ where ρ_{AB} denotes the default correlation coefficient and σ_{AB} denotes the default covariance.

Table 1. Payoffs of a bilaterally defaultable contract

This table displays all possible payoffs at time T . In the case of $X_T > 0$, there are a total of four possible states at time T : i) Both A and B survive with probability p_{00} . The contract value is equal to the payoff X_T . ii) A defaults but B survives with probability p_{10} . The contract value is $\bar{\varphi}_B X_T$, where $\bar{\varphi}_B$ represents the non-default recovery rate¹. $\bar{\varphi}_B = 0$ represents the one-way settlement rule, while $\bar{\varphi}_B = 1$ represents the two-way settlement rule. iii) A survives but B defaults with probability p_{01} . The contract value is $\varphi_B X_T$, where φ_B represents the default recovery rate. iv) Both A and B default with probability p_{11} . The contract value is $\varphi_{AB} X_T$, where φ_{AB} denotes the joint recovery rate when both parties A and B default simultaneously. A similar logic applies to the case of $X_T < 0$.

State		$Y_A = 0, Y_B = 0$	$Y_A = 1, Y_B = 0$	$Y_A = 0, Y_B = 1$	$Y_A = 1, Y_B = 1$
Comments		$A \ \& \ B$ survive	A defaults, B survives	A survives, B defaults	$A \ \& \ B$ default
Probability		p_{00}	p_{10}	p_{01}	p_{11}
Payoff	$X_T > 0$	X_T	$\bar{\varphi}_B X_T$	$\varphi_B X_T$	$\varphi_{AB} X_T$
	$X_T < 0$	X_T	$\varphi_A X_T$	$\bar{\varphi}_A X_T$	$\varphi_{AB} X_T$

Suppose that a financial contract that promises to pay a X_T from party B to party A at maturity date T , and nothing before date T where $T > t$. The payoff X_T may be positive or

¹ There are two default settlement rules in the market. The *one-way payment rule* was specified by the early ISDA master agreement. The non-defaulting party is not obligated to compensate the defaulting party if the remaining market value of the instrument is positive for the defaulting party. The *two-way payment rule* is based on current ISDA documentation. The non-defaulting party will pay the full market value of the instrument to the defaulting party if the contract has positive value to the defaulting party.

negative, i.e. the contract may be either an asset or a liability to each party. All calculations are from the perspective of party A.

At time T , there are a total of four ($2^2 = 4$) possible states shown in Table 1. The risky value of the contract is the discounted expectation of the payoffs and is given by the following proposition.

Proposition 3: *The bilateral risky value of the single-payment contract is given by*

$$V(t) = E[K(t, T)X_T | \mathcal{F}_t] = E[D(t, T)(1_{X_T \geq 0} k_B(t, T) + 1_{X_T < 0} k_A(t, T))X_T | \mathcal{F}_t] \quad (16a)$$

where

$$k_B(t, T) = p_B(t, T)p_A(t, T) + \varphi_B(T)q_B(t, T)p_A(t, T) + \bar{\varphi}_B(T)p_B(t, T)q_A(t, T) + \varphi_{AB}(T)q_B(t, T)q_A(t, T) + \sigma_{AB}(t, T)(1 - \varphi_B(T) - \bar{\varphi}_B(T) + \varphi_{AB}(T)) \quad (16b)$$

$$k_A(t, T) = p_B(t, T)p_A(t, T) + \varphi_A(T)q_A(t, T)p_B(t, T) + \bar{\varphi}_A(T)p_A(t, T)q_B(t, T) + \varphi_{AB}(T)q_B(t, T)q_A(t, T) + \sigma_{AB}(t, T)(1 - \varphi_A(T) - \bar{\varphi}_A(T) + \varphi_{AB}(T)) \quad (16c)$$

Proof: See the appendix.

We may think of $K(t, T)$ as the risk-adjusted discount factor. Proposition 3 tells us that the bilateral risky price of a single-payment contract can be expressed as the present value of the payoff discounted by a risk-adjusted discount factor that has a switching-type dependence on the sign of the payoff.

Proposition 4: *The bilateral risky value of the multiple-payment contract is given by*

$$V(t) = \sum_{i=1}^m E\left[\left(\prod_{j=0}^{i-1} K(T_j, T_{j+1})\right)X_i | \mathcal{F}_t\right] \quad (17a)$$

where $t = T_0$ and

$$K(T_j, T_{j+1}) = D(T_j, T_{j+1})(1_{(X_{j+1} + V(T_{j+1})) \geq 0} k_B(T_j, T_{j+1}) + 1_{(X_{j+1} + V(T_{j+1})) < 0} k_A(T_j, T_{j+1})) \quad (17b)$$

where $k_A(T_j, T_{j+1})$ and $k_B(T_j, T_{j+1})$ are defined in Proposition 3.

Proof: The proof is similar to Proposition 2 by replacing $F(T_j, T_{j-1})$ with $K(T_j, T_{j-1})$.

Proposition 4 says that the pricing process of a multiple-payment contract has a backward nature since there is no way of knowing which risk-adjusted discounting rate should be used

without knowledge of the future value. Only on the maturity date, the value of the contract and the decision strategy are clear. Therefore, the evaluation must be done in a backward fashion, working from the final payment date towards the present. This type of valuation process is referred to as backward induction.

The bilateral CVA of the multiple-payment contract can be expressed as

$$CVA(t) = V^F(t) - V(t) = \sum_{i=1}^m \left\{ E[D(t, T_i) X_i | \mathcal{F}_t] - E \left[\left(\prod_{j=0}^{i-1} K(T_j, T_{j+1}) \right) X_i | \mathcal{F}_t \right] \right\} \quad (18)$$

3. Implementation

We develop a practical framework for calculating bilateral CVA at counterparty portfolio level based on the theory described above. The framework incorporates netting and margin agreements, and captures right/wrong way risk.

One core element of the trading credit risk modeling is the Monte Carlo scenario generation (market evolution). This must be able to run a large number of scenarios for each risk factor with flexibility over parameterization of processes and treatment of correlation between underlying factors. Credit exposure may be calculated under real probability measure, while CVA or pricing counterparty credit risk should be conducted under risk-neutral probability measure.

For ease of illustration, we choose a vanilla interest rate swap, as interest rate swaps collectively account for around two-thirds of both the notional and market value of all outstanding derivatives (FinPricing (2015))

For swaplet i , there are four important dates: the fixing date $t_{i,f}$, the starting date $t_{i,s}$, the ending date $t_{i,e}$ and the payment date $t_{i,p}$. In general, these dates are not coincidently at the simulation time buckets. The time relationship between swaplet i and the simulation time buckets is illustrated in Figure B1.

Figure B1: An interest rate swaplet

This figure illustrates the time relationship between an interest rate swaplet and the simulation time buckets. The floating leg of the swaplet is reset at the fixing date $t_{i,f}$ with the starting date $t_{i,s}$, the ending date $t_{i,e}$, and the payment date $t_{i,p}$. The simulation time buckets are $T_j, T_{j+1}, \dots, T_{k+1}$. The simulated interest rate curve is starting at $t_{i,f}$. Both fixed rate payments and floating-rate payments occur on the same payment dates.

The cash flow of swaplet i is determined at the fixing date $t_{i,f}$ that is assumed to be between the simulation time buckets T_j and T_{j+1} . First, we need to create an interest rate curve observed at $t_{i,f}$ by interpolating the interest rate curves simulated at T_j and T_{j+1} via either Brownian Bridge or linear interpolation. The linear interpolation is the expectation of the Brownian Bridge. Then we can calculate the payoff of swaplet i at scenario j as

$$\chi_{j,i} = N(F(t_{i,f}; t_{i,s}, t_{i,e}) - R)\delta(t_{i,s}, t_{i,e}) \quad (\text{B1})$$

where N denotes the notional; $F(t_{i,f}; t_{i,s}, t_{i,e})$ denotes the simply compounded forward rate reset at $t_{i,f}$ for the forward period $(t_{i,s}, t_{i,e})$; $\delta(t_{i,s}, t_{i,e})$ denotes the accrual factor or day count fraction for the period $(t_{i,s}, t_{i,e})$ and R denotes the fixed rate.

The cash flow amount calculated by (B1) is paid on the payment date $t_{i,p}$. This value should be allocated into *the nearest previous time bucket* T_k as:

$$\tilde{\chi}_{j,k,i} = \chi_{j,i} D(T_k, t_{i,p}) \quad (\text{B2})$$

where $D(T_k, t_{i,p})$ denotes the risk-free discount factor based on the interest rate curve simulated at T_k .

Cash flow generation for products without early-exercise provision is quite straightforward. For early-exercise products, one can use the approach proposed by Longstaff and Schwartz (2001) to obtain the optimal exercise boundaries and then the payoffs.

For netting, we add all cash flows together at the same scenario and the same time bucket to recognize offsetting. The aggregated cash flow under netting at scenario j and time bucket k is given by

$$\tilde{\chi}_{j,k} = \sum_i \tilde{\chi}_{j,k,i} \quad (\text{B3})$$

For nonnetting, we divided cash flows into positive and negative groups and add them separately. In other words, the offsetting is not recognized. The aggregated cash flows under nonnetting at scenario j and time bucket k are given by

$$\tilde{\chi}_{j,k} = \begin{cases} \sum_l \tilde{\chi}_{j,k,l} & \text{if } \chi_{j,k,l} \geq 0 \\ \sum_m \tilde{\chi}_{j,k,m} & \text{if } \chi_{j,k,m} < 0 \end{cases} \quad (\text{B4})$$

For a more detailed discussion on pricing collateralized contract/portfolio, see Xiao (2013b).

After aggregating all cash flows via netting, one can price a portfolio in the same manner as pricing a single deal. We assume that the reader is familiar with the least square Monte Carlo valuation model proposed by Longstaff and Schwartz (2001) and thus do not repeat some well-known procedures for brevity.

CVA is by definition the difference between the risk-free portfolio value and the true (or risky or defaultable) portfolio value.

4. Conclusion

This article presents a framework for pricing risky contracts and their CVAs. The model relies on the probability distribution of the default jump rather than the default jump itself, because the default jump is normally inaccessible. We find that the valuation of risky assets and their CVAs, in most situations, has a backward recursive nature and requires a backward induction valuation. An intuitive explanation is that two counterparties implicitly sell each other an option to default when entering into an OTC derivative transaction. If we assume that a default may occur at any time, the default options are American style options. If we assume that a default may only happen on the payment dates, the default options are Bermudan style options. Both Bermudan and American options require backward induction valuations.

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